# Commodity Prices and Production Networks in Small Open Economies*

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**Abstract**

We study the role of domestic production networks in the transmission of commodity price fluctuations in small open economies. First, we present a tractable model of a small open economy's production network to explain sectoral propagation patterns. We demonstrate that the domestic production network is crucial in shaping the propagation of commodity prices. Using a panel of 31 sectors across 9 small open economies, we empirically confirm the model's predictions. Next, we construct a dynamic model of a small open economy featuring a production network to study the macroeconomic importance of the network structure in shaping both aggregate and sectoral responses to commodity price shocks. We show that: (i) the network-adjusted labor share of the commodity sector, rather than the sector's size, is key to understanding the real wage's response to commodity price fluctuations; and (ii) non-unitary elasticities of substitution in production are crucial for understanding the cross-sectional implications of these fluctuations.

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# 1 Introduction

This paper analyzes the propagation of commodity price changes through domestic production networks in small open economies in terms of both quantities and prices. We take advantage of a key stylized fact. Commodity sectors (mining, agriculture, and food sectors) are central sectors in small open economies, both as suppliers and buyers of intermediate inputs, which gives them a potential role as a source of supply and demand shock propagation. Moreover, as commodity prices are exogenous to non-commodity sectors in the economies analyzed, we have an ideal scenario to study the propagation of sectoral commodity price changes along the production chain.

We guide our empirical analysis with a tractable small open economy model featuring production networks (domestic intermediates), inelastic labor, and imported intermediates. Labor is fully mobile across sectors of the economy. Both domestic factors and goods markets are competitive, and representative firms in each sector display constant returns to scale in production. Importantly, the commodity sector supplies goods to domestic firms and consumers at home and abroad and makes production decisions; that is, it uses labor, imported intermediates, and domestic intermediate inputs in production. The commodity price is exogenously determined in international markets.

Our model highlights five mechanisms by which commodity price changes propagate to non-commodity sectors via the domestic production network. There is one supply-side channel and four demand-side channels. We label the supply side component *cost push channel* because costs for non-commodity producers using the commodity as input increase following an increase in the commodity price. Consequently, the prices and the real wage of these downstream sectors increase, generating further downstream propagation of the initial shock to other producers. The second mechanism is the *input-output substitution channel*, where all sectors reallocate their demand towards (away from) other sectors in response to changes in good and factor prices. The latter channel crucially depends on the elasticities of substitution at the consumer and producer levels and the production network structure. The third is the foreign substitution channel. An increase in the commodity price induces foreign consumers to reduce their expenditure on commodity output.

The fourth channel is the *domestic demand channel*: changes in commodity prices affect the total available expenditure for domestic consumers, affecting demand for all non-commodity sectors, akin to the well-known wealth effect of commodity prices. The extent to which each non-commodity sector is affected by this channel depends on its exposure to domestic consumer expenditure changes, which considers both direct and indirect linkages through production networks. The fifth channel is the *foreign demand channel*, where higher commodity exports, induced by an increase in foreign demand for the commodity sector, increase the commodity sector's demand for factors and intermediate inputs in production, pushing up production in non-commodity sectors. The key to this channel is the commodity sector's role as a buyer of non-commodity sectors, both directly and indirectly, via domestic production networks.

We test the model's implications in a panel of nine small open economies with 31 non-commodity sectors from 1995-2009. We provide empirical evidence of a strong upstream propagation of quantities—to sectors providing intermediate inputs to commodity sectors—of commodity price changes. We also show that while commodity price increases increase the price of downstream sectors—that is, those sectors buying from the commodity sector to produce their output—they have no real effect on the output of downstream sectors. Thus, commodity price fluctuations appear to propagate mainly as a demand-side shock in small open economies.

We then develop a dynamic small open economy model featuring a domestic production network to quantify commodity price shocks' macroeconomic and sectoral implications. The household in the economy borrows or lends from international markets at the world interest rate. Foreign assets are denominated in units of the commodity good and are subject to adjustment costs. Sectoral production functions display non-unitary elasticity of substitution between inputs. To highlight the different margins of substitutability, we assume there are three different elasticities of substitution. One elasticity of substitution between value-added inputs (e.g., labor) and the bundle of intermediates. An elasticity of substitution between domestic and imported intermediates and an elasticity of substitution among domestic intermediate inputs. To close the model, we assume that commodity prices follow an exogenous autoregressive process and are subject to supply and foreign demand shocks. Foreign demand also follows an exogenous autoregressive process.

The quantitative model results illustrate the importance of the domestic production network in shaping the response of wages in units of importable goods to a 1% commodity price shock. We show that a sufficient statistic for the response of wages is the inverse of the commodity sector's network-adjusted labor share, which considers all indirect linkages.

Sectors indirectly buy labor from other sectors via intermediate inputs. The network-adjusted labor share takes this notion into account. When this share is high, wages in units of importable goods respond less. This effect is quantitatively important. For example, facing the same 1% mining price shock, wages in Australia would increase by 1.11%, while wages in Bulgaria would increase by 1.35%. The reason for this is that the mining sector in Australia has a larger network-adjusted labor share (0.89) than the mining sector in Bulgaria (0.74).

The mechanisms that operate at the cross-sectional level in our tractable framework are also present in the dynamic model.

Using Australia as a laboratory, we shock the mining sector price and study its cross-sectional implications. We show that the cross-sectional dispersion of quantity responses across non-commodity sectors heavily depends on the value of the elasticities of substitution. Therefore, we estimate the value of the production elasticities using *indirect inference*. In particular, we simulate series from our model, for different combinations of elasticities, and minimize the distance between the model-implied coefficients of an auxiliary regression and the empirical estimates of the same auxiliary regression. The estimated elasticity of substitution between intermediates and value-added inputs is significantly larger than one, consistent with the results in Miranda-Pinto (2021) and Huneeus (2020). The elasticity between domestic intermediates and imported intermediates and the elasticity among domestic intermediates are both below one, consistent with the results in Boehm et al. (2019) and Atalay (2017). These estimated elasticities are key for the model to be closer to our estimated results on quantities where upstream propagation is strong but downstream propagation is muted.

Lastly, we conduct exercises where we replace the mining sector as a commodity sector with either the wood and cork sector or base metals" sector. The appeal of these sectors is that they coincide with the mining sector in terms of size (Basic metals) or network-adjusted labor share (Wood and Cork). The results underscore the role of the network-adjusted labor share of the commodity sector, as opposed to the Domar weight of the commodity sector, in amplifying or mitigating the aggregate effects of commodity price shocks.

**Related literature and contribution.** This paper contributes to two strands of literature. We relate to the now extensive literature on the propagation and macroeconomic effects of commodity price fluctuations (e.g. Corden and Neary, 1982; Mendoza, 1995; Kose, 2002; Drechsel and Tenreyro, 2018; Benguria et al., 2023; Cao and Dong, 2020; Allcott and Keniston, 2018; Kohn et al., 2021; Romero, 2022; González, 2022; Di Pace et al., Forthcoming).

We contribute to this literature by providing empirical evidence on the role of domestic production networks in propagating commodity price changes to other sectors of the economy. On the quantitative side, our paper introduced a domestic production network allowing for 34 sectors into an otherwise standard dynamic small open economy model, extending earlier contributions, such as Cao and Dong (2020), that considered at most 8 sectors.

The closest papers to ours are Allcott and Keniston (2018) and Benguria et al. (2023), which study the effects of commodity booms on manufacturing industries located upstream and downstream of commodities. We contribute to these papers on the following fronts. First, while these papers define manufacturing industries upstream or downstream to commodities using indicator variables, we consider all direct and indirect production linkages across all producers, including service sectors. Second, we consider a broader set of commodities and countries. Third, we study the effects of short-run fluctuations of commodity prices on non-commodity sectors' prices and production, while their focus is on low-frequency fluctuations in commodity prices (Benguria et al., 2023) and commodity endowments (Allcott and Keniston, 2018). In looking separately at prices and quantities, we can better dissect the transmission mechanisms of commodity prices. Fourth, unlike previous papers that only control for production linkages outside the model, our theoretical model directly speaks to and interprets the implications of our empirical results. An important implication of our model is that standard production network centrality measures used in the literature—namely the Leontief inverse—are not sufficient statistics to understand commodity price fluctuations. Opening the economy brings about imported intermediate inputs as a second factor, shaping the relevant Leontief inverse one should care about.

The tractable version of our model precisely highlights the mechanisms by which commodity price fluctuations can affect the cross-sectional distribution of gross output and prices with an arbitrary production network structure. In particular, our model sheds light on the role of non-unitary production elasticities in amplifying the upstream propagation and dampening the downstream propagation of commodity price changes on sectoral output. To rationalize the data, our model suggests that elasticities of substitution among inputs ought to be larger than one. At the annual frequency, which is the frequency we consider, several studies suggest that elasticities are larger than one (e.g. Carvalho et al., 2021; Miranda-Pinto, 2021; Huneeus, 2020; Nakano and Nishimura, 2023).[1] Although recent estimates of elasticities

[1] Atalay (2017) shows that the elasticity of substitution between labor and intermediates is close to

are below one (e,g., Boehm et al., 2019; Barrot and Sauvagnat, 2016), these elasticities are usually short-run estimates at the monthly or quarterly frequency.[2] Further, we show that the well-known wealth effect of commodity price fluctuations has an important (upstream) network propagation component.

We also contribute to the literature on production networks and business cycle fluctuations (e.g., Horvath, 1998; Foerster et al., 2011; Acemoglu et al., 2012; Atalay, 2017; Baqaee and Farhi, 2019, 2024; Miranda-Pinto, 2021; vom Lehn and Winberry, 2020; Carvalho et al., 2021), which has mainly focused on closed-economy models. We highlight that commodity price changes in a small open economy can have important effects on prices and output quantities and are largely propagated through input-output linkages. As in Carvalho et al. (2021) and Luo (2020) that emphasize the upstream and downstream propagation of productivity shocks and financial shocks, respectively, we show that commodity prices can strongly propagate to upstream and downstream sectors.

Finally, our paper emphasizes that the cross-sectional distribution of output can respond to commodity price changes while real gross domestic product — measured at constant prices — can stay constant. Thus, from a macroeconomic perspective, our paper exploits the result that commodity price fluctuations generate movements along the production possibility frontier but do not shift it. The particular point in the production possibility frontier the economy ends up after the commodity price shock depends on elasticities of substitution and network exposures and is what this paper is concerned about. In addition, our paper provides a laboratory to explore the role of elasticities of substitution among inputs in matching salient facts of the transmission of shocks via domestic production networks in line with recent literature and our discussion above (e.g., Boehm et al., 2019; Miranda-Pinto, 2021; Carvalho et al., 2021; Miranda-Pinto and Young, 2022).

one, while the elasticity among intermediate inputs is closer to zero. Miranda-Pinto (2021) uses the same data in Atalay (2017) and finds that production elasticities are highly heterogeneous among sectors. While manufacturing industries have elasticities similar to those estimated by Atalay (2017), service sectors present elasticities that exceed one.

[2] Boehm et al. (2023) and Peter and Ruane (2023) highlights the importance of the horizon in estimating trade elasticities.

## 2 Empirical Analysis

In this section, we present a stylized fact regarding commodity sectors: they are central buyers and suppliers in the domestic production network of small open economies. To that end, we combine commodity goods' exports data from Fernández et al. (2018) and input-output data from the World Input-Output Database (WIOD). We use the WIOD data as, unlike the OECD input-output data, it contains sectoral information on production and prices separately. Please refer to our Appendix A for more details on data sources and definitions. We match each commodity good to one of the 34 industries in the WIOD. Table B6 in our Appendix B provides a detailed mapping between goods and sectors in the WIOD data.[3] The three commodity sectors in the WIOD are Agriculture, Forestry, and Fishing; Mining and Quarrying; and Food Products, Beverages, and Tobacco.

***Commodity sectors are central sectors in the production network.*** We now provide empirical evidence that commodity sectors are central suppliers and buyers in the production network. We compute commodity sectors' centrality measures using a notion of upstream and downstream propagation as in Acemoglu et al. (2016). These refer to how shocks propagate through the network structure and not by the sectors' position.[4]

Figure 1 visually represents these notions. There are two sectors $k$ and $i$. Sector $k$ supplies to sector $i$. Shocks to sector $k$ (the supplier) propagate *downstream*, from seller to buyer. Shocks to sector $i$ (the buyer) propagate *upstream*, from buyer to seller.

We measure the supplier or *downstream* centrality of a given sector $i$ as

$$Supplier_i = \sum_{j=1}^{N} \Psi_{ji}, \tag{1}$$

[3] Appendix A also provides information on the sample of countries we use from the WIOD and the definition of the variables.

[4] These definitions are slightly different from the notions of downstreamness and upstreamness highlighted in the global value chains literature (see Antras and Chor, 2021). Their measure of upstreamness shows how important other sectors are as buyers to a given sector $i$. In our case, customer centrality comes from the importance of sector $i$ as a buyer to other sectors. This difference is expected because we focus on how shocks propagate, as in Acemoglu et al. (2016), while Antras and Chor (2021) focuses on the distance of each sector to final demand and primary factors. Our concept is closer to the Katz-Bonacich centrality used in the production networks literature. See Carvalho (2014) for an overview, especially footnote 11.

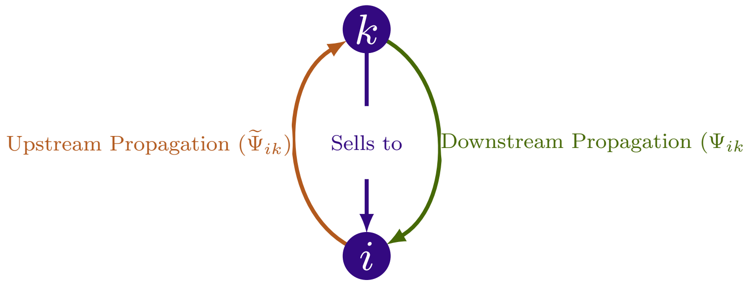



**Figure 1.** Upstream and Downstream Propagation

*Note:* This figure shows the propagation of shocks along the production network where we remove all other nodes and focus on total propagation (both direct and indirect). Downstream propagation from seller $k$ to buyer $i$ ($\Psi_{ik}$) and upstream propagation from buyer $i$ to seller $k$ ($\tilde{\Psi}_{ik}$). This illustrates the construction of measures in equations (1) and (3).

where $\Psi_{ij}$ is an element of the Leontief-Inverse matrix defined as

$$\boldsymbol{\Psi} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} = \sum_{s=0}^{\infty} \boldsymbol{\Omega}^s \tag{2}$$

where $\boldsymbol{I}$ is an identity matrix of size equal to the size of $\boldsymbol{\Omega}$. An element of $\boldsymbol{\Omega}$ is $\Omega_{ji} = P_i M_{ji}/P_j Q_j$. This represents the share of intermediates that sector $i$ supplies to sector $j$ ($P_i M_{ji}$) as a fraction of sector $j$'s sales ($P_j Q_j$). This shows the direct importance of producer $j$ as a supplier to producer $i$. An element $\Psi_{ji}$ records the importance of producer $i$ as a direct and *indirect* supplier to producer $j$. This intuition is precisely highlighted by the last equality in Equation (2), where $\boldsymbol{\Psi}$ is an infinite sum of direct and indirect linkages across producers. $Supplier_i$ adds across all buyers of good $i$ and measures the producer's $i$ importance as a *supplier* to the economy after considering direct and indirect linkages.

Similarly, we measure the customer or *upstream* centrality of a sector $i$ as

$$Customer_i = \sum_{j=1}^{N} \widetilde{\Psi}_{ij}, \tag{3}$$

where $\widetilde{\Psi}_{ij}$ is an element of the following matrix

$$\widetilde{\boldsymbol{\Psi}} = (\boldsymbol{I} - \boldsymbol{M})^{-1} = \sum_{s=0}^{\infty} \boldsymbol{M}^s$$

where $\boldsymbol{I}$ is an identity matrix of size equal to the size of $\boldsymbol{M}$. An element of $\boldsymbol{M}$ is $m_{ij} =$

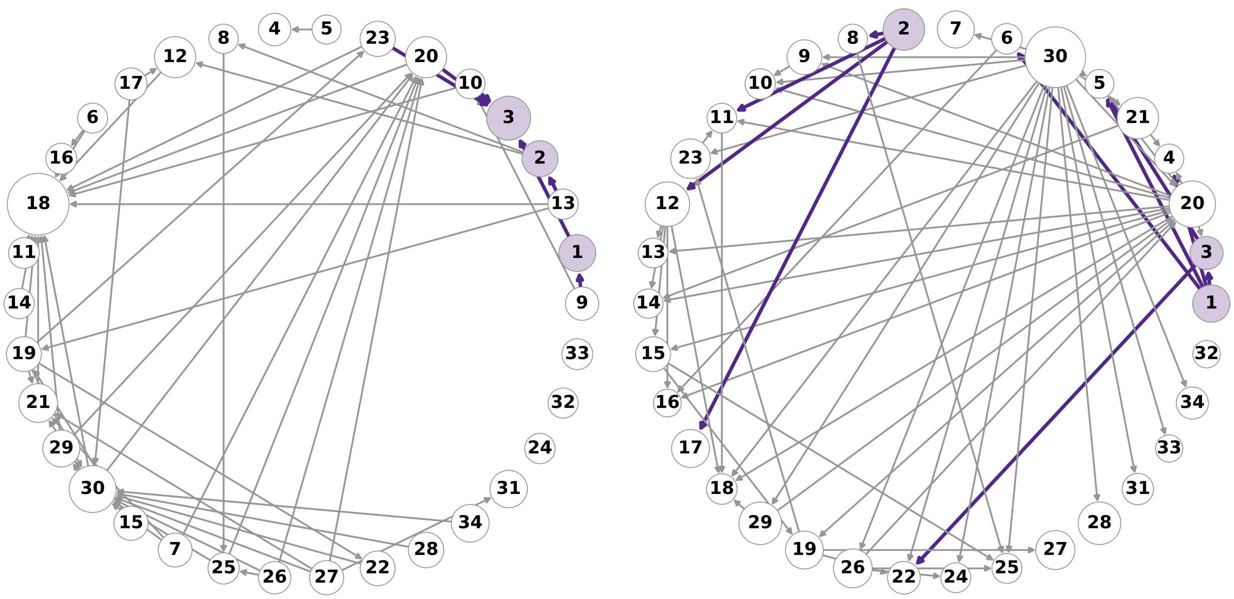
$P_jM_{ij}/P_jQ_j$. This represents the share of the sector's $j$ sales for which the sector $i$ accounts. This shows the direct importance of producer $i$ as a buyer to producer $j$. An element $\widetilde{\Psi}_{ij}$ records the importance of producer $i$ as a *buyer* to producer $j$ after considering direct and indirect linkages. $Customer_i$ adds across all suppliers to sector $i$. It measures sector $i$'s importance as a *buyer* to the economy after considering direct and indirect linkages.

To get a sense of how the production network looks like, Figure 2 plots the domestic network structure of Australia in 1995, using input-output data from the WIOD database. Each node (circle) is a different sector in the economy, and the node's size represents how important that sector is in the network based on the network centralities defined above. Panel (a) shows the network in which each node's size describes the customer centrality of the sector—this is, how much output of other sectors a given sector uses, directly and indirectly—, while in panel (b), the node size is based on each sector's supplier centrality—how much of a given sector output is used as input by other sectors, directly and indirectly.

We observe in Figure 2 that commodity sectors were central sectors in the domestic production network of Australia in 1995. Panel (a) shows that sector 3 (food) is one of the sectors with the largest customer centrality. Panel (b) also shows that mining is one of the most central sectors in its direct and indirect supply of intermediate inputs.

To describe the relative importance of commodity sectors in the domestic production network of small open economies, we report in Table 1 the ranking of the customer and supplier propagation centrality for the three commodity sectors, with respect to all the other sectors in the economy (a total of 34 in the WIOD data). The main takeaway from Table 1 is that for all the countries in our sample, at least one of the commodity sectors (many times 2 of them) is a central customer and/or a central supplier (top-10) in the domestic production network.

**Figure 2.** Domestic Production Network Australia




(a) Customer Centrality

(b) Supplier Centrality

*Note:* This figure shows the domestic production network of Australia (WIOD Input-Output data) for 1995 at the sector level (ISIC rev. 3). Each node (circle) is a different sector in the economy, and the size of the node represents how important that sector is as a direct and indirect buyer (panel a) and supplier (panel b) of intermediate inputs. Node labels are in Table B5 of our Appendix. To ease the exposition, we removed links that accounted for less than 5% of total sales either as a buyer or as a seller. Arrows pointing toward a sector imply that the sector is a buyer. Conversely, an arrow starting from a sector implies that the sector is a seller. In both cases, resources flow from the seller to the buyer.

## 2.1 A Simple Framework

Here, we provide a stylized static model to inform our empirical analysis of the sectoral propagation of commodity price changes.

**Setup.** Our model features a representative consumer that consumes $N+1$ goods and an imported good in a static setting. Each sector produces using a constant returns to scale production function. All sectors use labor, domestic intermediate inputs, and an imported intermediate input. Importantly, the commodity sector price is *exogenously* given.

**Notation.** We index non-commodity sectors with $i = 1, 2, ..., N$ and the commodity sector as $i = N+1$. We use **bold** to denote vectors and matrices. For any matrix $\boldsymbol{X}$, $\boldsymbol{X}^T$ is

**Table 1.** Ranking of Network Centrality of Commodity Sectors in 1995

| | *Customer Centrality* | | | *Supplier Centrality* | | |
|---|---|---|---|---|---|---|
| Country | Agric. | Mining | Food | Agric. | Mining | Food |
| Australia | 10 | 11 | 3 | 13 | 6 | 17 |
| Bulgaria | 2 | 8 | 1 | 2 | 9 | 13 |
| Brazil | 14 | 25 | 2 | 7 | 14 | 10 |
| Canada | 6 | 18 | 3 | 4 | 10 | 15 |
| Denmark | 6 | 33 | 1 | 8 | 17 | 11 |
| India | 9 | 25 | 6 | 3 | 9 | 23 |
| Lithuania | 1 | 33 | 3 | 2 | 34 | 9 |
| Mexico | 10 | 18 | 1 | 7 | 1 | 15 |
| Russia | 3 | 6 | 2 | 5 | 3 | 14 |
| **Average** | **7** | **20** | **2** | **6** | **11** | **4** |

*Note:* This table presents, for each country and commodity sector, the customer and supplier network centrality. Source: WIOD Input-Output database, 1995.

**Table 2.** Definitions

| Object | Typical Element | Computation | Indices | Definition |
|---|---|---|---|---|
| $\boldsymbol{\Omega}$ | $\{\Omega_{ij}\}$ | $P_j M_{ij}/P_i Q_i$ | $i = 1, 2, ..., N+1$ <br> $j = 1, 2, ..., N+1$ | Expenditure share on good $j$ by $i$ |
| $\boldsymbol{\Psi}$ | $\{\Psi_{ij}\}$ | $\{(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}\}_{ij}$ | $i = 1, 2, ..., N+1$ <br> $j = 1, 2, ..., N+1$ | Leontief-inverse of input-output matrix |
| $\boldsymbol{a}$ | $\{a_i\}$ | $WL_i/P_i Q_i$ | $i = 1, 2, ..., N+1$ | Labor share of producer $i$ |
| $\boldsymbol{\eta}$ | $\{\eta_i\}$ | $P_M M_{iM}/P_i Q_i$ | $i = 1, 2, ..., N+1$ | Imported intermediate input share of producer $i$ |
| $\boldsymbol{b}$ | $\{b_i\}$ | $P_i C_i/E$ | $i = 1, 2, ..., N+1$ | Home expenditure share on good $i$ |
| $\boldsymbol{\lambda}$ | $\{\lambda_i\}$ | $P_i Q_i/nGDP$ | $i = 1, 2, ..., N+1$ | Size of sector $i$ |
| $E$ | | $\sum_{i=1}^{N+1} P_i C_i + P_M C_M$ | | Home expenditure |
| $nGDP$ | | $\sum_{i=1}^{N+1} WL_i$ | | Home nominal GDP |

*Note:* This table presents the notation we use throughout the paper.

its transpose. Table 2 describes the objects that we use throughout the text.

### 2.1.1 Agents and Equilibrium

**Representative Consumer.** The representative consumer consumes all domestic goods ($i = 1, 2, ..., N + 1$) and the imported good. It aggregates these under a Cobb-Douglas utility function.

**Production.** There are $N + 1$ producing sectors. Each sector combines labor, domestic intermediate inputs, and imported intermediate inputs using a constant returns to scale production function. Given prices and wages, they minimize costs subject to their production function.

**Foreign demand.** The foreign economy demands commodities from the small open economy under a CES demand

$$X_{N+1} = \left(\frac{P^*_{N+1}}{P^*}\right)^{-\chi} C^* = (P^*_{N+1})^{-\chi} D^*, \tag{4}$$

where $P^*_{N+1}$ is the commodity price denominated in units of foreign currency and $D^* = (P^*)^{\chi} C^*$ is an exogenous demand shifter. This demand for commodity output is exogenous from the small open economy perspective, as it does not depend on any endogenous objects of the model.

We use $P^*_{N+1}$ and $D^*$ to capture the possibility of export changes coming from supply and demand forces. For example, an increase in the commodity price *decreases* exports of the small open economy for *given* $D^*$ according to Equation (4). In this case, changes in commodity prices can be considered supply-driven. Similarly, increases in $D^*$ raise commodity exports for *given* commodity price $P^*_{N+1}$. We say that a commodity price change is demand-driven when the increase in $D^*$ has a stronger effect on $X_{N+1}$ than the change in the commodity price.

**Law of one price.** The commodity sector price ($P_{N+1}$) and the imported good prices ($P_M$) in local currency satisfy the law of one price

$$P_M = P^*_M \mathcal{E}; \qquad P_{N+1} = P^*_{N+1} \mathcal{E},$$

where $\mathcal{E}$ is the nominal exchange rate between the small open economy and the foreign economy. The nominal exchange rate is units of home currency per unit of foreign currency. An increase in $\mathcal{E}$ is a depreciation of the home currency. An $^*$ over a variable denotes a price in *foreign currency*. Thus, $P_M^*$ and $P_{N+1}^*$ are exogenous for the small open economy.

**Equilibrium.** The equilibrium definition is as usual:

(i) Given prices and wages, the household maximizes utility subject to its budget constraint.

(ii) Given prices and wages, firms minimize costs subject to their production function.

(iii) Goods and labor markets clear.

(iv) Trade is balanced.

### 2.1.2 Comparative Statics

We now consider a small change in the commodity price in *foreign currency*, $\mathrm{d}\log P_{N+1}^*$, and study how this affects good prices, wage, and quantities $(\boldsymbol{P}, W, \boldsymbol{Q})$. All comparative exercises below assume that technology and factor supplies are fixed, $\mathrm{d}\log \boldsymbol{Z} = \mathbf{0}$ and $\mathrm{d}\log \bar{L} = 0$. This means that real GDP in our economy does not change with changes in commodity prices. This is a generalization of the two-sector result in Kehoe and Ruhl (2008). We formally derive this statement in Proposition 3 of Appendix E.3. Under this setup, we can focus on the cross-sectional responses of output and prices in different sectors to commodity price changes and their propagation throughout the production network.

Our first result states that up to a first-order, commodity prices propagate downstream to non-commodity prices:

**Proposition 1** (Price Responses to a Commodity Price Change)**.** Consider a perturbation of the commodity price, $\mathrm{d}\log P_{N+1}^*$. Up to a first-order approximation, changes in good prices satisfy

$$\mathrm{d}\log P_i = \frac{\widetilde{a}_i}{\widetilde{a}_{N+1}} \mathrm{d}\log P_{N+1}^*, \tag{5}$$

where $\widetilde{a}_i = \sum_{j=1}^{N+1} \Psi_{ij} a_j$ are the network-adjusted labor share of producer $i$ and $\Psi_{ij}$ represents how important is sector $j$ as a supplier, both directly and indirectly, to sector $i$.

**Proof.** See Appendix E.1. ■

Proposition 1 states that all prices increase proportionally to their wage exposure. Intuitively, a rise in the commodity sector price has two effects. First, it raises the marginal cost of all producers that use commodities to produce. Second, it raises the commodity sector demand for labor, putting upward pressure on wages. Since other sectors also use labor to produce, marginal costs rise everywhere. The relevant exposure to these changes is the network-adjusted labor shares, $\tilde{a}_i$. It considers how much labor each sector ultimately uses after taking the production network into account, effectively combining the two effects.

When the commodity sector is an important supplier to other sectors (measured by $\Psi_{j,N+1}$), it raises these sector exposures to labor and thus increases their sensitivity to commodity price changes. This provides an amplification mechanism. On the contrary, when the commodity sector is a highly intensive user of intermediate inputs, wages in equilibrium react less. Intuitively, a higher commodity price implies that the commodity sector increases demand for its inputs, putting upward pressure on all input prices. These increases in input prices feed back to the commodity sector's marginal cost. However, since in equilibrium the change in the marginal cost is exogenously given and equal to the commodity price increase, the commodity sector cannot increase its price in response. As a result, it has to decrease its demand for all inputs, putting downward pressure on input prices relative to the initial impulse. This downward pressure is stronger for wages, the higher the labor intensity (in a network sense) of the commodity sector is.[5]

We also emphasize another important result. Our model-implied measure of commodity sector supplier centrality is not exactly that in Equation (1). The standard Leontief measure of supplier centrality in Equation (1) applies for economies with single factors. In our economy, besides labor, firms have access to imported intermediates, which play a similar role to that of a second factor of production. This is why our adjusted Leontief inverse in Equation (5) adjusts for each producer $i$'s labor share and for the commodity-sector direct and indirect use of labor. Intuitively, because of the presence of imported intermediates, fluctuations in the real wage are important as they affect the use of imported intermediates and, therefore, the use of domestic intermediates once again.

[5]We acknowledge this dampening effect is not a novel result in itself. For example, Romero (2022) emphasizes this mechanism in a simplified version of the model we posit here. We highlight that the relevant statistic for such a dampening effect is the network-adjusted labor share of the commodity sector $\tilde{a}_{N+1}$. To our knowledge, this is not emphasized in Romero (2022) and other works in the literature.

**Testable Implication 1: Fluctuations in commodity prices increase non-commodity sector prices via intermediate input cost and real wage cost.**

$$\Delta \log P_{it} = \phi_p^{Down} \Delta \tilde{P}_{ikt}^{Down} + \delta_t + \alpha_i + \epsilon_{it},$$

where $\Delta \log P_{it}$ is the log deviation of the price of sector $i$ with respect to its steady-state value, $\Delta \tilde{P}_{ikt}^{Down} = \frac{\widetilde{a}_i}{\widetilde{a}_k} \mu_{kt}$ where $\mu_{kt}$ is a shock to the price of commodity sector $k$ at time $t$, $\delta_t$ is a time fixed-effect, and $\alpha_i$ a sector fixed-effect.

**Proposition 2** (Changes in Gross Output)**.** Up to a first-order approximation, changes in gross output of sector $i$ ($Q_i$), $\mathrm{d}\log S_i$, following a commodity price shock $\mathrm{d}\log P^*_{N+1}$, satisfy

$$\mathrm{d}\log Q_i = \left( \underbrace{\sum_{j=1}^{N+1} \frac{S_j}{S_i} \Phi^j(i, \mathrm{d}\log P^*_{N+1})}_{\text{Input-Output Substitution}} + \underbrace{(1-\alpha_i)(1-\chi)}_{\text{Foreign Substitution}} + \underbrace{\frac{\alpha_i}{\tilde{a}_{N+1}}}_{\text{Domestic Demand}} \right) \mathrm{d}\log P^*_{N+1}$$
$$+ \underbrace{(1-\alpha_i)\mathrm{d}\log D^*}_{\text{Foreign Demand}} - \underbrace{\frac{\widetilde{a}_i}{\widetilde{a}_{N+1}}}_{\text{Cost Push}} \mathrm{d}\log P^*_{N+1} \tag{6}$$

where

$$\alpha_i = \frac{\sum_{k=1}^{N+1} S_k^C \Psi_{ki}}{S_i}, \quad (1-\alpha_i) = \frac{S^*_{N+1}\Psi_{N+1,i}}{S_i},$$

represents the fraction of sales that satisfy domestic final demand ($\alpha_i$) and foreign final demand ($1-\alpha_i$), respectively, and

$$\Phi^j(i, \mathrm{d}\log P^*_{N+1}) = \sum_{k=1}^{N+1} \left( \sum_{h=1}^{N+1} (\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh}) \frac{\tilde{a}_h}{\tilde{a}_{N+1}} + (\theta^j_{kL} - 1) \frac{a_j}{\tilde{a}_{N+1}} \right) \Omega_{jk} \Psi_{ki}$$

is a version of the input-output substitute operator in Baqaee and Farhi (2019), where $\delta_{kh} = 1$, whenever $k = h$, and zero otherwise. $\theta^j_{kh}$ is the Allen-Uzawa elasticity of substitution for producer $j$ between any two pair of inputs $(k, h)$.

**Proof.** See Appendix E.2. ■

The last proposition follows by differentiating the market clearing condition of each good. To get this result, we impose Cobb-Douglas preferences for domestic consumers. Five terms govern changes in gross output. The first term on the right-hand side represents *substitution* that occurs at the level of the firm/sector that is then propagated *upstream* to other sectors, i.e., from buyers to suppliers. These substitution patterns are captured by the input-substitution operator $\Phi^j(i, \mathrm{d}\log P^*_{N+1})$, which can be considered a measure of *expenditure switching*. This input-output substitution operator comprises three steps that occur when a commodity price shock hits the economy, which we explain next.

The first step is that following a positive shock to the commodity sector price, the price of good $h$ increases by $\widetilde{a}_h/\widetilde{a}_{N+1}$.[6] This initial price change occurs because of downstream propagation on costs, as we already showed in Proposition 1.

In response to a change in the price of good $h$, each producer $j$ may substitute away/towards other intermediate goods or to factors of production. If, for example, it substitutes away/towards to some other intermediate good $k$, it does so by

$$\Omega_{jk}(\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh}),$$

which represents how much the expenditure share of producer $j$ on $k$ respond to a change in the price of good $h$ ($\partial\Omega_{jk}/\partial\log P_h$). This depends on the direct exposure of producer $j$ to both $k$ and $h$ and the elasticity of substitution between $k$ and $h$, a term captured by the Allen-Uzawa elasticity of substitution, $\theta^j_{kh}$. If goods have a high degree of substitutability ($\theta^j_{kh} > 1$) an increase in the price of good $h$ increases the expenditure share of producer $j$ on good $k$: producer $j$ substitutes from producer $h$ towards $k$. If goods $k$ and $h$ have a low degree of substitutability ($\theta^j_{kh} < 1$), then producer $j$ cannot reallocate its input demand much, which in turn decreases its expenditure share in good $k$. In other words, it cannot get away from the price increase in good $h$, and it is forced to decrease its expenditure share on good $k$ as a result. This is the second step.

How does this ultimately affect producer $i$? The third step answers this question by tying the substitution that each producer $j$ is doing towards/away other producers $k$ in the economy. The key object in this final step is the element of the Leontief-inverse $\Psi_{ki}$ that represents how important producer $i$ is as a supplier to producer $k$. This last term shows

[6] When good $h$ is labor ($L$), this measure is simply $1/\widetilde{a}_{N+1}$ as the network-adjusted labor share of the commodity sector is enough to pin down wage changes in general equilibrium.

the *upstream propagation* of substitution since it goes from $k$ (the buyer) to $i$ (the seller). While the initial shock propagates downstream, these different demand substitution patterns propagate upstream in the production network.

The second term represents how changes in domestic expenditure propagate upstream to sector $i$. An increase in domestic expenditure raises, up to a first order, the final demand for all goods. These demand increases translate into increases in intermediate input demand by all sectors. This means the relevant statistic for exposure to changes in final demand is not the direct exposure of sector $i$ to final demand ($S_i^C/S_i$) but rather a network-adjusted measure

$$\alpha_i = \sum_{k=1}^{N+1} S_k^C \Psi_{ki}/S_i,$$

that consider how changes in demand for other sectors also affect sector $i$ via input-output linkages.

The third term represents a foreign substitution channel. In response to a change in the commodity price, the foreign economy reduces its expenditure on commodity output by $(1-\chi)$ provided that $\chi > 1$. This mechanism reduces demand for all non-commodity sectors. Quantitatively, sector $i$'s sales decline by

$$(1-\alpha_i) = \frac{S_{N+1}^* \Psi_{N+1,i}}{S_i},$$

how much of $i$'s sales ultimately satisfy the foreign consumer demand.

The fourth term represents a foreign demand channel. It captures all other factors that induce the foreign consumer to demand more of the commodity sector in the small open economy for a given commodity price $P_{N+1}^*$. As in the case of the foreign substitution channel, this propagates upstream, affecting sectoral sales according to $(1-\alpha_i)$.

The final term, the response good $i$'s price to the commodity price change, fully characterizes the supply side response of this market. Since the marginal cost of sector $i$ pins down the price of good $i$ in general equilibrium, conditional on factor prices and technology, it encompasses all relevant information on the supply of good $i$.

Proposition 2 underscores the complex interaction between production elasticities of substitution and the cross-sectional effects of commodity price fluctuations in non-commodity sectors' output. The next auxiliary regression will be used in the empirical and quantitative section to infer the empirically relevant values of production elasticities of substitution.[7]

**Auxiliary regression: Fluctuations in commodity prices can affect the gross output of sectors that are located upstream and downstream from the commodity sector.**

$$\Delta \log Q_{it} = \phi_Q^{Up} \Delta \tilde{P}_{ikt}^{Up} + \phi_Q^{Down} \Delta \tilde{P}_{ikt}^{Down} + \delta_t + \alpha_i + \epsilon_{it}, \tag{7}$$

where $\Delta \log Q_{it}$ is the log deviation of sector $i$'s gross output with respect to its steady state value, $\Delta \tilde{P}_{ikt}^{Up} = \tilde{\Psi}_{ki} \cdot \mu_{kt}$, $\Delta \tilde{P}_{ikt}^{Down} = \frac{\tilde{a}_i}{\tilde{a}_k} \mu_{kt}$, $\mu_{kt}$ is a shock to the price of commodity sector $k$ at time $t$, $\delta_t$ is a time fixed-effect, and $\alpha_i$ a sector fixed-effect. Unlike our testable implication 1, the coefficients $\phi_Q^{Up}$ and $\phi_Q^{Down}$ are a function of the elasticities of substitution in production, via the input-output substitution operator in Proposition 2.

## 2.2 Commodity prices via production networks

In this section, we test the predictions from Proposition 1 (testable implication 1) and estimate the auxiliary model implied by Proposition 2. We focus on the network effects of commodity price changes on non-commodity sectors.[8] Our identification assumption is that commodity prices determined in international markets are exogenous to non-commodity sectors in these small open economies. In particular, we follow Schmitt-Grohé and Uribe (2018) and assume that commodity prices in each country follow an autoregressive process. The residual from that process is assumed to be the surprise to commodity price fluctuations for domestic non-commodity sectors.[9] Motivated by the implications from the simple model but extended to a panel of countries, the empirical specification is

$$\Delta \log Y_{ict} = \delta_t + \alpha_{i,c} + \delta_{c,t} + \phi_1 \Delta \tilde{P}_{ict}^{Up} + \phi_2 \Delta \tilde{P}_{ict}^{Down} + \boldsymbol{\nu}' \boldsymbol{X}_{ict-1} + \epsilon_{ict}, \tag{8}$$

[7]Under CES production technologies, the Allen-Uzawa elasticities of substitution in production can be expressed as a function of the nested CES elasticities and input shares.

[8]While there is important literature emphasizing the different effects of demand-side vs. supply-side shocks to commodity prices (e.g., Kilian, 2009; Aastveit et al., 2023), our goal is to study the propagation mechanisms of changes in commodity prices, regardless of the source of shock. We explore the consequence of demand and supply shocks in Section 3.

[9]Schmitt-Grohé and Uribe (2018) use such a shock in a SVAR setting to study the macroeconomic effects of terms of trade shocks. We report the resulting series in the appendix Figure B8.

where $\Delta \log Y_{ict}$ is the log change sector $i$'s output or prices in country $c$ at time $t$, with respect to the steady state. For estimation purposes, we assume that in 1995, these small open economies were at their steady-state equilibrium. To control for additional shocks and potential trends, we include year fixed effects $\delta_t$, country-sector fixed-effects $\alpha_{i,c}$, and a full set of country-time fixed effects $\delta_{c,t}$. $\Delta\tilde{P}^{Up}_{ict}$ and $\Delta\tilde{P}^{Up}_{ict}$ are our network spillover measures described in our previous section.[10] $\boldsymbol{X}_{ict-1}$ is a $H \times 1$ vector of lagged controls, including the dependent variable and our network spillover measures. Finally, $\epsilon_{ict}$ is an error term.[11]

Our theory has predictions for the values of $\phi_1$ and $\phi_2$. When the dependent variable is sectoral prices, as in Proposition 1, $\phi_2$ should be positive and equal to 1. Commodity prices raise output prices and wages, which then propagates to downstream firms' marginal costs. $\phi_1$ should be zero as changes in sectoral demand for inputs do not affect other sectors' prices, conditional on marginal costs, due to our assumption of constant returns to scale. On the other hand, when the dependent variable is sectoral gross output, as in Proposition 2, the values for $\phi_1$ and $\phi_2$ are ambiguous. If we assume Cobb-Douglas production technologies and Cobb-Douglas foreign demand ($\chi = 1$), the input-output substitution operator ($\Phi^j$) and the foreign demand channel are zero, implying that $\phi_1 = 1$, via domestic demand, and $\phi_2 = -1$ via cost push channel. However, under CES production technologies the value, and potentially the sign, of $\phi_1$ and $\phi_2$ are not clear. In our quantitative section, we will use *indirect inference* to obtain the values of the production elasticities that better resemble the empirical estimates from Equation (7).

### 2.2.1 Network propagation of commodity price fluctuations

We now present empirical evidence on the transmission mechanism of commodity price fluctuations via production networks using data from the WIOD database. The WIOD database has an important advantage over the OECD database: it reports sectoral quantity and price indexes, allowing us to study better the channels in which commodity price changes affect quantities and prices. Instead, the OECD data only reports nominal data (in US dollars) for sales, value-added, and intermediate input use. To construct the network exposures defined

[10] Notice that we omit the commodity sector subscripts ($k$) as there is only one measure at the country-sector-time level since we collapse all commodity sectors into one index.

[11] To make the regressions comparable to the model, in which there is one commodity sector, we measure the sectoral exposure to commodity as in $\Delta\tilde{P}^{Down}_{ikt} = \tilde{a}_i \left(\frac{\mu_{kt}}{\tilde{a}_k}\right)$, with $\left(\frac{\mu_{kt}}{\tilde{a}_k}\right) = \sum_{h\in\mathcal{K}} \frac{\mu_{ht}/\tilde{a}_h}{\sum_{k'\in\mathcal{K}}(1/\tilde{a}'_k)}$, where $\mathcal{K}$ is the set of commodity sectors. Hence, it is a weighted average exposure across commodity sectors.

in Proposition 1 and Proposition 2, we used the input-output structure in 1995. Appendix A describes the data sources and the process of constructing sectoral indexes of commodity prices for different countries.

Table 3 presents the results of estimating Equation (8) using quantity and price indexes for gross output. All regressions include one lag of the dependent variable. We first focus on the effects on sectors selling to the commodity sector ($\Delta\tilde{P}^{Up}_{ict}$). Columns (1) to (3) show that real commodity price fluctuations positively affect the gross output of non-commodity sectors. In particular, in column (3)—where we control for a year, country-sector, and country-year fixed effects—a 1% increase in $\Delta\tilde{P}^{Up}_{ict}$ generates a 0.44% increase in the sectoral gross output quantity index, on impact. We find no evidence of downstream ($\Delta\tilde{P}^{Down}_{ict}$) effects on quantities. Columns (4) to (6) show that, despite the muted downstream effect on quantities, we observe a positive downstream propagation of commodity prices to the price of non-commodity sectors, with no upstream propagation. A 1% increase in $\Delta\tilde{P}^{Down}_{ict}$ generates a 0.16% impact increase in the sectoral gross output price index of non-commodity sectors downstream to commodities.

**Robustness.** In Table B7 and Table B8 in the Appendix, we provide results using two alternative strategies. First, we show that the same results hold using log differences as the dependent variable, as in Acemoglu et al. (2016). Second, we use instrumental variables to control for the potential endogeneity in commodity prices for these small open economies. While there are no exogenous instruments for all the commodity prices we use in our sectoral commodity price indexes, we use oil supply shocks from Baumeister and Hamilton (2019) and Känzig (2021) as well as harvest shocks from Winne and Peersman (2021). The results support the main findings in Table 3.[12]

The empirical evidence in this section points to strong upstream propagation of commodity prices, alongside a muted downstream propagation, on the quantity produced by non-commodity industries. At the same time, we find a strong increase in the price of industries downstream from the commodity sector, with no effect on upstream industries. In the next section, we build a dynamic small open economy with production networks and a commodity sector that rationalizes the findings we document in this section. In particular, we ask how can a small open economy model with production networks, in which the commodity sector

[12]The results using instrumental variables in Table B8 suffer from weak instruments, except for columns (3) and (6), when we include country-year fixed effects.

**Table 3.** Network Effects of Commodity Price Changes on Non-Commodity Sectors

| | Panel (a): Quantities | | | Panel (b): Prices | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $\Delta\tilde{P}^{Up}_{ict}$ | 0.2769** | 0.5342*** | 0.4389*** | -0.4094 | 0.1827 | 0.0214 |
| | (0.1337) | (0.1274) | (0.1431) | (0.2717) | (0.2007) | (0.1132) |
| $\Delta\tilde{P}^{Down}_{ict}$ | 0.1740*** | 0.0794 | 0.1168 | -0.0106 | 0.5053*** | 0.1652** |
| | (0.0665) | (0.0632) | (0.0898) | (0.1349) | (0.1004) | (0.0708) |
| Observations | 3906 | 3906 | 3906 | 3906 | 3906 | 3906 |
| Within $R^2$ | 0.929 | 0.794 | 0.784 | 0.960 | 0.746 | 0.715 |
| Year F.E. | Yes | Yes | Yes | Yes | Yes | Yes |
| Country × Sector F.E. | | Yes | Yes | | Yes | Yes |
| Country × Year F.E. | | | Yes | | | Yes |

*Note:* This table presents OLS regressions using sectoral log quantity (columns 1 to 3) and log price index (columns 4 to 6) as the dependent variable. $\Delta\tilde{P}^{Up}_{ict}$ and $\Delta\tilde{P}^{Down}_{ict}$ are constructed using sectoral commodity price shocks for each country and non-commodity sectors' input-output connections to each country's commodity sector using the results in Proposition 1 and Proposition 2. The independent variables also include a lag of $\Delta\tilde{P}^{Up}_{ict}$ and $\Delta\tilde{P}^{Down}_{ict}$, on top of a lag of the dependent variable. Newey-West standard errors are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

is a central supplier and user of intermediate inputs (as documented in Table 1), rationalize the large upstream propagation and muted downstream propagation of commodity price fluctuations.

# 3 A Quantitative Small Open Economy with Production Networks

The goal of this section is to embed the static model outlined in Section 2.1 into an otherwise standard small open economy model. In the interest of space, we relegate details to Appendix D.

## 3.1 Household

### 3.1.1 Intertemporal

The household's intertemporal problem is as follows

$$\max_{\{C_t,B_t\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\rho}-1}{1-\rho}$$

subject to

$$P_t C_t + P_{N+1,t}(B_t + g(B_t)) \le W_t \bar{L} + (1+r) P_{N+1,t} B_{t-1}, \tag{9}$$

$$\text{given } B_{-1} \tag{10}$$

where $\rho$ is the (inverse) intertemporal elasticity of substitution, $\beta$ is a discount factor, $P_t$ is the price index of the aggregate consumption bundle ($C_t$), $B_t$ is the foreign assets position that we assume is denominated in units of the commodity good ($P_{N,t+1}$) (as in Di Pace et al. (Forthcoming)), $g(B_t)$ is a bond holding adjustment cost function (to be specified below), $W_t$ is the wage, $r$ is the interest rate on the foreign asset and $\bar{L}$ is the inelastic labor supply that we assume constant over time.

We assume the bond holding adjustment cost function takes the following form

$$g(B_t) = \frac{\psi}{2}(B_t - \bar{B})^2, \tag{11}$$

where $\psi > 0$ is a parameter and $\bar{B}$ is the steady-state level of debt. The existence of $\bar{B}$ allows us to examine the possibility of trade imbalances at the steady state.

### 3.1.2 Intratemporal

Given a path for aggregate consumption $\{C_t\}_{t=0}^{\infty}$, the intratemporal problem is the solution to the following program

$$PC = \min_{\left\{\{C_i\}_{i=1}^{N+1}, C_M\right\}} \sum_{i=1}^{N+1} P_i C_i + P_M C_M$$

subject to

$$\left(\prod_{i=1}^{N+1} \left(\frac{C_i}{\beta_i}\right)^{\beta_i}\right) \left(\frac{C_M}{\beta_M}\right)^{\beta_M} \geq C, \tag{12}$$

where $\sum_{i=1}^{N+1} \beta_i + \beta_M = 1$.

## 3.2 Production

As in the static model, $N + 1$ sectors in the economy produce using labor and intermediate inputs. As firms do not make intertemporal decisions, we omit the time subscript. In what follows, a bar over a variable denotes its steady-state value.

The production function for good $i$ is of the calibrated CES form

$$\frac{Q_i}{\bar{Q}_i} = Z_i \left(a_i \left(\frac{L_i}{\bar{L}_i}\right)^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - a_i)\left(\frac{M_i}{\bar{M}_i}\right)^{\frac{\sigma_i - 1}{\sigma_i}}\right)^{\frac{\sigma_i}{\sigma_i - 1}}, \tag{13}$$

where $a_i$ is the labor share in production and $(1 - a_i)$ is the intermediate input share. $\sigma_i$ is the elasticity between labor and the intermediate input bundle.

The intermediate input bundle contains both domestic and imported intermediates. It has the following CES aggregator

$$\frac{M_i}{\bar{M}_i} = \left(\omega_i^D \left(\frac{M_i^D}{\bar{M}_i^D}\right)^{\frac{\epsilon_i - 1}{\epsilon_i}} + (1 - \omega_i^D)\left(\frac{M_{iM}}{\bar{M}_{iM}}\right)^{\frac{\epsilon_i - 1}{\epsilon_i}}\right)^{\frac{\epsilon_i}{\epsilon_i - 1}}, \tag{14}$$

where $\omega_i^D$ is the expenditure share on domestic intermediate goods as a fraction of total intermediate input expenditure. Conversely, $(1 - \omega_i^D)$ is the imported intermediate input share. $\epsilon_i$ is the elasticity of substitution between domestic and imported intermediates.

The domestic intermediate input bundle aggregates intermediate input demand from all domestic sectors according to the following CES layer

$$\frac{M_i^D}{\bar{M}_i^D} = \left( \sum_{j=1}^{N+1} \omega_{ij} \left( \frac{M_{ij}}{\bar{M}_{ij}} \right)^{\frac{\epsilon_i^D - 1}{\epsilon_i^D}} \right)^{\frac{\epsilon_i^D}{\epsilon_i^D - 1}}, \tag{15}$$

where $\omega_{ij}$ is the expenditure share on domestic good $i$ as a fraction of total domestic intermediate input expenditure that satisfies $\sum_{j=1}^{N+1} \omega_{ij} = 1$. $\epsilon_i^D$ is the elasticity of substitution among domestic intermediate inputs.

## 3.3 Foreign demand

The foreign demand for the commodity sector is such that

$$X_t = \beta_{N+1}^* (P_{N+1,t}^*)^{-\chi} D_t^* \tag{16}$$

where $\beta_{N+1}^*$ is the expenditure share on the exported good by the foreign economy, $P_{N+1,t}^*$ is the commodity price in foreign currency units (to be defined below), $\chi$ is the elasticity of foreign demand to the commodity price and $D_t^*$ is an exogenous demand shifter.

## 3.4 Exogenous processes

We consider AR(1) processes (in logs) for the commodity price in foreign currency units, $P_{N+1,t}^*$, and the foreign demand shifter, $D_t$,

$$\log P_{N+1,t}^* = \rho_{N+1} \log P_{N+1,t-1}^* + \phi \log D_t^* + \varepsilon_{N+1,t}, \tag{17}$$

$$\log D_t^* = \rho_{N+1} \log D_{t-1}^* + \varepsilon_{D,t}. \tag{18}$$

We assume both exogenous processes have the same persistence, $\rho_{N+1}$. We posit a positive relationship between the commodity price process and foreign demand shifter. This is to capture in reduced form the fact that commodity prices are endogenous to global demand. We parametrize this relationship by $\phi \geq 0$.

### 3.5 Market clearing

The market clearing conditions are as follows

$$Q_{i,t} = C_{i,t} + \sum_{j=1}^{N+1} M_{ji,t}, \qquad \text{for } i = 1, 2, ..., N \tag{19}$$

$$Q_{N+1,t} = C_{N+1,t} + X_t + \sum_{j=1}^{N+1} M_{j,N+1,t}, \tag{20}$$

$$\bar{L} = \sum_{i=1}^{N+1} L_{i,t}, \tag{21}$$

$$B_t = (1+r)B_{t-1} - g(B_t) + \underbrace{X_t - \frac{P_{M,t}}{P_{N+1,t}} \left( \sum_{i=1}^{N+1} M_{iM,t} + C_{M,t} \right)}_{\text{Trade Balance}}. \tag{22}$$

Equation (19) is the market clearing condition for non-commodity sectors, equation (20) is the market clearing condition for the commodity sector, equation (21) is the labor market clearing condition, and equation (22) is the evolution of net foreign assets.

**Numeráire and exogenous commodity price.** To close the model in general equilibrium, we need to define a numeráire. Since the law of one price holds for the commodity good and the importable good, we have

$$P_{N+1,t} = \mathcal{E}_t P^*_{N+1,t}; \qquad P_{M,t} = \mathcal{E}_t P^*_{M,t}. \tag{23}$$

This implies that the commodity price in *units of the importable good* is exogenous since

$$\frac{P_{N+1,t}}{P_{M,t}} = \frac{\mathcal{E}_t P^*_{N+1,t}}{\mathcal{E}_t P^*_{M,t}} = \frac{P^*_{N+1,t}}{P^*_{M,t}}, \tag{24}$$

where both $P^*_{N+1,t}$ and $P^*_{M,t}$ are exogenous.

To proceed, we set $P^*_{M,t} = 1$ for all $t$ and let $\mathcal{E}_t$ be the numeráire. This is equivalent to setting the imported good price in local currency units as the numeráire since $P_{M,t} = \mathcal{E}_t P^*_{M,t} = \mathcal{E}_t$. Therefore, in our model, commodity price changes are relative price changes.

### 3.6 Calibration and Estimation

We take a two-step approach to choosing the values of the model's parameters. Our first step involves calibrating the household preference parameters, bond-holding adjustment costs, steady-state level of debt, shock processes, sectoral consumption shares, and input-output shares. We then estimate the value of the elasticities of substitution in production using *indirect inference*.

The discount factor is $\beta = 0.961$ to get an annual interest rate $r$ equal to 4%. The intertemporal elasticity of substitution is $\rho = 2$, in line with most of the small open economy literature (see Uribe and Schmitt-Grohé, 2017). The bond holdings adjustment cost parameter is $\psi = 0.000742$ following Schmitt-Grohé and Uribe (2003).

To calibrate all expenditure share parameters we target the details of the production structure of the Australian economy. In particular, we use information from the Input-Output tables year 1995 to calibrate

$$(\{a_i, \omega_i^D, \beta_i\}_{i=1}^{N+1}, \{\omega_{ij}\}_{i,j=1}^{N+1}, \beta_M, \beta_{N+1}^*).$$

In the steady state of the model, given the household and the firms' optimality conditions, these parameters correspond to the observed expenditure shares. For example, $a_i$ equals the share of labor input in total gross output $\frac{wL_i}{P_iQ_i}$.

We then choose $\bar{B}$ to get a -0.8% trade balance to GDP ratio at the steady state. This is consistent with the average Australian trade balance to GDP ratio between 1995 and 2019 according to data from the International Financial Statistics database of the International Monetary Fund.[13]

The persistence of the commodity price shock process is $\rho_{N+1} = 0.53$ following the median value reported in Table 7.1 of Uribe and Schmitt-Grohé (2017) that considers a sample of 51 economies. The elasticity of commodity prices to changes in foreign demand is $\phi = 0.15$, consistent with the elasticity estimates in Baumeister and Hamilton (2019). The elasticity of export demand to commodity price changes is of the Cobb-Douglas type $\chi = 1$. In doing so, we remove the foreign substitution channel from the quantitative model. We chose this parameter configuration to get a positive correlation between commodity price changes and exports, a typical feature of commodity exporters like Australia.[14]

[13] We compute trade balance as exports minus imports, which include goods and services.

[14] In particular, for export demand to increase in response to a commodity price change and foreign demand

To estimate the elasticities of substitution $\sigma_i$, $\epsilon_i$ and $\epsilon_i^D$ we use *indirect inference*. In particular, using the auxiliary regression in Equation (7), we obtain the model-implied estimates for $\phi_Q^{Up}$ and $\phi_Q^{Down}$, using simulated data from the model. As emphasized in Proposition 2 these coefficients are a function of the elasticities of substitution in the model.

Hence, the approach entails setting up a grid of values for the elasticities $((\sigma, \epsilon, \epsilon_D) \in (0.2, 5))$ and then minimizing the following loss function

$$\mathcal{L}(\sigma, \epsilon, \epsilon^D) = (\phi_Q^{Up}(data) - \phi_Q^{Up}(\sigma, \epsilon, \epsilon_D))^2 + (\phi_Q^{Down}(data) - \phi_Q^{Down}(\sigma, \epsilon, \epsilon_D))^2,$$

where $\phi_Q^{up}(data), \phi_Q^{Down}(data)$ are the estimated coefficients from Equation (7) in Table 3. The coefficients $\phi_Q^{Up}(\sigma, \epsilon, \epsilon_D)$ and $\phi_Q^{Down}(\sigma, \epsilon, \epsilon_D)$ are those estimated by the model for that specific combination of $(\sigma, \epsilon, \epsilon_D)$. The estimated values are $\sigma = 3$, $\epsilon = 0.6$, and $\epsilon^D = 0.2$. We plot the values of the loss function for the different combinations of $\sigma, \epsilon, \epsilon_D$ in Figure B11. For computational purposes we choose a relatively coarse grid. However, one can clearly see that the data favours a high elasticity of substitution between intermediates and value added inputs ($\sigma = 3$), consistent with Miranda-Pinto (2021) and Huneeus (2020); a low elasticity of substitution between domestic intermediates and imported intermediates ($\sigma = 0.6$), as in Boehm et al. (2019); and an even lower elasticity of substitution among domestic intermediate inputs ($\sigma^D = 0.2$), as in Atalay (2017).

A summary of these parameters and their sources are in Table 4. We solve the model using a first-order approximation around the deterministic steady state.

## 3.7 Results

We now turn to the results. Section 3.7.1 studies the aggregate effects of a commodity price change. We then study the cross-sectional impact of a commodity price change in section 3.7.2. Throughout this section, all impulse response functions depict the response of variables to a 1% increase in the international price of commodities.

### 3.7.1 Aggregate effects of commodity price shock

In this subsection, we study the aggregate effects of a commodity price change on the real wage response and the average quantity response of domestic sectors.

shift, we require $\mathrm{d}\log X = (1 - \chi\phi)\mathrm{d}\log D^* > 0 \rightarrow \chi < \frac{1}{\phi}$.

**Table 4.** Calibrated Parameters

| Parameter | Value | Description | Source/Note |
|---|---|---|---|
| $\beta$ | 0.961 | Discount rate | Match interest rate $r = 4\%$ |
| $\rho$ | 2 | Intertemporal elasticity of substitution | Uribe and Schmitt-Grohé (2017) |
| $\psi$ | 0.000742 | Bond holdings cost adjustment parameter | Schmitt-Grohé and Uribe (2003) |
| $\rho_{N+1}$ | 0.53 | Persistence of commodity price process | Median value in Uribe and Schmitt-Grohé (2017) |
| $\phi$ | 0.15 | Elast. of commodity price to demand shifter | Baumeister and Hamilton (2019) |
| $\chi$ | 1 | Elast. of export demand to commodity price changes | Cobb-Douglas Foreign Demand |
| $\bar{B}$ | 0.2 | Steady-state asset level | Matched trade balance to GDP ratio |
| $\sigma_i$ | 3 | Elast. btw. labor and intermediate inputs | Indirect inference |
| $\epsilon_i$ | 0.6 | Elast. btw. domestic and imported int. inputs | Indirect inference |
| $\epsilon_i^D$ | 0.2 | Elast. across domestic int. inputs | Indirect inference |

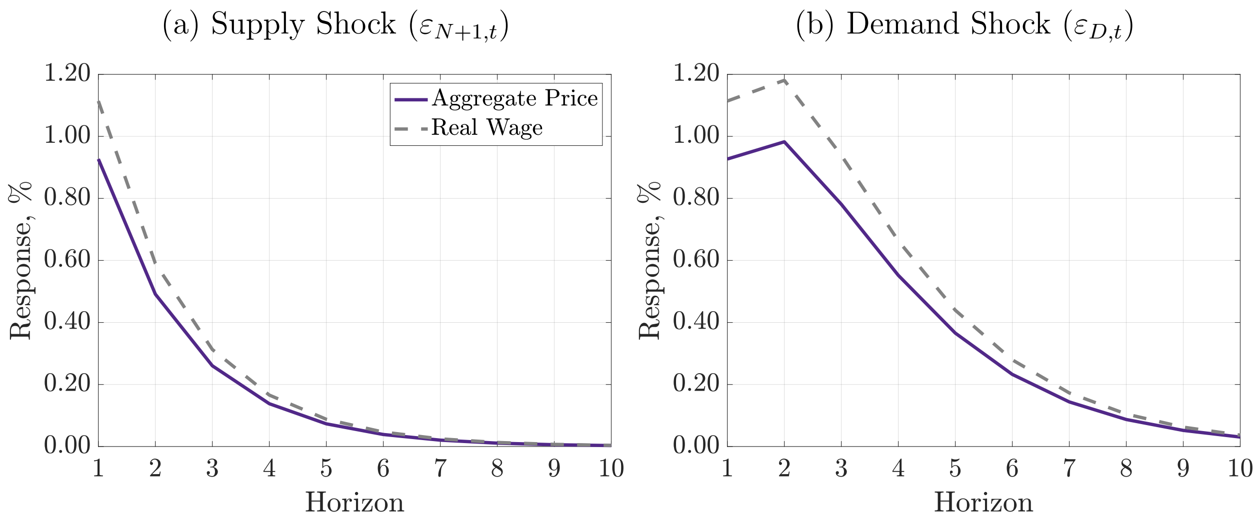
*Note:* This table shows our estimated and calibrated parameters.

**Real wage responses.** As shown in Proposition 1, through its downstream linkages, an increase in commodity prices raises sectoral prices and, therefore, the aggregate price index. Second, it raises the commodity sector demand for labor, which puts pressure on wages.

Figure 3 shows that, despite the rise in the aggregate price index, the real wage of the household, in units of the importable good, increases more than proportional to the shock. The left panel shows the effect of a supply shock to commodity prices and the right panel shows the effect of a demand shock to commodity prices. The effects are qualitatively the same. Figure B9 in our Appendix depicts the responses of aggregate consumption, trade balance, and the current account.

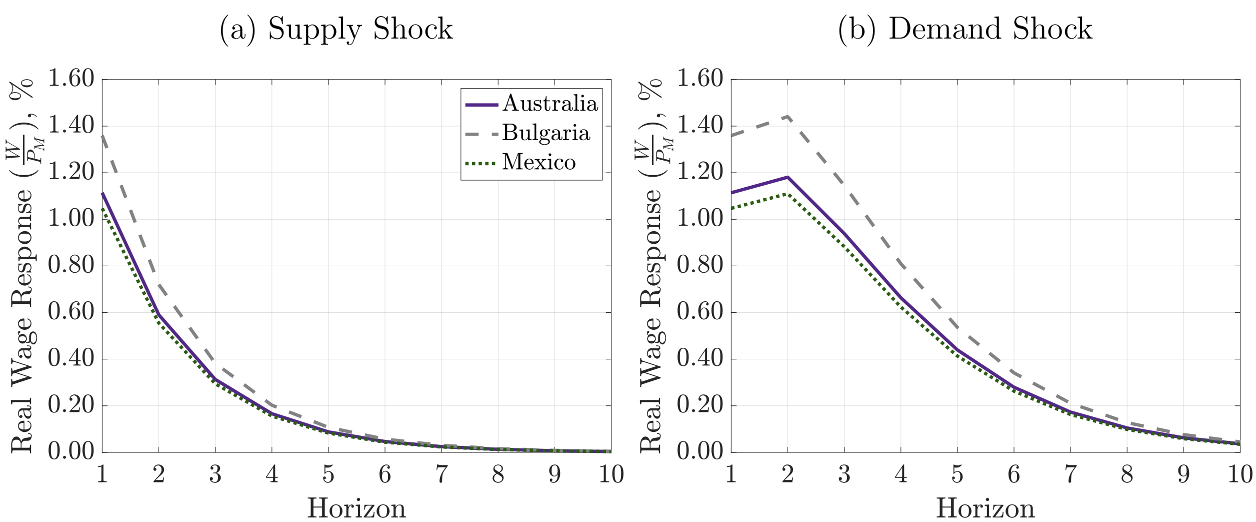
In Figure 4, we illustrate the importance of the network structure in shaping the aggregate effects of a commodity price shock. We recalibrate the model and change Australia's network structure to match Bulgaria's and Mexico's. Interestingly, in response to a 1% increase in mining prices, if the Australian mining sector were connected to other sectors, as in Bulgaria, the real wage would increase by 1.4% rather than by 1.1%. This is a 27% increase in the real wage's elasticity to commodity prices. Note that Bulgaria's mining sector size does not drive this result. Indeed, the mining sector in Bulgaria is smaller than the mining sector in Australia ($5\% < 8.7\%$). The difference in the response of the real wage is explained by the fact that the mining sector in Bulgaria is less labor intensive, in a network sense, than the mining sector in Australia ($\widetilde{a}_{N+1}^{AUS} = 0.90 > \widetilde{a}_{N+1}^{BUL} = 0.74$). The opposite holds for the case of Mexico as $\widetilde{a}_{N+1}^{AUS} = 0.90 < \widetilde{a}_{N+1}^{MEX} = 0.96$.

**Figure 3.** Real wage and aggregate price




*Note:* This figure shows the response of the aggregate price index and the real wage, in units of the importable good price, to a 1% increase in commodity prices due to a supply shock $\varepsilon_{N+1,t}$ (left panel) and due to a demand shock $\varepsilon_{D,t}$ (right panel).

**Figure 4.** Real wage response for different network structures




*Note:* This figure shows the response of the real wage, in units of the importable good price, to a 1% increase in commodity prices due to a supply shock $\varepsilon_{N+1,t}$ (left panel) and due to a demand shock $\varepsilon_{D,t}$ (right panel). Australia corresponds to the calibration. Bulgaria imposes the input-output structure of Bulgaria to the benchmark economy, and Mexico imposes the input-output structure of Mexico to the benchmark economy.

**Production elasticities and average quantity response.** We now emphasize the role played by production elasticities. Panel I of Figure 5 shows the average quantity response across non-commodity sectors in response to a commodity price due to a supply shock, $\varepsilon_{N+1,t}$. Panel II, in contrast, shows the average quantity change in response to a demand shock, $\varepsilon_{D,t}$. Each sub-panel varies a different elasticity while keeping other elasticity values at 0.2. In each panel, one of the combinations corresponds to our estimated values of the elasticities.

Sub-panels a) show that the elasticity of substitution between value-added inputs (labor) and intermediates is quantitatively key in shaping the output response to shocks in commodity prices. The higher the elasticity, the higher the increase in the average output of non-commodity sectors. Intuitively, as the rise in the commodity price makes intermediate inputs more expensive, higher substitutability towards labor allows non-commodity firms to partially shield themselves against the rise in the cost of materials.

Sub-panels b) of Figure 5 highlights a different role for the elasticity of substitution between domestic intermediates and imported intermediates ($\epsilon$). Conditional on having low substitutability between intermediate and labor ($\sigma$) and among domestic intermediates ($\epsilon_D$), the higher the elasticity of substitution between domestic and imported intermediates the larger the decline in the output of the average domestic non-commodity sector. The increase in the relative price of the commodity sector is a decrease in the relative price of imported intermediates. Hence, a higher elasticity of substitution between domestic and imported intermediates is associated with larger output declines in domestic non-commodity sectors.

Sub-panels c) of Figure 5 shows the role of the elasticity of substitution between domestic intermediates $\epsilon_D$. The increase in the price of the commodity sector shifts demands towards domestic sectors with smaller downstream exposure to the commodity shock. Hence, as is the case with $\sigma$, the average domestic sector increases output more the higher the substitutability among domestic intermediates.

### 3.7.2 Cross-sectional effects of commodity price shock

In this subsection, we study the cross-sectional effects of commodity price shocks.

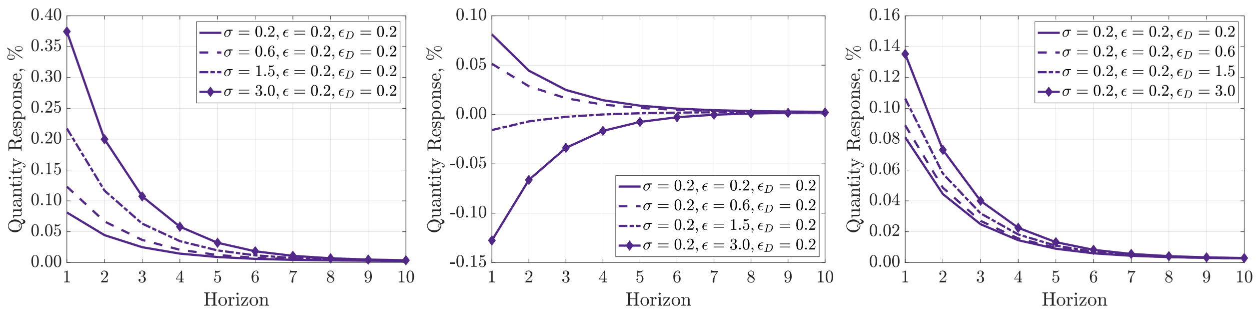
Proposition 2 stated that the sign of the output change in non-commodity sectors can be negative for sectors that are mainly downstream to commodities, and it can be positive for sectors that are mainly upstream to commodities. Figure 6 depicts this result quantitatively for the Australian economy.

We observe that commodity price shocks generate highly heterogeneous effects on non-commodity sectors' output depending on the value of elasticities of substitution. The output

**Figure 5.** Role of Production Elasticities

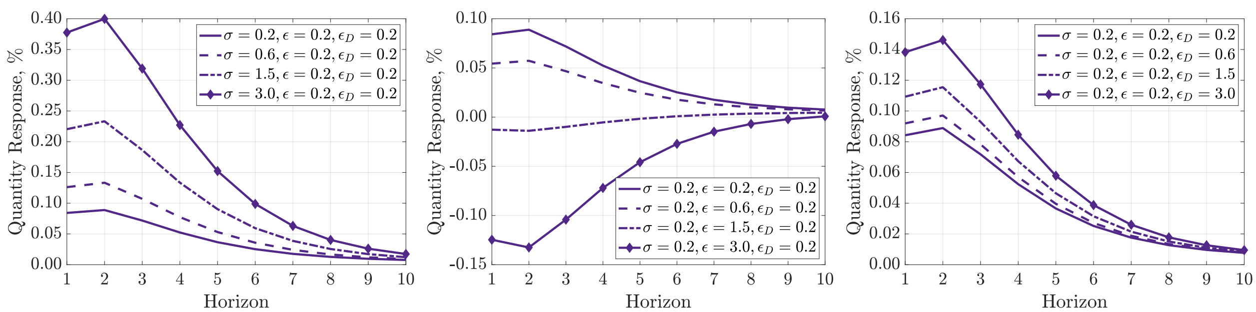
Panel I: Supply Shocks

(a) Role of $\sigma$ (b) Role of $\epsilon$ (c) Role of $\epsilon^D$




Panel II: Demand Shocks

(a) Role of $\sigma$ (b) Role of $\epsilon$ (c) Role of $\epsilon^D$




*Note:* This figure shows the average output response of non-commodity sectors to a 1% increase in commodity prices, for different values of production elasticities, due to a supply shock $\varepsilon_{N+1,t}$ (Panel I) and due to a demand shock $\varepsilon_{D,t}$ (Panel II). $\sigma$ is the elasticity between labor and intermediate inputs. $\epsilon$ is the elasticity between domestic and imported intermediate inputs. $\epsilon^D$ is the elasticity among domestic intermediate inputs.

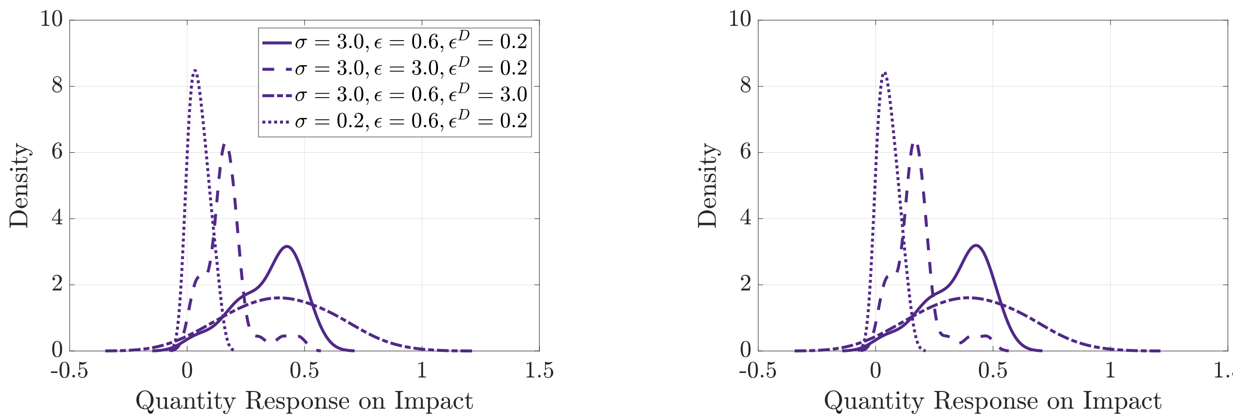
response has a higher mean and dispersion when either the elasticity between value added and intermediate inputs are high ($\sigma = 3, \epsilon = 0.6, \epsilon^D = 0.2$ in our estimated benchmark) or when there is high substitutability among domestic intermediates ($\sigma = 3, \epsilon = 0.6, \epsilon^D = 3$). Notably, in the case with low input substitutability ($\sigma = 0.2, \epsilon = 0.6, \epsilon^D = 0.2$), there is very limited sectoral reallocation in response to a commodity price change.

The next figure, Figure 7, shows that the heterogeneity in reallocation greatly depends on the production linkages between non-commodity sectors and the commodity sector.[15] In this

[15] Notice that the impact effects of a supply and demand shock are the same. This is because we calibrate the supply and the demand shock to have the same impact effect on commodity prices and, therefore, on domestic prices and output. If we were to plot the cross-sectional effects along the transition, they would inherit the differences in supply and demand shocks observed in Figure 3.

**Figure 6.** Cross-sectional quantity response for different elasticities

(a) Supply Shock (b) Demand Shock




*Note:* This figure shows the distribution of non-commodity sectoral output response, for different values of production elasticities, to a 1% increase in commodity prices due to a supply shock $\varepsilon_{N+1,t}$ (left panel) and due to a demand shock $\varepsilon_{D,t}$ (right panel).

case, we use the estimated elasticities in Table 4.

Panel a) of Figure 7 plots the output response to a commodity price shock in the y-axis and the downstream centrality of a given sector to the commodity sector in the x-axis. As expected, relatively more labor-intensive sectors, after accounting for indirect linkages, exhibit a lower quantity response as their price reacts more to commodity price changes. Hence, there is a negative correlation between quantity and downstream exposures.[16]

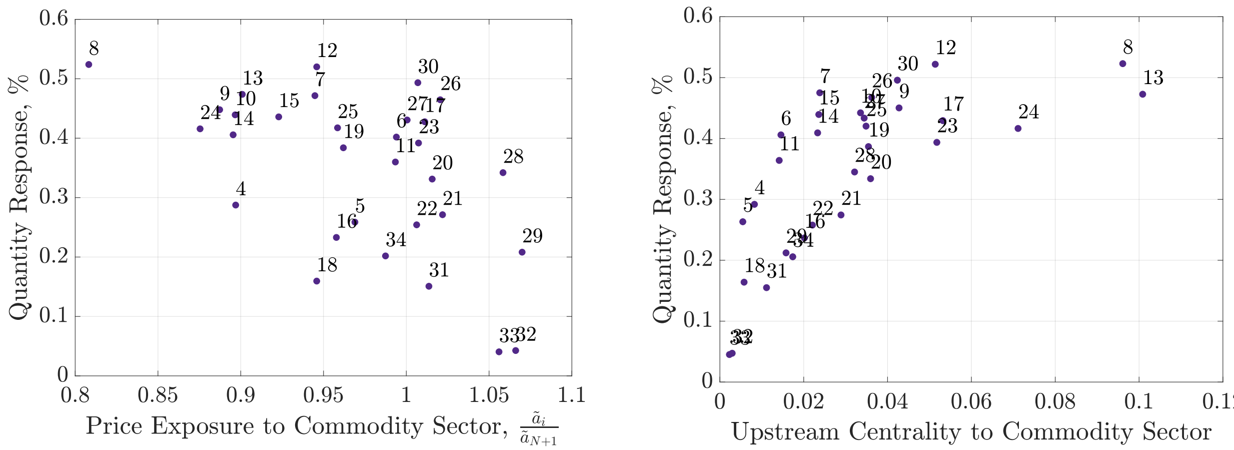
On the other hand, the right panel of Figure 7 shows the positive relationship between the sectoral upstream connection to the commodity sector and the output change of that sector. For example, basic metals and fabricated metal (sector 12) and renting of m&eq and other business activities (sector 30) are sectors for which the mining sector is an important buyer directly and indirectly via the production network. These sectors also have some of the most significant output increases after a positive commodity price shock.

**Centrality of commodities compared to other sectors.** Here we return to our motivation to show that commodity sectors are important in the network structure. We

[16]We chose not to plot the price counterpart of these figures in the main text as the model has a sharp prediction for it. Namely, when plotting the downstream exposure and the price response, they should coincide exactly, while there should be no correlation between upstream centrality and the price response. For completeness, we provide this plot in Figure B10 of the appendix. The figure confirms the tractable model predictions.

**Figure 7.** Quantity response, upstream and downstream exposure: Cross-Section

(a) Downstream (b) Upstream




*Note:* This figure plots the relationship between the non-commodity sectors' downstream (left panel) and upstream (right panel) exposure to the commodity sector (x-axis) and the non-commodity sectoral output response to a 1% increase in commodity prices (y-axis) due to a supply shock $\varepsilon_{N+1,t}$.

conduct two additional exercises, both of which involve re-estimating all plots of this section but using different sectors. The first exercise is to consider as a commodity sector one that exhibits the same size as the mining sector in Australia, which is roughly 8.7% of GDP. To that end, we pick the basic metals and fabricated metal sector, whose size is 8.7% of GDP, identical to that of the mining sector. Its inverse network-adjusted labor share ($\frac{1}{\tilde{a}_{N+1}}$), on the other hand, is about 6% larger than that of the mining sector—this is, the mining sector is more labor intensive than the base metal sector once we adjust for indirect network linkages. Hence, we expect a shock to base metals to have larger implications on aggregate prices and the real wage.[17] Figure B18 confirms the predictions of the model. The aggregate effect on the price index and the real wage are larger for the base metal sector than for the mining in Figure 3. Figure B17 depicts the aggregate behavior of other aggregates. While the qualitative results are similar to that of Figure B9, they differ quantitatively. For example, aggregate consumption rises by around 0.06% on impact when we shock the price of base metals, but it is ≈ 0.16% when we shock the mining sector, even though they are the same

[17] Proposition 1 indicates that the relevant statistic for downstream propagation of commodity prices on non-commodity prices is $\frac{\tilde{a}_i}{\tilde{a}_{N+1}}$, with $\tilde{a}_i = \sum_{j=1}^{N+1} \Psi_{ij} a_j$. In the data $1/\tilde{a}_{metals} = 1.06 \cdot 1/\tilde{a}_{mining}$. Note that we did not use the base metal sector as one of our commodity sectors because none of the economies in our sample exports significant amounts of base metals.

size.

Figure B19 to Figure B21 explores the cross-sectional implication of a shock to base metals. While patterns are qualitatively similar, they are quantitatively different and have different implications for different sectors. For example, the quantity response of electricity, gas, and water supply (sector 17) in response to a price shock in mining is around 0.43% but increases to 0.53% with the same sized shock to the base metals price (see Figure B21). Again, this underscores the quantitative importance of network linkages: even though sectors have roughly the same size, their aggregate and cross-sectional implications can quantitatively differ.

The second exercise considers a sector with a similar network-adjusted labor share as the mining sector and a smaller size. To that end, we picked the wood and cork sector, which exhibits a network-adjusted labor share about the same as the mining sector. The size of this sector, however, is around 1.3% of GDP. These two sectors provide a nice balance of examples of the forces at play. The results in Figure B13 show that the aggregate effects of a price shock to the wood and cork sector are similar to the effects of a shock to the mining sector. Figure B12 plots the dynamic of other aggregate variables when shocking the wood and cork sector price. In contrast to the case of base metals, the responses of trade balance, consumption, and the current account are qualitatively and quantitatively similar when we shock the wood and cork sector price. This suggests that what matters for aggregate outcomes is the network-adjusted labor share of the shocked sector and not its size.

Figure B14 to Figure B16 shows the cross-sectional implications of a shock to the wood and cork sector price. These cross-sectional responses are also qualitatively similar to our baseline model but can differ quantitatively, often substantially. For example, in response to a wood and cork sector price shock, the machinery sector (sector 13) quantity response on impact is around 0.41%, while it is 0.47% when the shocked sector price is mining, a 0.06 percentage point difference. Here, the quantity response of the machinery sector reacts more when the shock is to the mining sector price. In contrast, the inland transport sector (sector 23) quantity response is 0.42% when we shock the wood and cork sector price, and it is 0.39% when we shock the mining sector price (see Figure B16). In this case, gross output in the inland transport sector reacts more when the shock is to the wood and cork sector price.

Overall, these exercises suggest that sectoral size does not determine the aggregate and cross-sectional responses of quantities and prices. Rather, it is the production network structure economy, as our tractable model has already highlighted.

# 4 Conclusion

We study how sectoral commodity price fluctuations propagate through domestic production networks in small open economies. We provide a tractable model to highlight the key mechanisms of propagation of commodity prices on non-commodity sectors' prices and gross output. Evidence for a sample of 9 small open economies and 31 non-commodity sectors, provides support for the role of domestic production networks in propagating commodity price shocks.

Empirically, we find that the gross output of non-commodity upstream sectors, those sectors supplying intermediate inputs to commodity sectors, largely respond to commodity price shocks. For this mechanism to be at play, elasticities of substitution among inputs play a key role. In contrast, we find evidence of muted downstream propagation on quantities, to those buying intermediate inputs from commodity sectors. On the price front, we provide evidence of downstream propagation of commodity prices to non-commodity sectoral prices but nil upstream propagation.

We then embed this simple model into a dynamic quantitative small open economy model to assess whether the mechanisms from the simple model survive in a more complex environment. We show that the intuition from the simple model carries through the dynamic quantitative model. Our counterfactual experiments illustrate the importance of the domestic network structure in shaping the aggregate effects and the sectoral effects of commodity price shocks. We show that the size of the commodity sector is not a sufficient statistic to gauge the aggregate effects of commodity price shocks. The network-adjusted labor share of the commodity sector is the key to understanding the response of the real wage to a commodity price shock.

All in all, our results highlight the importance of taking into account the structure of the domestic production network to understand the propagation and amplification of commodity price fluctuations throughout the economy.

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# ONLINE APPENDIX

## A Data sources and Definitions

### Commodity Prices

The disaggregated commodity price data, and the export shares used to construct price indexes, are obtained from Fernández et al. (2018). We constructed the sectoral indexes of commodity prices for different countries as follows.

(i) We use the export data Fernández et al. (2018) and calculate, for each country, the share of each commodity good in its sectoral group, be it agriculture, mining, or food sectors. Then, we multiply each sector-country weight by the monthly commodity price.

(ii) The outcome from step (i) is a matrix of country-specific monthly commodity price index that we deflate using the US Consumer Price Index (CPI).

(iii) We take the average across months within each quarter by year.

### Input-Output Table Database

Our main database is the World Input-Output database (Timmer et al., 2015), release 2013. It provides information on intersectoral and cross-country final and intermediate flows for 40 countries and 35 sectors classified according to the International Standard Industrial Classification Revision 3 (ISIC Rev. 3). These tables match the 1993 version of the SNA. We use the sectoral data on quantities (gross output, value-added, number of employees, and capital) and price indexes for the period 1995-2011(2009) in the National IO tables. The sample of small open economies with data on commodity prices and WIOD input-output data includes the following countries: Australia, Bulgaria, Brazil, Canada, Denmark, India, Lithuania, Mexico, and Russia.

This dataset is freely available here https://www.rug.nl/ggdc/valuechain/wiod/wiod-2013-release.

**Commodity Linkages Data**

To measure sectoral linkages to the *commodity sector* we use detailed information on each country's commodity bundle composition from Fernández et al. (2018). There is a total of 44 commodities classified according to the Harmonized System (HS) 1992 – 4 digits. We separate commodities into 3 groups: Agriculture, Hunting, Forestry, and Fishing; Mining and Quarrying; and Food Products, Beverages, and Tobacco.

# B Additional Tables and Figures

## B.1 Tables

**Table B5.** Sectors in WIOD Database

| Sector Number | Sector Name |
|---|---|
| 1 | Agriculture, Hunting, Forestry and Fishing |
| 2 | Mining and Quarrying |
| 3 | Food, Beverages, and Tobacco |
| 4 | Textiles and Textile Products |
| 5 | Leather, Leather, and Footwear |
| 6 | Wood and Products of Wood and Cork |
| 7 | Pulp, Paper, Paper, Printing, and Publishing |
| 8 | Coke, Refined Petroleum and Nuclear Fuel |
| 9 | Chemicals and Chemical Products |
| 10 | Rubber and Plastics |
| 11 | Other Non-Metallic Mineral |
| 12 | Basic Metals and Fabricated Metal |
| 13 | Machinery, Nec |
| 14 | Electrical and Optical Equipment |
| 15 | Transport Equipment |
| 16 | Manufacturing, Nec; Recycling |
| 17 | Electricity, Gas and Water Supply |
| 18 | Construction |
| 19 | Sale, Maintenance and Repair of Motor Vehicles and Motorcycles |
| 20 | Wholesale Trade and Commission Trade |
| 21 | Retail Trade |
| 22 | Hotels and Restaurants |
| 23 | Inland Transport |
| 24 | Water Transport |
| 25 | Air Transport |
| 26 | Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies |
| 27 | Post and Telecommunications |
| 28 | Financial Intermediation |
| 29 | Real Estate Activities |
| 30 | Renting of M&Eq and Other Business Activities |
| 31 | Public Admin and Defense; Compulsory Social Security |
| 32 | Education |
| 33 | Health and Social Work |
| 34 | Other Community, Social and Personal Services |

**Table B6.** Commodities and WIOD Industries.

| Commodity | HS Code | Industry |
|---|---|---|
| Beef | 201 | Agriculture, hunting, forestry and fishing |
| Pork | 203 | Agriculture, hunting, forestry and fishing |
| Lamb | 204 | Agriculture, hunting, forestry and fishing |
| Chicken | 207 | Agriculture, hunting, forestry and fishing |
| Fish | 301 | Agriculture, hunting, forestry and fishing |
| Fish Meal | 304 | Agriculture, hunting, forestry and fishing |
| Shrimp | 306 | Agriculture, hunting, forestry and fishing |
| Bananas | 803 | Agriculture, hunting, forestry and fishing |
| Coffee | 901 | Agriculture, hunting, forestry and fishing |
| Tea | 902 | Agriculture, hunting, forestry and fishing |
| Wheat | 1001 | Agriculture, hunting, forestry and fishing |
| Barley | 1003 | Agriculture, hunting, forestry and fishing |
| Corn | 1005 | Agriculture, hunting, forestry and fishing |
| Rice | 1006 | Agriculture, hunting, forestry and fishing |
| Soybeans | 1201 | Agriculture, hunting, forestry and fishing |
| Groundnuts | 1202 | Agriculture, hunting, forestry and fishing |
| Wool | 1505 | Agriculture, hunting, forestry, and fishing |
| Sugar | 1701 | Agriculture, hunting, forestry and fishing |
| Cocoa | 1801 | Agriculture, hunting, forestry and fishing |
| Natural Rubber | 4001 | Agriculture, hunting, forestry, and fishing |
| Hides | 4101 | Agriculture, hunting, forestry and fishing |
| Hard Log | 4401 | Agriculture, hunting, forestry and fishing |
| Soft Log | 4403 | Agriculture, hunting, forestry and fishing |
| Hard Swan | 4407 | Agriculture, hunting, forestry and fishing |
| Soft Swan | 4408 | Agriculture, hunting, forestry and fishing |
| Cotton | 5201 | Agriculture, hunting, forestry and fishing |
| Iron | 2601 | Mining and quarrying |
| Copper | 2603 | Mining and quarrying |
| Nickel | 2604 | Mining and quarrying |
| Aluminum | 2606 | Mining and quarrying |
| Lead | 2607 | Mining and quarrying |
| Zinc | 2608 | Mining and quarrying |
| Tin | 2609 | Mining and quarrying |
| Coal | 2701 | Mining and quarrying |
| Crude Oil | 2709 | Mining and quarrying |
| NatGas | 2711 | Mining and quarrying |
| Uranium | 2844 | Mining and quarrying |
| Gold | 7108 | Mining and quarrying |
| Soybean Meal | 1208 | Food products, beverages and tobacco |
| Soy Oil | 1507 | Food products, beverages and tobacco |
| Olive Oil | 1509 | Food products, beverages and tobacco |
| Palm Oil | 1511 | Food products, beverages and tobacco |
| Sun Oil | 1512 | Food products, beverages and tobacco |
| Coconut Oil | 1513 | Food products, beverages and tobacco |

**Table B7.** Network Effects of Commodity Price Changes on Non-Commodity Sectors in Log Differences (previous period)

| | Panel (a): Δ log(Quantity) | | | Panel (b): Δ log(Price) | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $\Delta\tilde{P}_{ict}^{Up}$ | 0.1460 | 0.2709** | 0.2936** | 0.2510 | 0.1835 | 0.0557 |
| | (0.1235) | (0.1226) | (0.1382) | (0.2472) | (0.2519) | (0.1076) |
| $\Delta\tilde{P}_{ict}^{Down}$ | 0.0943 | 0.0671 | 0.1879** | -0.1374 | -0.0577 | 0.1366** |
| | (0.0617) | (0.0608) | (0.0861) | (0.1236) | (0.1258) | (0.0666) |
| Observations | 3627 | 3627 | 3627 | 3627 | 3627 | 3627 |
| Within $R^2$ | 0.115 | 0.053 | 0.035 | 0.178 | 0.108 | 0.014 |
| Year F.E. | Yes | Yes | Yes | Yes | Yes | Yes |
| Country × Sector F.E. | | Yes | Yes | | Yes | Yes |
| Country × Year F.E. | | | Yes | | | Yes |

*Note:* This table presents OLS regressions using sectoral log quantity (columns 1 to 3) and log price index (columns 4 to 6) as the dependent variable. The independent variables also include one lag of the dependent variable. Newey-West standard errors in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

**Table B8.** Network Effects of Commodity Price Changes on Non-Commodity Sectors IV approach

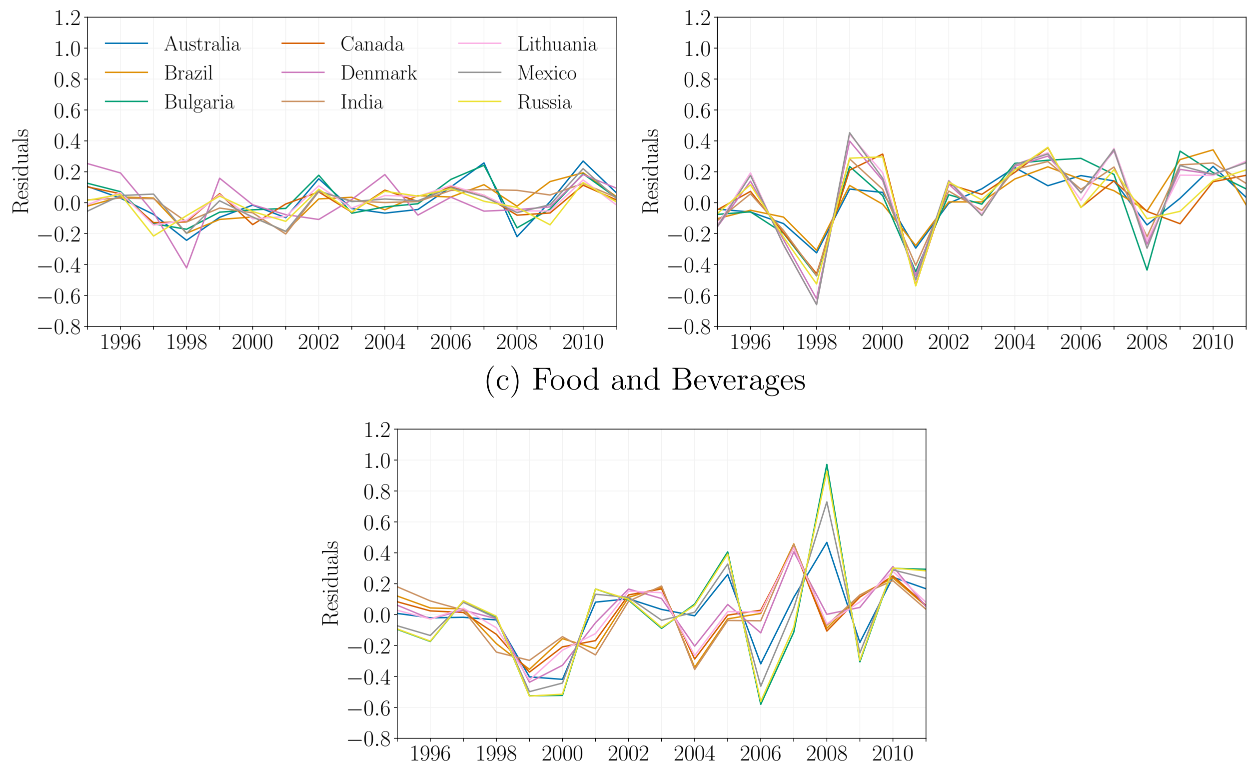
| | Panel (a): Quantities | | | Panel (b): Prices | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $\Delta\tilde{P}_{ict}^{Up}$ | 2.3192** | 3.7298*** | 1.7747** | -9.8643*** | -2.8672* | 1.2513** |
| | (1.0806) | (1.0755) | (0.7882) | (2.4796) | (1.6034) | (0.6221) |
| $\Delta\tilde{P}_{ict}^{Down}$ | 0.4534* | 0.4020 | 0.3225 | 0.1505 | 0.9656** | 0.4909*** |
| | (0.2636) | (0.2788) | (0.2088) | (0.6042) | (0.4110) | (0.1655) |
| Observations | 3906 | 3906 | 3906 | 3906 | 3906 | 3906 |
| Within $R^2$ | 0.925 | 0.752 | 0.780 | 0.946 | 0.732 | 0.707 |
| Year F.E. | Yes | Yes | Yes | Yes | Yes | Yes |
| Country × Sector F.E. | | Yes | Yes | | Yes | Yes |
| Country × Year F.E. | | | Yes | | | Yes |
| First-stage F stat. | 9.78 | 9.26 | 23.13 | 9.83 | 8.67 | 23.35 |

*Note:* This table presents instrumental variable regressions using sectoral log quantity (columns 1 to 3) and log price index (columns 4 to 6) as the dependent variable. The independent variables also include one lag of the dependent variable and one lag of the upstream and downstream measures. We use three sets of shocks to construct upstream and downstream instruments. We use oil supply shocks from Baumeister and Hamilton (2019) and Känzig (2021) and Harvest shocks from Winne and Peersman (2021). Newey-West HAC standard errors are in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. The first stage F-statistic for weak identification test is the Kleibergen-Paap rk Wald F statitic for multiple endogenous regressors. The Stock-Yogo weak ID critical values are 15.72, 9.48, and 6.08 for 5%, 10%, and 20% maximal IV relative bias, respectively.

## B.2 Figures

**Figure B8.** Commodity Price Changes

(a) Agriculture and Forestry (b) Mining and Quarrying

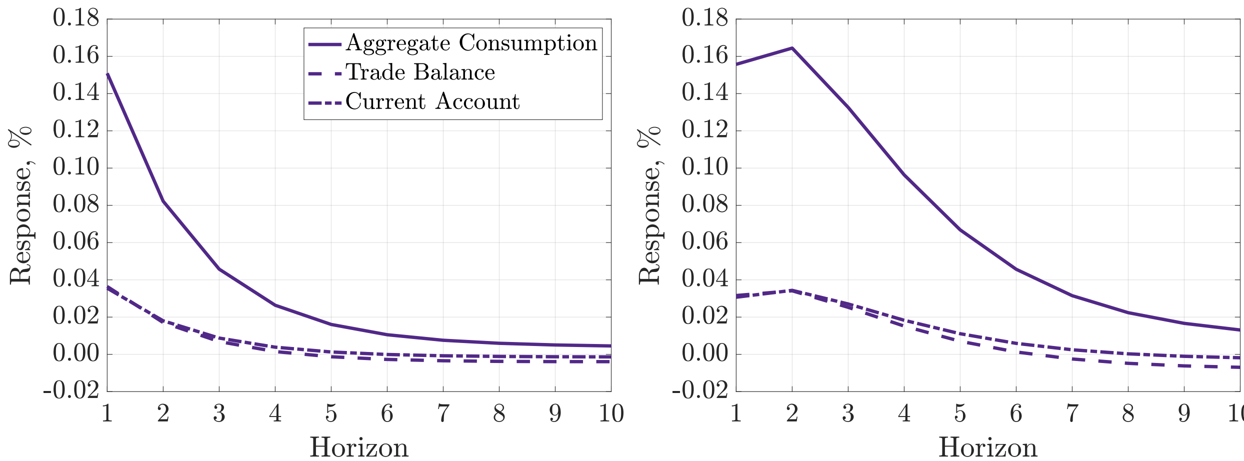



(c) Food and Beverages

*Note:* This figure plots the residuals from the AR(1) specification for each commodity price in our sample of 9 countries in the WIOD database.

**Figure B9.** Aggregates: Mining Sector as Commodity

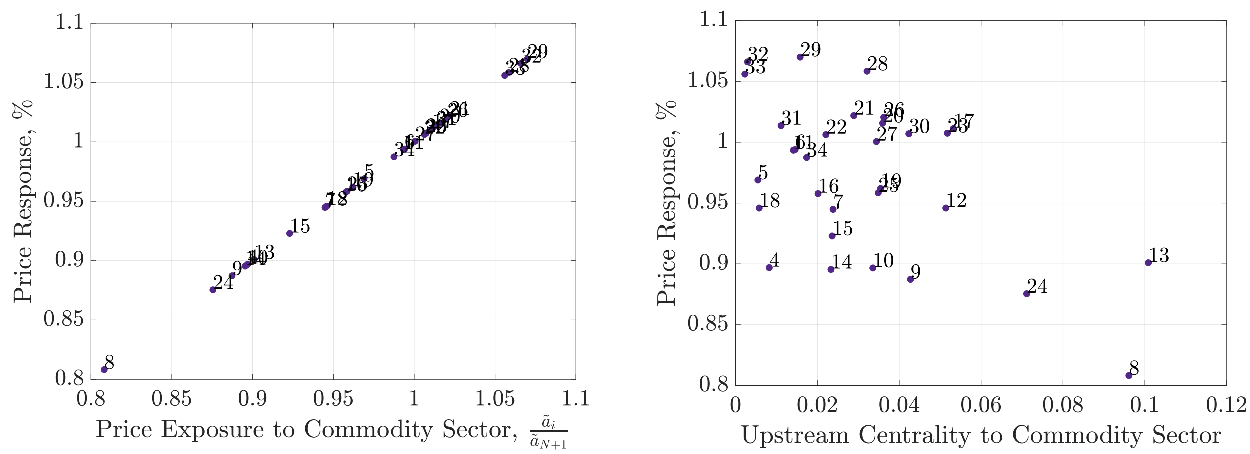
(a) Supply Shock (b) Demand Shock




*Note:* This figure depicts the response of aggregate consumption, trade balance (as a fraction of GDP), and the current account (also as a fraction of GDP) in response to a 1% increase in the mining sector price in Australia.

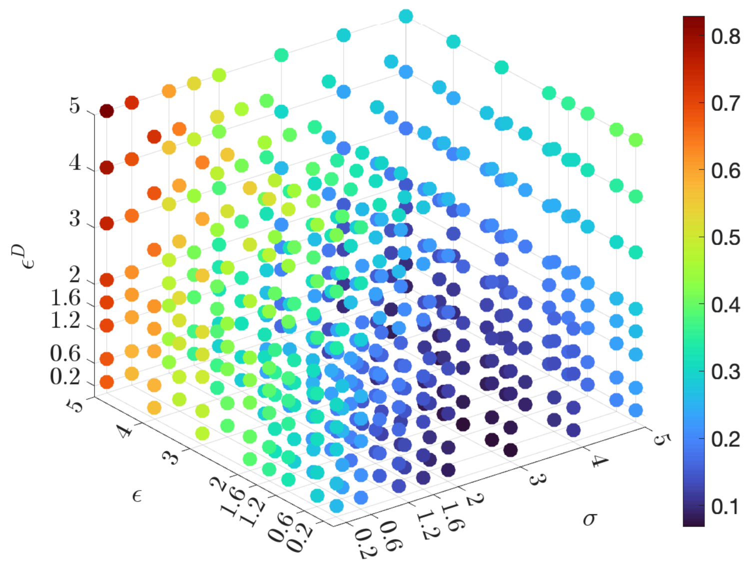
**Figure B10.** Price responses, upstream and downstream exposure: Cross-Section

(a) Downstream (b) Upstream




*Note:* This figure plots the relationship between the non-commodity sectors' downstream (left panel) and upstream (right panel) exposure to the commodity sector (x-axis) and the non-commodity sectoral price response to a 1% increase in commodity prices (y-axis) due to a supply shock $\varepsilon_{N+1,t}$.

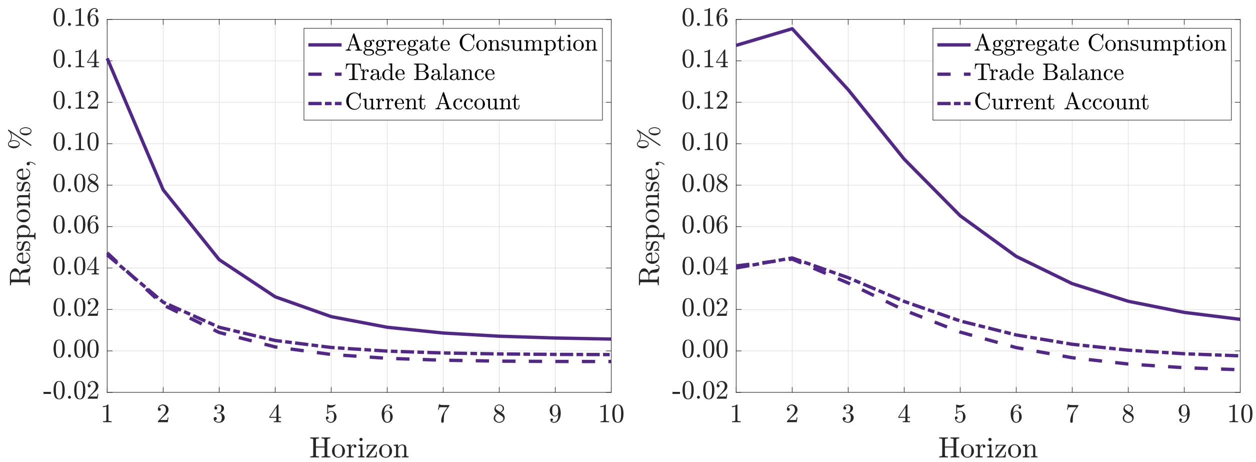
**Figure B11.** Loss function for different elasticity combinations




*Note:* This figure plots the loss function between the model and data estimated parameters for different elasticities. The x-axis is the elasticity between value-added and intermediates. The y-axis is the elasticity between imported and domestic intermediates. The z-axis is the elasticity among domestic intermediates. The color scale plots the loss-function values for each combination. Lower values represent a better fit between the model and data. We estimate the model and run the same regression as in our empirical section. Our loss function takes the form $\mathcal{L}(\sigma, \epsilon, \epsilon^D) = (\phi_Q^{Up}(data) - \phi_Q^{Up}(\sigma, \epsilon, \epsilon_D))^2 + (\phi_Q^{Down}(data) - \phi_Q^{Down}(\sigma, \epsilon, \epsilon_D))^2$. $\phi_Q^{up}(data), \phi_Q^{Down}(data)$ are the estimated coefficients from Equation (7) in Table 3. We take column (3) as our preferred specification.

**Figure B12.** Aggregates: Wood and Cork Sector as Commodity

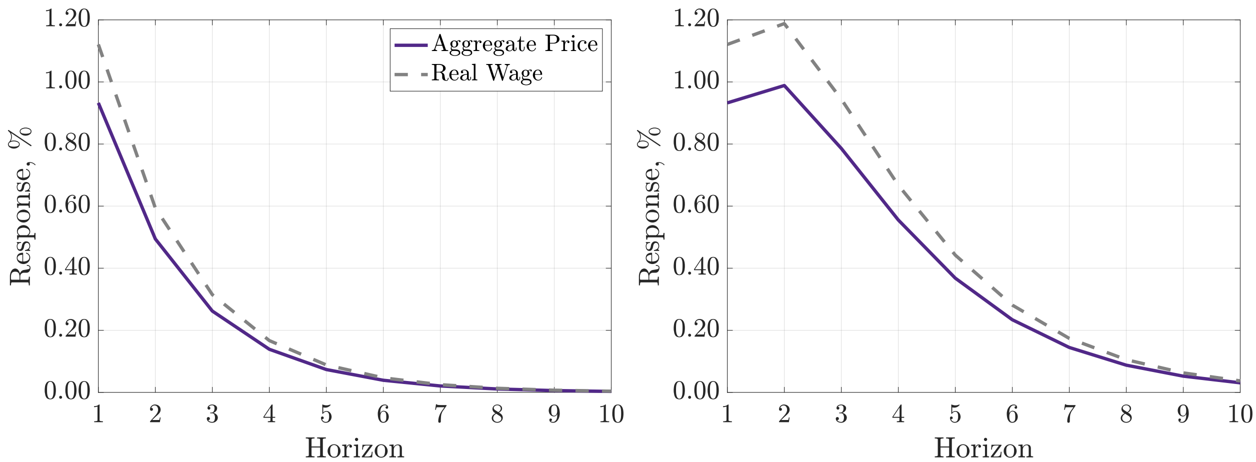
(a) Supply Shock (b) Demand Shock




*Note:* This figure depicts the response of aggregate consumption, trade balance (as a fraction of GDP), and the current account (also as a fraction of GDP) in response to a 1% increase in the wood and cork sector price in Australia.

**Figure B13.** Real wage and aggregate price: Wood and Cork Sector as Commodity

(a) Supply Shock ($\varepsilon_{N+1,t}$) (b) Demand Shock ($\varepsilon_{D,t}$)

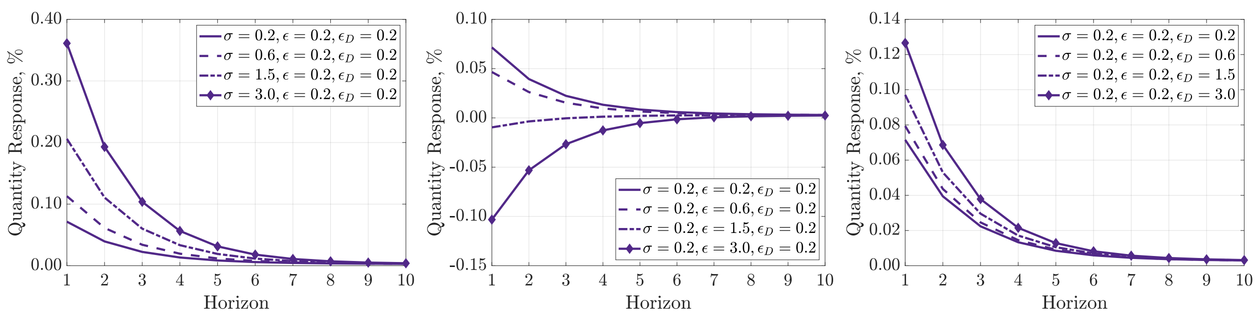



*Note:* This figure shows the response of the aggregate price index and the real wage, in units of the importable good price, to a 1% increase in the wood and cork sector price due to a supply shock $\varepsilon_{N+1,t}$ (left panel) and due to a demand shock $\varepsilon_{D,t}$ (right panel).

**Figure B14.** Role of Production Elasticities: Wood and Cork Sector as Commodity

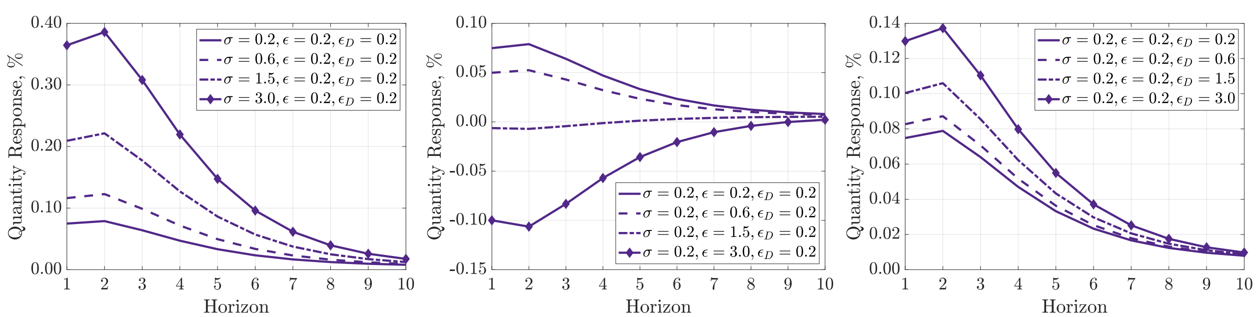
Panel I: Supply Shocks

(a) Role of $\sigma$ (b) Role of $\epsilon$ (c) Role of $\epsilon^D$




Panel II: Demand Shocks

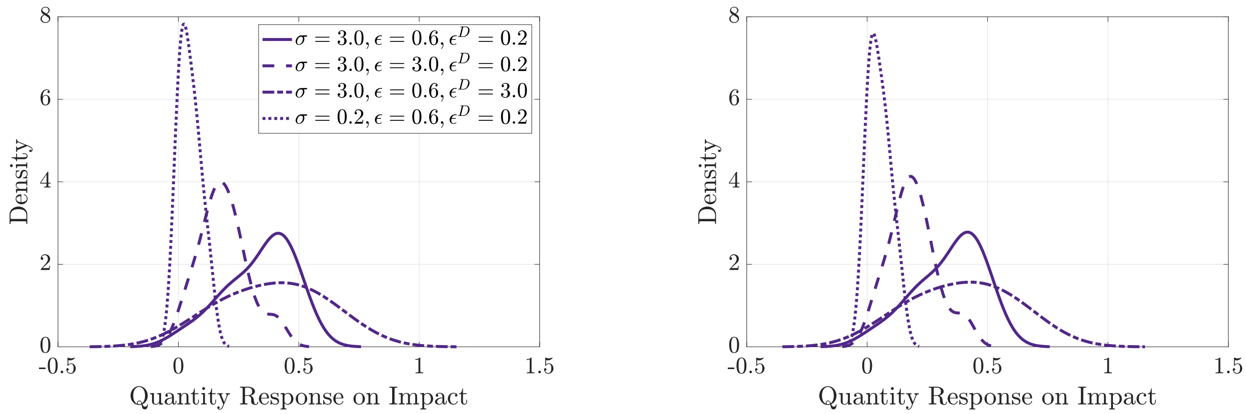
(a) Role of $\sigma$ (b) Role of $\epsilon$ (c) Role of $\epsilon^D$




*Note:* This figure shows the average output response of non-commodity sectors to a 1% increase in the wood and cork sector prices, for different values of production elasticities, due to a supply shock $\varepsilon_{N+1,t}$ (Panel I) and due to a demand shock $\varepsilon_{D,t}$ (Panel II). $\sigma$ is the elasticity between labor and intermediate inputs. $\epsilon$ is the elasticity between domestic and imported intermediate inputs. $\epsilon^D$ is the elasticity among domestic intermediate inputs.

**Figure B15.** Cross-sectional quantity response for different elasticities: Wood and Cork Sector as Commodity

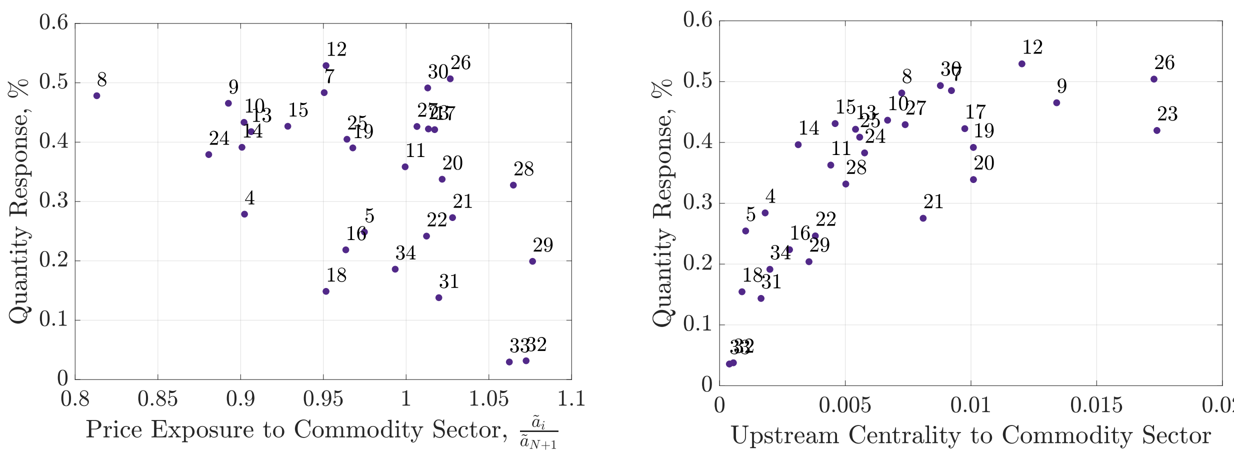
(a) Supply Shock (b) Demand Shock




*Note:* This figure shows the distribution of non-commodity sectoral output response, for different values of production elasticities, to a 1% increase in the wood and cork sector price due to a supply shock $\varepsilon_{N+1,t}$ (left panel) and due to a demand shock $\varepsilon_{D,t}$ (right panel).

**Figure B16.** Quantity response, upstream and downstream exposure: Cross-Section, Wood and Cork Sector as Commodity

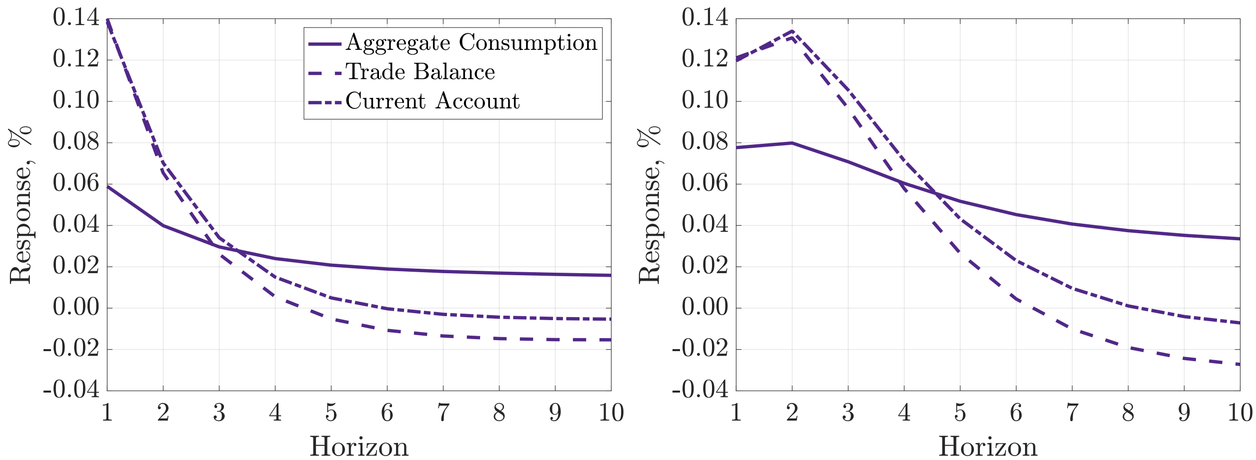
(a) Downstream (b) Upstream




*Note:* This figure plots the relationship between the non-commodity sectors' downstream (left panel) and upstream (right panel) exposure to the commodity sector (x-axis) and the non-commodity sectoral output response to a 1% increase in the wood and cork sector price (y-axis) due to a supply shock $\varepsilon_{N+1,t}$.

**Figure B17.** Aggregates: Base Metals as Commodity

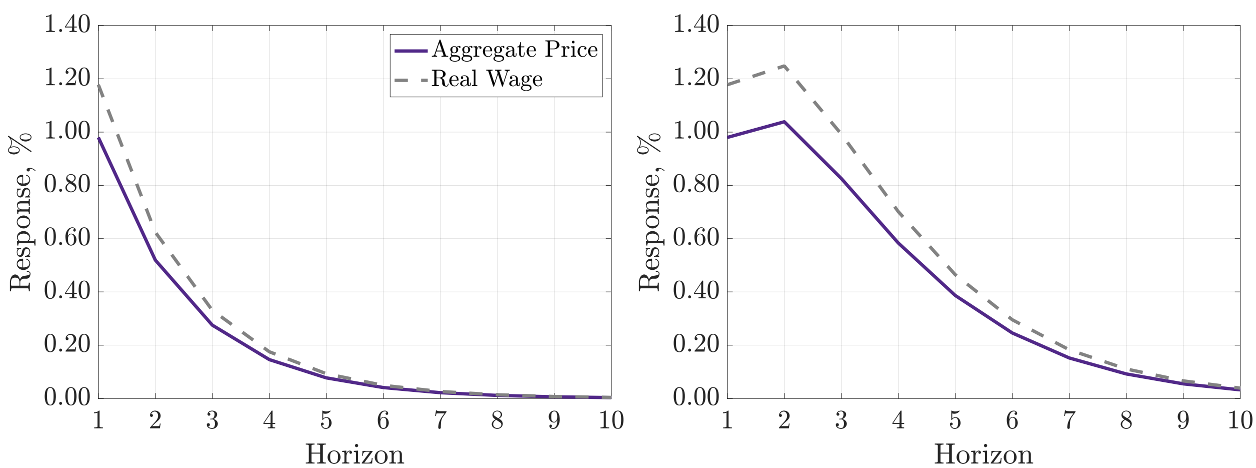
(a) Supply Shock (b) Demand Shock




*Note:* This figure depicts the response of aggregate consumption, trade balance (as a fraction of GDP), and the current account (also as a fraction of GDP) in response to a 1% increase in the base metals sector price in Australia.

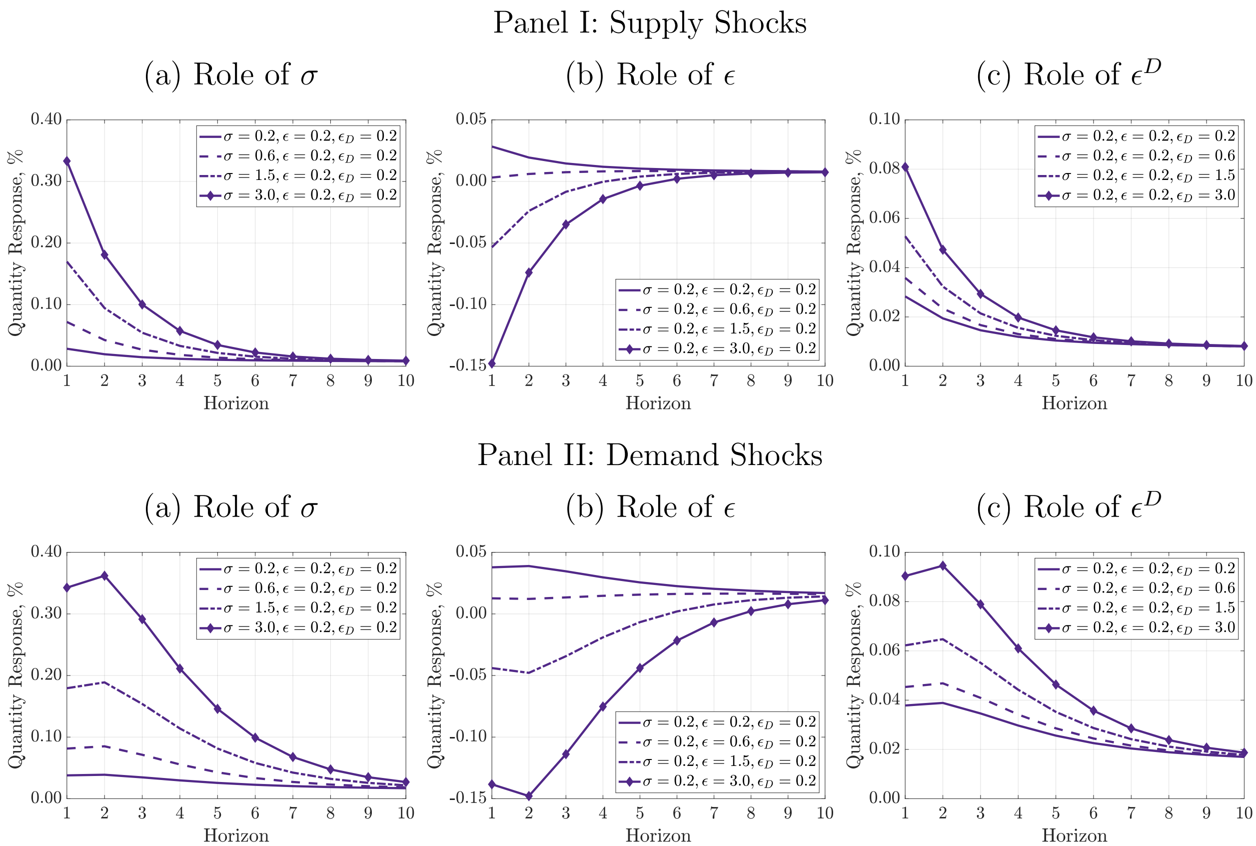
**Figure B18.** Real wage and aggregate price: Base Metals Sector as Commodity

(a) Supply Shock ($\varepsilon_{N+1,t}$) (b) Demand Shock ($\varepsilon_{D,t}$)




*Note:* This figure shows the response of the aggregate price index and the real wage, in units of the importable good price, to a 1% increase in the mining sector price due to a supply shock $\varepsilon_{N+1,t}$ (left panel) and due to a demand shock $\varepsilon_{D,t}$ (right panel).

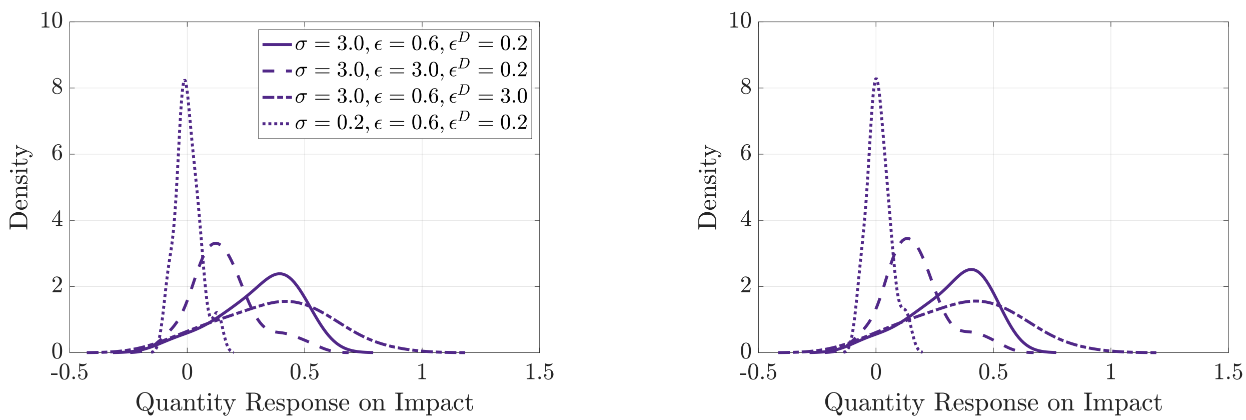
**Figure B19.** Role of Production Elasticities: Base Metals Sector as Commodity




*Note:* This figure shows the average output response of non-commodity sectors to a 1% increase in base metals sector prices, for different values of production elasticities, due to a supply shock $\varepsilon_{N+1,t}$ (Panel I) and due to a demand shock $\varepsilon_{D,t}$ (Panel II). $\sigma$ is the elasticity between labor and intermediate inputs. $\epsilon$ is the elasticity between domestic and imported intermediate inputs. $\epsilon^D$ is the elasticity among domestic intermediate inputs.

**Figure B20.** Cross-sectional quantity response for different elasticities: Base Metals Sector as Commodity

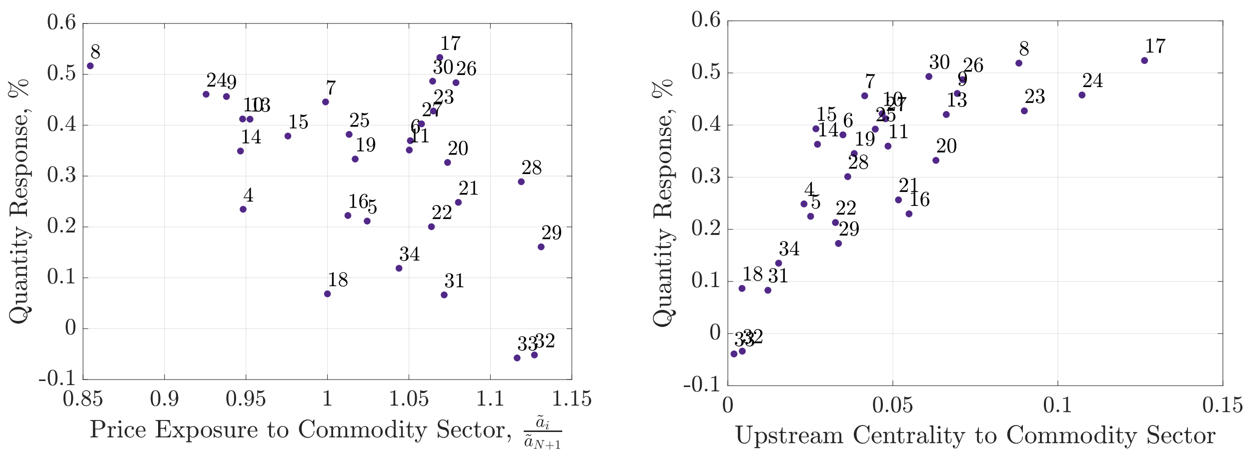
(a) Supply Shock (b) Demand Shock




*Note:* This figure shows the distribution of non-commodity sectoral output response, for different values of production elasticities, to a 1% increase in the base metals sector price due to a supply shock $\varepsilon_{N+1,t}$ (left panel) and due to a demand shock $\varepsilon_{D,t}$ (right panel).

**Figure B21.** Quantity response, upstream and downstream exposure: Cross-Section, Base Metals Sector as Commodity

(a) Downstream (b) Upstream




*Note:* This figure plots the relationship between the non-commodity sectors' downstream (left panel) and upstream (right panel) exposure to the commodity sector (x-axis) and the non-commodity sectoral output response to a 1% increase in base metals sector prices (y-axis) due to a supply shock $\varepsilon_{N+1,t}$.

# C Static Model Details

This appendix provides details on the model used to motivate the results in Section 2.1.

## C.1 Representative Consumer

There is a representative consumer. She owns the labor and supplies it inelastically. We denote this inelastic supply as $\bar{L}$. She has a utility function: $U(\boldsymbol{C}) = U(\{C_i\}_{i=1}^{N+1}, C_M)$. This includes consumption on domestically produced goods ($\{C_i\}_{i=1}^{N+1}$) and imported goods ($C_M$). The utility function satisfies typical regularity conditions and is homogeneous of degree one in its arguments.

Taking good and factor prices ($\{P_i\}_{i=1}^{N+1}, P_M, W$) as given the representative consumer solves the following program

$$\max_{\boldsymbol{C}} \quad U(\boldsymbol{C}) \quad \text{s.t.} \quad \sum_{i=1}^{N+1} P_i C_i + P_M C_M \leq W\bar{L} = E, \tag{C.1}$$

where $E$ is *total expenditure*. The solution to this program delivers consumption schedules that are a function of prices and the additional income source i.e. $\boldsymbol{C} = \boldsymbol{C}(\boldsymbol{P}_D, P_M, W; \bar{L})$.

## C.2 Production

Gross output in sector $i$, $Q_i$, is produced according to the following production function

$$Q_i = Z_i F_i(L_i, \{M_{ij}\}_{j=1}^{N+1}, M_{iM}), \tag{C.2}$$

where $Z_i$ is a producer-specific shock, $F_i(.)$ is a constant-returns to scale function. We use a subscript $i$ to index this production function to allow for the possibility of different production functions across producers. $L_i$ is the labor demand of producer $i$. We label $L_i$ as labor but interpret it more broadly as a value-added construct. $M_{ij}$ is intermediate demand for good $j = 1, 2, ..., N+1$ by producer $i$. $M_{iM}$ is the demand for imported intermediate inputs by producer $i$.

Cost-minimization implies that the marginal cost of production, $MC_i$ can be written as

$$P_i = MC_i(W, \boldsymbol{P}_D, P_M; Z_i). \tag{C.3}$$

The marginal cost is a function of the wage rate, the price of all goods $\boldsymbol{P} = (P_1, P_2, ..., P_{N+1})$, the imported input price, $P_M$, and its own productivity, $Z_i$. Its equality to its good price, $P_i$, then follows from profit maximization.

To get conditional demands for labor and each intermediate input, we differentiate the marginal cost function

$$L_i = Q_i \frac{\partial MC_i(.)}{\partial W}, \quad M_{ij} = Q_i \frac{\partial MC_i(.)}{\partial P_j}, \quad M_{iM} = Q_i \frac{\partial MC_i(.)}{\partial P_M}.$$

## C.3 Equilibrium

The following conditions close the model

$$Q_i = C_i + \sum_{j=1}^{N+1} M_{ji} \qquad \forall i = 1, ..., N, \tag{C.4}$$

$$Q_{N+1} = C_{N+1} + X_{N+1} + \sum_{j=1}^{N+1} M_{j,N+1}, \tag{C.5}$$

$$\bar{L} = \sum_{i=1}^{N+1} L_i, \tag{C.6}$$

$$WL = \sum_{i=1}^{N+1} P_i C_i + P_M C_M, \tag{C.7}$$

Equation (C.4) is the market clearing condition in non-tradable goods markets. Equation (C.5) is the market clearing for the commodity sector. We follow Adão et al. (2022) and assume that both $(P^*_{N+1}, X_{N+1})$ are exogenous objects. Since we are interested in the effect of commodity price changes on cross-sectional responses within a small open economy, this is sufficient to highlight the main mechanisms through which commodity prices affect those responses. Equation (C.6) is the labor market clearing condition. Finally, Equation (C.7) is the consumer's budget constraint. Note that equations (C.4) to (C.7) combined implies balanced trade: $P_{N+1}X_{N+1} = P_M \left( C_M + \sum_{i=1}^{N+1} M_{iM} \right)$.

# D Dynamic Model Details

This appendix provides more details on the quantitative model in Section 3.

## D.1 Representative Consumer

**Intertemporal.** The optimality conditions imply the Euler equation

$$\frac{C_t^{-\rho}}{P_t} = \beta \frac{(1+r)}{\left(1 + \frac{\partial g(B_t)}{\partial B_t}\right)} \mathbb{E}_t \left( \frac{C_{t+1}^{-\rho}}{P_{t+1}} \frac{P_{N,t+1}}{P_{N,t}} \right). \tag{D.1}$$

Coupled with the appropriate transversality condition and an initial condition for the net foreign asset position ($B_{-1}$), equation (D.1) delivers the solution paths $\{C_t, B_t\}_{t=0}^{\infty}$.

**Intratemporal.** Solving this problem in Equation (12) implies that the consumer spends a constant fraction of its expenditure on each good

$$C_i = \beta_i \frac{PC}{P_i} \qquad \text{for all } i = 1, 2, ..., N+1, \tag{D.2}$$

$$C_M = \beta_M \frac{PC}{P_M}, \tag{D.3}$$

where

$$P = \left( \prod_{i=1}^{N+1} P_i^{\beta_i} \right) P_M^{\beta_M} \tag{D.4}$$

is the ideal price index of the household.

## D.2 Production

We can solve these problems separately. The upper layer minimizes costs over labor and intermediate inputs. The mid-layer minimizes costs over domestic and imported intermediate inputs. Finally, the bottom layer minimizes costs across domestic intermediate inputs. We state each of these problems for completeness below.

$$\frac{MC_iQ_i}{\bar{M}C_i\bar{Q}_i} = \min_{\{L_i,M_i\}} \quad \frac{W_iL_i}{\bar{W}_i\bar{L}_i} + \frac{P_i^M M_i}{\bar{P}_i^M \bar{M}_i} \quad \text{s.t} \quad \frac{Q_i}{\bar{Q}_i} \geq \tilde{Q}_i \tag{D.5}$$

$$\frac{P_i^M M_i}{\bar{P}_i^M \bar{M}_i} = \min_{\{M_i^D, M_{iM}\}} \quad \frac{P_i^D M_i^D}{\bar{P}_i^D \bar{M}_i^D} + \frac{P_M M_{iM}}{\bar{P}_M \bar{M}_{iM}} \quad \text{s.t} \quad \frac{M_i}{\bar{M}_i} \geq \tilde{M}_i \tag{D.6}$$

$$\frac{P_i^D M_i^D}{\bar{P}_i^D \bar{M}_i^D} = \min_{\{M_{ij}\}_{j=1}^{N+1}} \quad \sum_{j=1}^{N+1} \frac{P_j M_{ij}}{\bar{P}_j \bar{M}_{ij}} \quad \text{s.t} \quad \frac{M_i^D}{\bar{M}_i^D} \geq \tilde{M}_i^D \tag{D.7}$$

The solution to these problems deliver the following price indices

$$\frac{MC_i}{\bar{M}C_i} = \left( a_i \left( \frac{W_i}{\bar{W}_i} \right)^{1-\sigma_i} + (1 - a_i) \left( \frac{P_i^M}{\bar{P}_i^M} \right)^{1-\sigma_i} \right)^{\frac{1}{(1-\sigma_i)}} \tag{D.8}$$

$$\frac{P_i^M}{\bar{P}_i^M} = \left( \omega_i^D \left( \frac{P_i^D}{\bar{P}_i^D} \right)^{1-\epsilon_i} + (1 - \omega_i^D) \left( \frac{P_{iM}}{\bar{P}_{iM}} \right)^{1-\epsilon_i} \right)^{\frac{1}{(1-\epsilon_i)}} \tag{D.9}$$

$$\frac{P_i^D}{\bar{P}_i^D} = \left( \sum_{j=1}^{N+1} \omega_{ij} \left( \frac{P_j}{\bar{P}_j} \right)^{1-\epsilon_i^D} \right)^{\frac{1}{(1-\epsilon_i^D)}} \tag{D.10}$$

# E Proofs

## E.1 Proof of Proposition 1

**Proof of Proposition 1.** Starting from price changes, we have

$$\mathrm{d}\log P_i = a_i \mathrm{d}\log W + \sum_{j=1}^{N+1} \Omega_{ij} d\log P_j + \eta_i \mathrm{d}\log P_M - d\log Z_i \qquad \text{for all } i = 1, 2, ..., N+1, \tag{E.1}$$

where

$$a_i = \frac{WL_i}{P_iQ_i} = \frac{WL_i}{TC_i}; \qquad \eta_i = \frac{P_M M_{iM}}{TC_i}; \qquad \Omega_{ij} = \frac{P_j M_{ij}}{TC_i} \qquad \text{for all } i = 1, 2, ..., N+1,$$

is how much producer $i$ spends on either labor, imported intermediate input, and domestic intermediate inputs as a fraction of its sales, $P_iQ_i$, which, due to the constant returns to scale assumption of the production function, equals total costs ($TC_i$), $P_iQ_i = TC_i$.

The system in Equation (E.1) is $N+1$ equations in $N+2$ unknowns. Up to choosing a numeraire, we can solve for domestic price changes as a function of commodity price changes. Let $P^*_M = 1$ and the nominal exchange rate be the numèraire. Hence, all prices are expressed in units of foreign currency, $\mathrm{d}\log P_i - \mathrm{d}\log \mathcal{E} = \mathrm{d}\log P_i$. Stacking the system into matrix/vector form, we have

$$\mathrm{d}\log \boldsymbol{P} = \boldsymbol{\Omega}\mathrm{d}\log \boldsymbol{P} + \boldsymbol{a}\mathrm{d}\log W - \mathrm{d}\log \boldsymbol{Z}$$

Setting $\mathrm{d}\log \boldsymbol{Z} = \boldsymbol{0}$ and inverting the system we arrive at

$$d\log \boldsymbol{P} = \boldsymbol{\Psi}\boldsymbol{a}\mathrm{d}\log W \Longrightarrow d\log P_i = \left(\sum_{h=1}^{N+1} \Psi_{ih}a_h\right)\mathrm{d}\log W \text{ for all } i = 1, 2, ..., N+1 \quad \text{(E.2)}$$

Note that we can write the above expression as

$$\mathrm{d}\log \boldsymbol{P} = \widetilde{\boldsymbol{a}}\mathrm{d}\log W$$

where we define the typical element of $\widetilde{\boldsymbol{a}} = \{\widetilde{a}_i\} = \left\{\sum_{h=1}^{N+1} \Psi_{ih}a_h\right\}$, that represents the *network-adjusted* labor share of producer $i$.

We now make use of the fact that $\mathrm{d}\log P_{N+1} - \mathrm{d}\log \mathcal{E} = \mathrm{d}\log P^*_{N+1}$ is exogenously given to express changes in wages, $\mathrm{d}\log W$, as an explicit function of it since

$$\mathrm{d}\log P^*_{N+1} = \widetilde{a}_{N+1}\mathrm{d}\log W \Longrightarrow \mathrm{d}\log W = \frac{1}{\widetilde{a}_{N+1}}\mathrm{d}\log P^*_{N+1}$$

Replacing this expression into Equation (E.2), we get

$$\mathrm{d}\log P_i = \frac{\widetilde{a}_i}{\widetilde{a}_{N+1}}\mathrm{d}\log P^*_{N+1} \quad \text{(E.3)}$$

which completes the proof. ■

## E.2 Proof of Proposition 2

**Proof of Proposition 2.** To solve for changes in gross output, $\mathrm{d}\log Q_i$, we define real sales (in units of foreign currency),

$$S_i = P_i Q_i / \mathcal{E},$$

and totally differentiate it to get

$$\mathrm{d}\log Q_i = \mathrm{d}\log S_i - (\mathrm{d}\log P_i - \mathrm{d}\log \mathcal{E}). \tag{E.4}$$

Similarly, we define real sales for final domestic consumption and exports as

$$S_i^C = \frac{P_i C_i}{\mathcal{E}}, \qquad i = 1, 2, ..., N+1 \tag{E.5}$$

$$S_{N+1}^* = \frac{P_{N+1} X_{N+1}}{\mathcal{E}} = P_{N+1}^* X_{N+1} \tag{E.6}$$

We already know $\mathrm{d}\log P_i - \mathrm{d}\log \mathcal{E}$. We are left to determine changes in real sales, $\mathrm{d}\log S_i$.

Start from the market clearing conditions of all goods (in vector form)

$$\boldsymbol{S} = \boldsymbol{\Psi}^T \left( \boldsymbol{S}^C + \boldsymbol{S}^* \right) \tag{E.7}$$

Totally differentiating this expression

$$\mathrm{d}\boldsymbol{S} = \underbrace{\mathrm{d}\boldsymbol{\Psi}^T \left( \boldsymbol{S}^C + \boldsymbol{S}^* \right)}_{\text{Changes in IO matrix given final sales}} + \underbrace{\boldsymbol{\Psi}^T \left( \mathrm{d}\boldsymbol{S}^C + \mathrm{d}\boldsymbol{S}^* \right)}_{\text{Changes in sales given IO linkages}} \tag{E.8}$$

We now totally differentiate the definition of the Leontief-Inverse, $\boldsymbol{\Psi}$, to map its changes to changes in the IO matrix, $\boldsymbol{\Omega}$

$$\begin{aligned}
\boldsymbol{\Psi}^T &= (\boldsymbol{I} - \boldsymbol{\Omega}^T)^{-1} \\
\boldsymbol{\Psi}^T(\boldsymbol{I} - \boldsymbol{\Omega}^T) &= \boldsymbol{I} \\
\boldsymbol{\Psi}^T - \boldsymbol{\Psi}^T\boldsymbol{\Omega}^T &= \boldsymbol{I} \\
\mathrm{d}\boldsymbol{\Psi}^T - \mathrm{d}\boldsymbol{\Psi}^T\boldsymbol{\Omega}^T - \boldsymbol{\Psi}^T\mathrm{d}\boldsymbol{\Omega}^T &= \boldsymbol{0} \\
\mathrm{d}\boldsymbol{\Psi}^T(\boldsymbol{I} - \boldsymbol{\Omega}^T) &= \boldsymbol{\Psi}^T\mathrm{d}\boldsymbol{\Omega}^T \\
\mathrm{d}\boldsymbol{\Psi}^T &= \boldsymbol{\Psi}^T\mathrm{d}\boldsymbol{\Omega}^T\boldsymbol{\Psi}^T \\
\mathrm{d}\boldsymbol{\Psi}^T(\boldsymbol{S}^C + \boldsymbol{S}^*) &= \boldsymbol{\Psi}^T\mathrm{d}\boldsymbol{\Omega}^T \underbrace{\boldsymbol{\Psi}^T(\boldsymbol{S}^C + \boldsymbol{S}^*)}_{=\boldsymbol{S}} \\
\mathrm{d}\boldsymbol{\Psi}^T(\boldsymbol{S}^C + \boldsymbol{S}^*) &= \boldsymbol{\Psi}^T\mathrm{d}\boldsymbol{\Omega}^T\boldsymbol{S}
\end{aligned}$$

Using this expression into Equation (E.8)

$$\mathrm{d}\boldsymbol{S} = \boldsymbol{\Psi}^T\mathrm{d}\boldsymbol{\Omega}^T\boldsymbol{S} + \boldsymbol{\Psi}^T\left(\mathrm{d}\boldsymbol{S}^C + \mathrm{d}\boldsymbol{S}^*\right) \tag{E.9}$$

For a given producer $i$, we have

$$\mathrm{d}S_i = \sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}S_j\mathrm{d}\log\Omega_{jk} + \sum_{k=1}^{N+1}\Psi_{ki}\left(\mathrm{d}S_k^C + \mathrm{d}S_k^*\right) \tag{E.10}$$

We now focus on the last term on the right-hand side of the above equation. To focus on the propagation mechanisms from intermediate inputs, we assume that the home consumer has Cobb-Douglas preferences over goods. This means that consumption of each good $k$ as a share of total expenditure, $b_k$, is constant and independent of quantities and prices:

$$P_kC_k = b_kE. \tag{E.11}$$

From Equation (E.11) we can construct sales in units of foreign currency

$$S_k^C = \frac{P_kC_k}{\mathcal{E}} = b_k\frac{E}{\mathcal{E}}, \tag{E.12}$$

that only depends on $E/\mathcal{E}$ expenditure in units of the numèraire.

Log-differentiating Equation (E.12)

$$\mathrm{d}\log S_k^C = \mathrm{d}\log E/\mathcal{E}. \tag{E.13}$$

Therefore, $S_k^C$ moves in tandem with domestic expenditure.

Similarly, recall that exports in units of foreign currency can be written as $S_{N+1}^* = (P_{N+1}^*)^{1-\chi} D^*$ and so changes in export sales

$$\mathrm{d}\log S_{N+1}^* = (1-\chi)\mathrm{d}\log P_{N+1}^* + \mathrm{d}\log D^* \tag{E.14}$$

Using Equation (E.13) and (E.14) into Equation (E.10), we get

$$\mathrm{d}S_i = \sum_{k=1}^{N+1} \Psi_{ki} \sum_{j=1}^{N+1} \Omega_{jk} S_j \mathrm{d}\log\Omega_{jk} + \sum_{k=1}^{N+1} \Psi_{ki} S_k^C d\log S_k^C + \Psi_{N+1,i} S_{N+1}^* \mathrm{d}\log S_{N+1}^*$$

$$\mathrm{d}S_i = \sum_{k=1}^{N+1} \Psi_{ki} \sum_{j=1}^{N+1} \Omega_{jk} S_j \mathrm{d}\log\Omega_{jk} + \sum_{k=1}^{N+1} \Psi_{ki} S_k^C (\mathrm{d}\log E - \mathrm{d}\log\mathcal{E}) + \Psi_{N+1,i} S_{N+1}^* \big((1-\chi)\mathrm{d}\log P_{N+1}^* + \mathrm{d}\log D^*\big)$$

Upon rearranging

$$\begin{aligned}\mathrm{d}\log S_i = {} & \frac{1}{S_i}\left(\sum_{k=1}^{N+1} \Psi_{ki} \sum_{j=1}^{N+1} \Omega_{jk} S_j d\log\Omega_{jk} + \sum_{k=1}^{N+1} \Psi_{ki} S_k^C (\mathrm{d}\log E - \mathrm{d}\log\mathcal{E})\right) \\ & + \frac{1}{S_i}\Big(\Psi_{N+1,i} S_{N+1}^* \big((1-\chi)\mathrm{d}\log P_{N+1}^* + \mathrm{d}\log D^*\big)\Big)\end{aligned} \tag{E.15}$$

We are now ready to construct the input-output substitution operator. To economize on notation, let $\mathcal{I}_j$ be the set of inputs that producer $j$ uses in production. Write the changes in expenditure shares

$$\begin{aligned}\mathrm{d}\log\Omega_{jk} &= \mathrm{d}\log P_k + \sum_{h\in\mathcal{I}_j} (\theta_{kh}^j - 1)\Omega_{jh} \mathrm{d}\log P_h \\ &= \delta_{kh} \mathrm{d}\log P_h + \sum_{h\in\mathcal{I}_j} (\theta_{kh}^j - 1)\Omega_{jh} \mathrm{d}\log P_h \\ \mathrm{d}\log\Omega_{jk} &= \sum_{h\in\mathcal{I}_j} (\delta_{kh} + (\theta_{kh}^j - 1)\Omega_{jh}) \mathrm{d}\log P_h \end{aligned} \tag{E.16}$$

where $\delta_{kh}$ is the Kronecker delta, equal to 1 if $k = h$ and zero otherwise. In that expression, we also define

$$\theta^j_{kh} = \frac{\varepsilon^j_{kh}}{\Omega_{jh}} \tag{E.17}$$

$$\varepsilon^j_{kh} = \frac{\partial \log M_{jk}}{\partial \log P_h} \tag{E.18}$$

where Equation (E.17) is the Allen-Uzawa elasticity for producer $j$ between input $k$ and $h$. Note that input $h$ can be either a factor or an intermediate input, while input $k$ is always an intermediate good. Equation (E.18) represents the constant-output elasticity of input demand of producer $j$ of good $k$ with respect to a change in the price of good/factor $h$.

We now use the model's structure to simplify the above expression. In particular, we only have one factor (labor) and the imported intermediate input price is the numèraire, as we assume $P^*_M = 1$ and so $P_M = P^*_M \mathcal{E} = \mathcal{E}$. We can write Equation (E.16) as

$$\mathrm{d}\log \Omega_{jk} = \sum_{h=1}^{N+1} (\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh}) \mathrm{d}\log P_h + (\theta^j_{kL} - 1) a_j \mathrm{d}\log W$$

At this point, we can use the result in Proposition 1 that relates changes in prices to changes in the commodity price and the one that links changes in wages to changes in the commodity price as well. We rewrite those below

$$\mathrm{d}\log P_h = \frac{\widetilde{a}_h}{\widetilde{a}_{N+1}} \mathrm{d}\log P^*_{N+1} \tag{E.19}$$

$$\mathrm{d}\log W = \frac{1}{\widetilde{a}_{N+1}} \mathrm{d}\log P^*_{N+1} \tag{E.20}$$

Plugging these expressions into the above expression, we get

$$\mathrm{d}\log \Omega_{jk} = \left( \sum_{h=1}^{N+1} (\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh}) \frac{\widetilde{a}_h}{\widetilde{a}_{N+1}} + \frac{(\theta^j_{kL} - 1) a_j}{\widetilde{a}_{N+1}} \right) \mathrm{d}\log P^*_{N+1} \tag{E.21}$$

We can replace Equation (E.21) into the first term on the right-hand side of Equation (E.15), to get

$$\sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}S_j d\log\Omega_{jk}=\sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}S_j\left(\sum_{h=1}^{N+1}(\delta_{kh}+(\theta^j_{kh}-1)\Omega_{jh})\frac{\widetilde{a}_h}{\widetilde{a}_{N+1}}+\frac{(\theta^j_{kL}-1)a_j}{\widetilde{a}_{N+1}}\right)\mathrm{d}\log P^*_{N+1}$$
$$=\sum_{j=1}^{N+1}S_j\left(\sum_{k=1}^{N+1}\sum_{h=1}^{N+1}(\delta_{kh}+(\theta^j_{kh}-1)\Omega_{jh})\Psi_{ki}\Omega_{jk}\frac{\widetilde{a}_h}{\widetilde{a}_{N+1}}+\sum_{k=1}^{N+1}\Psi_{ki}\Omega_{jk}\frac{(\theta^j_{kL}-1)a_j}{\widetilde{a}_{N+1}}\right)\mathrm{d}\log P^*_{N+1}$$

Let's define the following objects

$$\Phi^j(\Psi_{(:,i)},\widetilde{\boldsymbol{a}})=\frac{1}{\widetilde{a}_{N+1}}\sum_{k=1}^{N+1}\sum_{h=1}^{N+1}(\delta_{kh}+(\theta^j_{kh}-1)\Omega_{jh})\Psi_{ki}\Omega_{jk}\widetilde{a}_h$$
$$\Phi^j(\Psi_{(:,i)},\boldsymbol{a})=\frac{1}{\widetilde{a}_{N+1}}\sum_{k=1}^{N+1}\Psi_{ki}\Omega_{jk}(\theta^j_{kL}-1)a_j$$
$$\Phi^j j(i,\mathrm{d}\log P^*_{N+1})=\Phi^j(\Psi_{(:,i)},\widetilde{\boldsymbol{a}})+\Phi^j(\Psi_{(:,i)},\boldsymbol{a})$$

Hence, we have

$$\sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}S_j\mathrm{d}\log\Omega_{jk}=\sum_{j=1}^{N+1}S_j\Phi^j(i,\mathrm{d}\log P^*_{N+1})\mathrm{d}\log P^*_{N+1}\tag{E.22}$$

where $\Phi^j(i,\mathrm{d}\log P^*_{N+1})$ is a version of the *Input-Substitution Operator* defined by Baqaee and Farhi (2019) applied to our small open economy environment. This operator captures how, in response to a change in the commodity price, producer $j$ substitutes away/towards producer $i$ both directly and indirectly through input-output linkages.

Therefore, changes in sales can be written as

$$\mathrm{d}\log S_i=\left(\sum_{j=1}^{N+1}\frac{S_j\Phi^j(i,\mathrm{d}\log P^*_{N+1})}{S_i}\mathrm{d}\log P^*_{N+1}+\sum_{k=1}^{N+1}\frac{\Psi_{ki}S^C_k}{S_i}(\mathrm{d}\log E-\mathrm{d}\log\mathcal{E})\right)$$
$$+\frac{1}{S_i}\left(\Psi_{N+1,i}S^*_{N+1}\left((1-\chi)\mathrm{d}\log P^*_{N+1}+\mathrm{d}\log D^*\right)\right),\tag{E.23}$$

To complete the proof, recall that trade balance implies $E=WL$ and therefore

$$\mathrm{d}\log E-\mathrm{d}\log\mathcal{E}=\mathrm{d}\log W-\mathrm{d}\log\mathcal{E}=\frac{1}{\tilde{a}_{N+1}}\mathrm{d}\log P^*_{N+1}.\tag{E.24}$$

Replacing Equation (E.24) in (E.23), we have

$$\mathrm{d}\log S_i = \left( \sum_{j=1}^{N+1} \frac{S_j \Phi^j(i, \mathrm{d}\log P^*_{N+1})}{S_i} \mathrm{d}\log P^*_{N+1} + \sum_{k=1}^{N+1} \frac{\Psi_{ki} S^C_k}{S_i} \frac{1}{\tilde{a}_{N+1}} \mathrm{d}\log P^*_{N+1} \right) + \left( \frac{\Psi_{N+1,i} S^*_{N+1}}{S_i} \left( (1-\chi)\mathrm{d}\log P^*_{N+1} + \mathrm{d}\log D^* \right) \right), . \tag{E.25}$$

Let $\alpha_i = \sum_{k=1}^{N+1} \frac{\Psi_{ki} S^C_k}{S_i}$ and $1 - \alpha_i = \sum_{k=1}^{N+1} \frac{\Psi_{N+1,i} S^*_{N+1}}{S_i}$. Rearrange Equation (E.25) to get

$$\begin{aligned} \mathrm{d}\log Q_i &= \mathrm{d}\log S_i - d\log P_i \\ &= \left( \sum_{j=1}^{N+1} \frac{S_j \Phi^j(i, \mathrm{d}\log P^*_{N+1})}{S_i} \mathrm{d}\log P^*_{N+1} + \frac{\alpha_i}{\tilde{a}_{N+1}} \mathrm{d}\log P^*_{N+1} \right) \\ &+ \left( (1-\alpha_i) \left( (1-\chi)\mathrm{d}\log P^*_{N+1} + \mathrm{d}\log D^* \right) \right) - d\log P_i \end{aligned}$$

Using Proposition 1, we can rewrite this as

$$\mathrm{d}\log Q_i = \left( \sum_{j=1}^{N+1} \frac{S_j \Phi^j(i, \mathrm{d}\log P^*_{N+1})}{S_i} + \frac{\alpha_i}{\tilde{a}_{N+1}} + (1-\alpha_i)(1-\chi) - \frac{\tilde{a}_i}{\tilde{a}_{N+1}} \right) \mathrm{d}\log P^*_{N+1} + (1-\alpha_i)\mathrm{d}\log D^* \tag{E.26}$$

This completes the proof. ■

## E.3 Proposition 3

**Proposition 3.** Changes in real GDP ($\mathrm{d}\log rGDP$) in the model satisfies

$$\mathrm{d}\log rGDP = \sum_{i=1}^{N+1} \lambda_i \mathrm{d}\log Z_i + \mathrm{d}\log \bar{L},$$

and are thus independent of commodity price changes.

**Proof of Proposition 3.**

Consider the definition of nominal GDP. In our particular model,

$$nGDP = \sum_{i=1}^{N+1} P_i Q_i - \sum_{i=1}^{N+1}\sum_{j=1}^{N+1} P_j M_{ij} - P_M \sum_{i=1}^{N+1} M_{iM}.$$

Changes in nominal GDP thus satisfies

$$\mathrm{d}\log nGDP = \underbrace{\sum_{i=1}^{N+1} \frac{P_i Q_i}{GDP}\left[\mathrm{d}\log Q_i - \sum_{j=1}^{N+1}\frac{P_j M_{ij}}{P_i Q_i}\mathrm{d}\log M_{ij} - \frac{P_M M_{iM}}{P_i Q_i}\mathrm{d}\log M_{iM}\right]}_{\text{Real GDP changes}}$$
$$+ \underbrace{\sum_{i=1}^{N+1} \frac{P_i Q_i}{GDP}\left[\mathrm{d}\log P_i - \sum_{j=1}^{N+1}\frac{P_j M_{ij}}{P_i Q_i}\mathrm{d}\log P_j - \frac{P_M M_{iM}}{P_i Q_i}\mathrm{d}\log P_M\right]}_{\text{GDP deflator changes}},$$

Define $\lambda_i = P_i Q_i/GDP$, $\Omega_{ij} = P_j M_{ij}/P_i Q_i$ and $\eta_i = P_M M_{iM}/P_i Q_i$. Then changes in real GDP, $\mathrm{d}\log rGDP$,

$$\mathrm{d}\log rGDP = \sum_{i=1}^{N+1} \lambda_i \left[\mathrm{d}\log Q_i - \sum_{j=1}^{N+1}\Omega_{ij}\mathrm{d}\log M_{ij} - \eta_i \mathrm{d}\log M_{iM}\right]. \tag{E.27}$$

Log-differentiating the production function

$$\mathrm{d}\log Q_i = \mathrm{d}\log Z_i + \sum_{j=1}^{N+1}\Omega_{ij}\mathrm{d}\log M_{ij} + \eta_i\mathrm{d}\log M_{iM} + a_i\mathrm{d}\log L_i \tag{E.28}$$

Replacing Equation (E.28) in Equation (E.27), we get

$$\mathrm{d}\log rGDP = \sum_{i=1}^{N+1}\lambda_i\left[\mathrm{d}\log Z_i + a_i\mathrm{d}\log L_i\right]. \tag{E.29}$$

The second term on the right-hand side can be simplified using the labor market clearing condition. First, note $\lambda_i a_i = (P_i Q_i/nGDP) * (W L_i/P_i Q_i) = W L_i/nGDP$. Log-differentiating the labor market clearing condition

$$\sum_{i=1}^{N+1} \frac{L_i}{\bar{L}} \mathrm{d}\log L_i = \mathrm{d}\log \bar{L}. \tag{E.30}$$

Therefore, rewrite the second term on the right-hand side of Equation (E.29)

$$\begin{aligned}
\sum_{i=1}^{N+1} \lambda_i a_i \mathrm{d}\log L_i &= \sum_{i=1}^{N+1} \frac{WL_i}{nGDP} \mathrm{d}\log L_i \\
&= \frac{W}{nGDP} \sum_{i=1}^{N+1} L_i \mathrm{d}\log L_i \\
&= \frac{W\bar{L}}{nGDP} \mathrm{d}\log \bar{L} \\
\sum_{i=1}^{N+1} \lambda_i a_i \mathrm{d}\log L_i &= \mathrm{d}\log \bar{L}, \qquad \text{(E.31)}
\end{aligned}$$

where we used the fact that $W\bar{L} = nGDP$.

Replacing Equation (E.31) into Equation (E.29) delivers an expression for real GDP

$$\mathrm{d}\log rGDP = \sum_{i=1}^{N+1} \lambda_i \mathrm{d}\log Z_i + \mathrm{d}\log \bar{L},$$

that is independent of the commodity price changes. Since we assume for our exercise that $\mathrm{d}\log Z_i = 0$ for all $i = 1, 2, ..., N+1$ and $\mathrm{d}\log \bar{L} = 0$, then $\mathrm{d}\log rGDP = 0$. This completes the proof. ■