Production Network Diversification and Economic Development*

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Abstract

We provide empirical evidence and a theoretical analysis of the influence of production network diversification on countries' economic performance, reflected in their GDP per-capita levels. Using a panel sample of 55 countries, we find a strong positive association between the number of active links in the input-output network of a country and its GDP per-capita over time, even after controlling for several country characteristics. To complement and scrutinize our empirical finding, we advance economic theory on the link between network diversity and economic development by proposing a multisector model with input-output linkages, non-unitary elasticity of substitution in production, and a love of diversification in the bundle of intermediate inputs that rationalize our empirical results. In the long run, when labor and intermediates are substitute inputs, denser production structures enjoy higher productivity in the intermediate input bundle and also amplify positive shocks more strongly than less connected networks. Hence, our model predicts that economies with denser production structures display higher income.

JEL codes: 011, 014, 041, 047

Keywords: Production network structure, GDP per capita, productivity, substitutability between inputs

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1 Introduction

One of the key features of a country's economy is how its production system operates and what it generates. Input-output (I-O) tables have been historically and widely used to map the interconnections of productive systems and to measure the economic impacts of changes in their structures. I-O data allow us to observe the interlinkages between industries and sectors, from which we can assess how well or poorly diversified the productive systems of an economy are. By analyzing these interlinkages, in this paper, we provide empirical and theoretical analyses of how a denser production network structure can support higher levels of economic output in a country. In particular, we explore the question: How important is the organization of production in explaining the large differences in income per capita observed across countries? This is a long standing question in the economic development literature (see, for example, Hirschman, 1958; Hidalgo, Klinger, Barabási, and Hausmann, 2007; Jones, 2011; McNerney, Savoie, Caravelli, Carvalho, and Farmer, 2022) because its answer has important implications for industrial policies and for the development of nations and regions (see, for example, Liu, 2019; Choi and Levchenko, 2021).

In this paper, we study the relationship between the diversification of intermediate inputs traded across economic sectors of a country and the country's economic performance measured by its level of GDP per capita. We start by exploring empirical evidence on a panel of 55 countries for the period 1995-2011; using this evidence, we then propose a formal theoretical model displaying a network of producers trading intermediate inputs. Our empirical results and theoretical construct present novel evidence on the effect that intermediate input diversification can have on GDP per capita levels. Empirically, we document a strong fact using different specifications: countries with higher production network density—that is, a higher number of non-zero intermediate input links among sectors—present higher levels of GDP per capita. To complement this empirical observation, we build a relatively simple theoretical model of a detailed network structure that shows, intuitively, that under certain conditions, in denser production networks, a given productivity shock could spill over more easily to the rest of the economy, generating higher output, as observed in the data.

In our empirical approach, we use different econometric specifications that aim to explore whether and to what extent production network density influences the GDP per capita level of a country. Our data are sourced from the OECD databases and contain detailed information on production linkages for 33 industries for each of the 55 countries in our sample. We calculate network density following influential network science research that aims to capture the complexity of network interconnections (see, for example, Gai, Haldane, and Kapadia, 2011; Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015; Herskovic, 2018). In particular, production network density is built to capture the proportional amount of active connections within the productive input-output structure of a country, which allows us to measure network diversification as in Miranda-Pinto (2021), while capturing the notion of economic sparsity analyzed in Dupor (1999), Acemoglu et al. (2012), and Acemoglu et al. (2015).

We test the relevance of network density to a country's economic performance using a number of cross-country fixed-effect panel regressions that include different sets of drivers of economic growth commonly used in the literature such as years of schooling, quality of institutions, service share, and an index of economic complexity, among others. Across our specifications, we find a consistent positive effect of network density on a country's level of GDP. Our most conservative estimate indicates that a 10% increase in network density (85 new links, among 1,056 possible links) is associated with a 3.5% increase in GDP per capita (on average, $726 PPP US dollars, which corresponds to Cambodia's GDP per capita in 1995). The results suggest that network connections among domestic industries drive most of the effect (77%) and that the number of imported intermediates also plays a role in accounting for differences in GDP per capita across countries.[1]

To support our empirical observation, we formally propose a theoretical model in which network density could generate higher levels of output in a country. In our model we assume that the topology of the production network is exogenous (active links), while the intensity of the connections is endogenous to changes in relative input prices. We use the model to study how different network structures affect the level of GDP via shaping the strength of the propagation and amplification of sectoral productivity shocks. To this end, we assume that sectoral productivity follows the same stochastic process across sectors and countries and that the elasticity of substitution between inputs is non-unitary and common across sectors and economies. Therefore, the main difference among economies is the production network structure—namely, the network density, sectoral intermediate input shares, and sectoral consumption shares.

Our main theoretical result indicates that the role of production network density in shaping the level of GDP in an economy depends on the specifics of sectoral production function. Under standard Cobb-Douglas production functions, which assume unitary elasticity of substitution between inputs, network density plays no role in shaping aggregate GDP. However, production network density does have a role in affecting GDP when sectoral production function displays non-unitary elasticities of substitution in production and a love of diversification in the bundle of intermediate inputs. We highlight two mechanisms in our model in which a higher network density results

[1] Kasahara and Rodrigue (2008) document that importing intermediate inputs is associated with higher productivity and, therefore, output, which is consistent with the results in our empirical section.

in higher output works as follows. First, within the intermediate input bundle of CES production technologies, the number of intermediate inputs used in production affects how productive a given mix of intermediate inputs is. The difference in intermediate input productivity embedded in the diversification of the intermediate input bundle then shapes firms' equilibrium input shares, as long as the elasticity of substitution between labor and intermediate inputs is different from one. If intermediate inputs and labor/capital are substitutes, the love of diversification effect makes intermediates more attractive in denser production networks, implying that denser networks are also more connected networks that display a larger input-output multiplier. In this case, a given level of sectoral productivity generates higher output in denser networks, all else equal.

In addition, our CES model displays non-linearities, as highlighted by Baqaee and Farhi (2019). We show that when inputs are gross substitutes, positive productivity shocks are amplified, and negative productivity shocks are mitigated. This asymmetry is larger in denser production structures. Therefore, while our first mechanism holds in steady state, this second mechanism operates for higher-order approximations of real GDP.

We conclude our theoretical construct with a numerical simulation and counterfactual exercises. We calibrate the model to match each of the 55 countries production structures in 1995. Our calibration also matches the empirical relationship between network density and log GDP per capita we document. In our counterfactual experiment, we study what Thailand or Indonesia's GDP would be if they displayed the production network diversification observed in Denmark or Australia. Our results show that these gains can be sizable. For instance, in our benchmark calibration, Thailand's GDP would be 23% larger if it displayed the diversified production structure of Denmark.

The rest of the paper is organized as follows. Section 2 discusses the literature on the role that production diversification and complexity play in shaping the economic performance of countries and regions. Section 3 describes data sources and methodology, including the design of our network density variable. Section 4 presents our empirical findings. Section 5 expands our empirical analysis by developing a theoretical framework that rationalizes our results and we propose and quantify the mechanism for the influence of network density on GDP per capita. Finally, Section 6 concludes, including a discussion of policy implications. Additional analysis, data considerations, and proofs are provided in the Appendix.

2 Related literature

Our paper contributes to the literature that studies the role that production network structures play in shaping countries' average income. In this sense, the papers that are closest to our study are Jones (2011), Bartelme and Gorodnichenko (2015), McNerney et al. (2022), and Fadinger et al. (2021).

Jones (2011) studies the role that intermediate input linkages and input complementarity play in amplifying distortions and depressing aggregate productivity and GDP (*misallocation*). Their model assumes a unitary elasticity of substitution between intermediate inputs and labor-capital, and the key production network moment shaping GDP is the share of intermediate inputs in production (assumed to be common across sectors). Jones (2011) empirically analyzes the cross-country correlation between the intermediate input share and GDP per capita, finding no significant association.

Bartelme and Gorodnichenko (2015), McNerney et al. (2022), and Fadinger et al. (2021) study the empirical association between the average input-output multiplier and aggregate productivity to account for the differences in cross-country income per capita. These papers find a positive correlation between the average input-output multiplier and log of GDP per capita. The theoretical framework proposed by these authors and Jones (2011) assumes unitary elasticity of substitution between intermediates and labor-capital, which implies that the intensity of input-output connections is the key metric in determining how productivity or frictions propagate and amplify along the production chain. In their framework, there is no role for the network density.

Different from Jones (2011), Bartelme and Gorodnichenko (2015), McNerney et al. (2022) and Fadinger et al. (2021), we emphasize the role played by a particular network structure—network density—in amplifying productivity shocks in the presence of non-unitary elasticity of substitution between intermediates and labor-capital, including a love of diversification in the bundle of intermediate inputs. In this sense, we provide empirical and theoretical support for the role of the proportion of positive I-O connections in shaping the input-output multiplier and, therefore, GDP.

There is a large literature within regional economics, developed in the 1990's, that have used input-output tables to study economic growth and stability (e.g., Siegel, Alwang, and Johnson, 1995; Wagner and Deller, 1998) —for a review, see Dissart (2003). More recently, studies in different disciplines have used the interconnections of input-output tables to also explore productive structures and their relation to economic performance (e.g., Bartelme and Gorodnichenko, 2015; Sonis and Hewings, 1998; Xu, Allenby, and Crittenden, 2011; Blöchl, Theis, Vega-Redondo, and Fisher, 2011). In the context of a single country, Choi and Levchenko (2021) study the successful role that industrial policies played in the development of Korea. In their model, the industrial policy generates manufacturing hubs in which firms become more productive due to having easier access to credit and learning by doing, thus generating higher diversification among the I-O structure of the country. In relation to this, although we do not model industrial policies (and their costs) or other factors that determine a country's network structure, we show in our theoretical framework that a higher number of active I-O links (denser network) alone can improve a country's output via multipliers.[2]

Explaining the intricacies and economic outcomes of input-output networks, Acemoglu and Azar (2020) model the evolution of the production network of a country to understand how the endogenous reshaping of networks can support long-term economic growth. The authors show that denser networks arise due to sectoral improvements in productivity. However, they also show that, given productivity, a higher density of the network increases output via providing more possibilities of efficient input combinations. In our case, the network structure in our theoretical analysis is exogenous, but the positive relationship between network density and income does arise from the fact that input diversification improves input productivity (love of diversification). In this way, our paper complements Acemoglu and Azar (2020) by documenting the empirical relationship between network density and GDP per capita and by rationalizing this relationship with a relatively simple extension of existing exogenous production network models with CES technologies, as in Papageorgiou and Saam (2008) and Atalay (2017).[3]

Hidalgo and Hausmann (2009) and Hausmann and Hidalgo (2011) propose novel ways to empirically study the influence of production structures in the economic performance of nations. The authors develop indexes of economic complexity based on exports diversification to understand how complexity supports economic development. Their measure of economic complexity is constructed using the number of different products a country exports and the uniqueness of those products compared to other exporters' products. Thus, their measure captures the ability to diversify shocks (number of products) and comparative advantage (the ability to produce a unique product). Different from these papers, our network density measure is more aggregated at the industry level and captures how shocks transmit along the production chain by considering the number of active industry connections. In addition, the relationship between production complexity and development in Hidalgo and Hausmann (2009) and Hausmann and Hidalgo (2011) resides in the fact that a more complex production structure is the result of unobserved capabilities. Expanding on this notion, our paper explicitly develops a model that displays a direct connection between the production network structure and GDP per capita.

Production and export diversification has also been widely studied as a feature of

[2] Choi and Levchenko (2021) do not evaluate this, as their model does not connect firms.

[3] It is worth noting that Acemoglu and Azar (2020) use a Cobb-Douglas production function. In addition, the love-of-diversification effect in their model is different from ours. In particular, in their model, the arrival of new varieties increases the amount of inputs in production and, all else equal, increases output. In our model, the CES function imposes weights on the intermediate inputs, which add up to one; therefore, when adding a new variety, the process does not necessarily increase output, as it requires reallocating other intermediates.

economic resilience, with researchers arguuing that diversified economies support a better buffer to shocks. Indeed, the 2008-09 global economic crisis spawned a number of studies on economic diversity, stability and the ability to recover from downturns (e.g., Han and Goetz, 2015; Deller and Watson, 2016). Looking at link volatility and development, Koren and Tenreyro (2013) explore the role of input diversification, although the authors do not offer a framework with input-output linkages and provide no empirical evidence linking network density and GDP per capita.[4] In a similar analysis, Krishna and Levchenko (2013) show a negative relationship between the number of intermediates used in production and GDP volatility; however, the authors do so only by exploring information from manufacturing industries and, thus, miss all the other interconnections that occur in an economy that can be also relevant to explaining growth. Miranda-Pinto (2021) expands on Acemoglu et al. (2012) and Krishna and Levchenko (2013) by exploring the empirical and theoretical relationship between the economy's wide intermediate input diversification and the service share in driving GDP volatility. We expand the econometric and theoretical approaches developed by Miranda-Pinto (2021), and, rather than focusing on the relationship between the production network structure and short-term macroeconomic volatility, we focus on the role of production structure in shaping the average income of a country.

3 Data and methodology

We piece together a cross-country panel dataset consisting of 55 countries over seventeen years (1995-2011), forming a strongly balanced panel of 935 observations.[5] To evaluate the economic effects of production network density, we construct panel models using GDP per capita as the dependent variable and a set of different drivers of economic development as independent variables. For the dependent variable, we collect data on GDP per capita at purchasing power parity (PPP) at current international dollars from the World Bank's World Development Indicators.[6] From this same database, we obtain data on population and years of schooling, this last a key determinant of income per capita according to Mankiw et al. (1992).

Motivated by the resource curse literature (e.g., Sachs and Warner, 1995; Fleming, Measham, and Paredes, 2015), on the one hand, we control for the size of the commodity sector in each economy (as a share of total output). On the other hand, motivated by

[4]The authors provide evidence for eight OECD economies showing that for the period 1970-2007, the diagonal shares of the input-output table have become smaller, on average, (indicating more reliance on other sectors.)

[5]We select these 55 countries because they are the only ones with available data, across our different sources. *Rule of Law* is the only variable where data is not available across all 17 years.

[6]data.worldbank.org

Calderón and Liu (2003) and Beck et al. (2014), who study the relationship between the size of the financial sector and economic growth, we use the output share of the financial sector as a control. Broadberry (1998) and Moro (2015) study the relationship between the service sector share and GDP growth. Hence, we also control for the service share within a country's economy. In addition, following Acemoglu et al. (2017), we control for the degree of sectoral dominance in the production network.[7] These four measures are obtained from the OECD input-output tables.

An open economy has long been touted as a necessary element to growing an economy as discussed in the seminal paper of Frankel and Romer (1999). Thus, we control for this by introducing the ratio *trade to GDP* in our dataset, where trade is the sum of a country's exports and imports. Additionally, high quality institutions have been shown to have significant effects on GDP per capita (Acemoglu and Robinson, 2012). Therefore, to control for the quality of institutions and governance, we use the *Rule of Law* index from the World Governance Indicators (also via the World Bank), which captures institutional quality of countries. In addition to trade and institutions, formative papers such as Lucas (1988) and Mankiw et al. (1992) have long well established the importance of physical capital in economic development. We capture this using a measure of physical capital stock provided by the Penn World Tables[8].

We also include as development driver the *Economic Complexity Index* (ECI+) (developed by Hidalgo and Hausmann, 2009; Albeaik, Kaltenberg, Alsaleh, and Hidalgo, 2017), which measures how sophisticated an economy's production is. ECI+ is assessed by taking into account the diversity of a nation's exports and the uniqueness of the products that are exported. This index is a preferred measure of economic complexity since it performs better than the original ECI as a predictor of growth (Albeaik et al., 2017). To complement the set of drivers of development, we also use OECD input-output tables to calculate forward and backward I-O linkages[9]. Forward and backward linkages are important factors to consider when analyzing the interdependence between different sectors of an economy and its effect on economic development. First conceptualized by Hirschman (1958), forward linkage is the instance when outputs from a given sector encourage their use as inputs in other sectors. Conversely, backward linkages are instances

[7] Acemoglu et al. (2017) show that economies with larger sectoral dominance display sharper downturns. Dominance is defined as the ratio between the largest Domar weight (supplier centrality) in the network and the variability in sectoral supplier importance. A symmetric and simple network with no dominant sector displays a value of 1, while a network with only one extremely important supplier of intermediates will display a very large value of dominance.

[8] www.rug.nl/ggdc/productivity/pwt

[9] Forward $= \frac{1}{N}(1^{'}(I-\Gamma)^{-1}1)$, Backward $= \frac{1}{N}(1^{'}(I-\tilde{\Gamma}^{'})^{-1}1)$, where Γ is an NxN matrix and an element γ_{ij} represents the importance of j as a supplier of i (i.e., forward linkage). Likewise, $\tilde{\Gamma}$ is an NxN matrix and an element $\tilde{\gamma}_{ij}$ represents the importance of j as a customer of i (i.e., backward linkage). $1^{'}$ and 1 are 1xN and Nx1 matrices, respectively. They help to sum up and collapse the matrices into a single measure.

when growth in a particular sector also encourages growth in other sectors which provide its inputs. Baqaee and Farhi (2019) demonstrated how distortions in such linkages can explain variations in productivity across countries. Summary statistics of these, and all variables used in our study, are provided in the Appendix (Table 3).

Finally, OECD data is also used to estimate our main variable of interest: Production network density ($Density$). Details on how $Density$ is calculated and analyzed is described next.[10]

3.1 Measuring Production Network Diversification

Following Acemoglu et al. (2015) and Miranda-Pinto (2021), we define production network diversification using network density. The production network density, $Density$, of a country measures how interconnected its industries are. To estimate this, we use the formula

$$Density = \frac{\sum_{i=1}^{N} \sum_{j}^{N} 1\left[\tilde{\omega}_{ij} > \underline{\omega}\right]}{N(N-1)}, \tag{1}$$

where $\tilde{\omega}_{ij}$ is an input-output share that can be observed. It captures the portion of intermediate input that originated from sector i and subsequently shipped to sector's j total expenditure on intermediates. These input-output relationships are not symmetrical; there may be situations, for example, in which sector i provides inputs to sector j, but sector j does not supply intermediates to sector i. $1\left[\tilde{\omega}_{ij} > 0\right]$ is a function that determines input-output connections that are larger than a tiny threshold of $\underline{\omega} \in [0.00001, 0.001]$. N is the number of sectors. If $Density$ is equivalent to 0, this indicates that no sectors in the economy rely on others for production. However, if $Density$ equals 1, then each sector relies on all other sectors for production purposes.

Table 1 displays descriptive statistics for total $Density$ (assuming $\underline{\omega} = 0.001$), its variation across countries and over time, and GDP per capita.[11] Across all our observations, $Density$ has a mean of 0.811, implying that the average country in our sample, on an average year, will have about 856 network connections out of a possible 1,056 connections (81.1 per cent of possible connections). Delving into the average standard deviation (SD) of $Density$ within countries (over time), we see an average of 0.022, which tells us that a single deviation from the mean within a given country would create roughly 20 extra net-

[10]The OECD input-output data have two different sources: first, the 'ISIC Revision 3' that covers 33 industries over the period 1995-2011; and, the 'ISIC Revision 4' that covers 35 industries from 2005 to 2015. For consistency, we decided to use the ISIC Revision 3, to have comparable network density measures over time. Source: `http://www.oecd.org/sti/ind/input-outputtables.htm`.

[11]Throughout the paper we use total $Density$, which includes domestic linkages and imported intermediates. In the Appendix we show how results hold for network density considering only domestic linkages.

work linkages. With regard to the average standard deviation of $Density$ across countries, we observe a variation of around 96 production network linkages. This demonstrates that there is a healthy variation in network densities throughout the 55 countries' data and over time. Focusing on GDP per capita (US PPP dollars), we see that there is also a decent amount of variability in income per capita across countries, given a standard deviation (\$13,219) that is more than half the average national income (\$20,743).

Table 1. Descriptive statistics

Variable	Obs.	Mean	Std. Dev.	Min.	Max.
$Density$ (Production network density)	935	0.811	0.094	0.498	0.980
SD of $Density$ across countries	55	0.094	0.003	0.088	0.098
SD of $Density$ within countries	17	0.022	0.016	0.003	0.091
GDP per capita ($ US dollars PPP)	935	20,743	13,219	789	75,113

Figure 1 provides an illustrative example of $Density$. The figure shows the production network structure of Thailand and Denmark in 1995. The size of each node represents the relative share of that particular sector in the economy. Each number represents a sector (see details in Table 7 in the Appendix). For example, sector s20 (Construction) and sector s21 (Wholesale trade) are among the largest sectors in both economies. There is also clear heterogeneity in sectoral composition, consistent with each country's stage of development. While the Agriculture and Textile sectors (sectors s1 and s4, respectively) are among the largest sectors in Thailand, the Real Estate and Health sectors (sector s26 and s32, respectively) are among the largest sectors in Denmark.

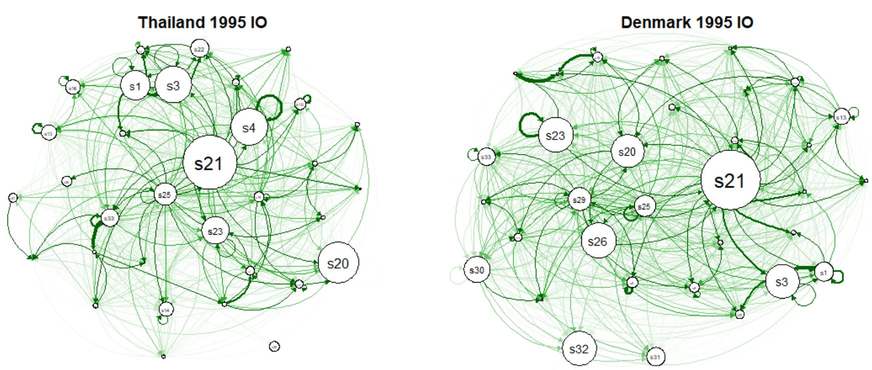
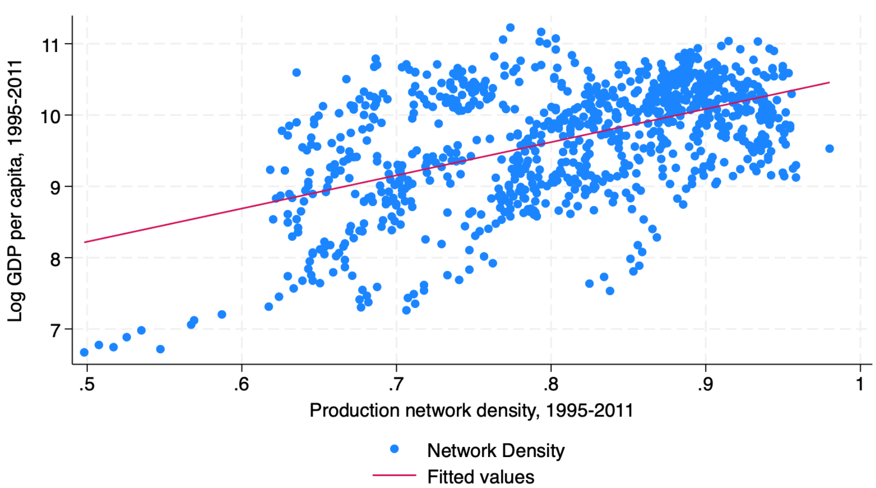


Figure 1. Input-output network in Thailand and Denmark in 1995

Note: This figure shows the production network of Thailand and Denmark in 1995 using the ISIC rev. 3 sectoral classification. Each node (circle) is a different sector in the economy, and the size of the node represents sectoral output shares (the labels in the nodes are linked to sectors in Table 7 of our Appendix). An arrow from sector i to sector j represents intermediate inputs flowing from i to j. The intensity of the arrow (darkness and width) indicates how much sector i is buying from j as a fraction of total intermediate input expenses. Source: Authors with I-O data from the OECD –see footnote 11.

A network link is represented by an edge from sector i pointing to sector j, which represents intermediate inputs supplied from sector i to sector j. The width of the edge represents the intensity of the connection (intermediate input purchases) as a share of sectors' total sales. Visually, one can see that Denmark has a denser production network than Thailand. Indeed, in 1995, 86% of sectoral connections in Denmark were non-zero ($Density = 0.86$), while in Thailand, only 62% of sectoral connections were active ($Density = 0.62$). To put these numbers into perspective, in 1995, the Danish economy had about 250 extra linkages compared to Thailand. Part of this difference comes from the highly connected service sector in Denmark. We can see that the Financial Intermediation sector (s25) resides in the center of the network, and, even though it does not rank among the largest sectors, it is one of the sectors with the largest number of edges pointing to other sectors. Two additional service sectors are central in the Danish network (but not in the Thai network). These sectors are the Real Estate sector (s26) and the R&D and Professional Services sector (s29).

Figure 2 presents a scatter plot showing our measure of production network density over the years against the respective country (log) GDP per capita. As the figure shows, there is a strong positive association between $Density$ and GDP per capita. On average, an increase in network density of 0.1 —about ten new links among sectors— is associated with a 4.65% higher GDP per capita. [12]



Note: OLS coefficient is 4.65 with a robust standard error of 0.28. R-squared is 0.28.

Figure 2. Production network density ($Density$) and development, all data points

To study the robustness of the association between $Density$ and GDP per capita in Figure 2, the next section specifies the econometric strategy we use to control for additional drivers of economic development, including unobserved and time-invariant

[12] As pointed out by one reviewer, the relationship seems to flatten at higher levels of GDP per capita. We checked this non-linearity and found that the squared value of $Density$ is negative and significant, but producing a turning point at around 0.95. In other words, it is driven by the extremes of distribution of network density (95 percent and above) and it is associated with negligible declines in GDP per capita. We checked non-linearity in the panel regressions performed below, which results were not statistically valid.

country characteristics.

3.2 Econometric strategy

We use within estimators in order to develop a robust understanding of the relationship between $Density$ and GDP per capita. This form of estimation, country fixed effects, helps us to deal with any endogeneity stemming from time-invariant country characteristics. Taking into account the insights from Figure 2, we estimate Equation 2:

$$Ln(Y_{ct}) = \alpha_0 + \alpha_1 Ln(Density_{ct-1}) + \gamma \Psi'_{ct-1} + \epsilon. \tag{2}$$

Here, c is a country and t a given year between 1995 and 2011. The dependent variable Y is GDP per capita PPP adjusted in international dollars. α_1 is our main coefficient of interest and captures the elasticity of $Density$ on GDP per capita. Ψ' is a vector that includes the drivers of economic development. As discussed above, these control variables include measures of institutional quality, market share of the financial, service and commodity sectors, the degree of sectoral dominance in the network, population growth, trade to GDP ratio, years of schooling, the economic complexity index (ECI+), capital stock, and backward and forward linkages. All covariates, including $Density$, are lagged one year to avoid reverse causality issues in the estimation. Year fixed effects are also considered in the estimation of equation (2).

4 Empirical Results

The results from the panel regressions are presented in Table 2. Column 1 shows the parsimonious model results (accounting only for country fixed effects) where *Density* reaches a statistically significant elasticity of 2.1. The within R-squared that measures the fit of the regression within a country over time is 7.3%.

Column 2 presents the results of a model including all additional drivers of economic development discussed above. Column 3 adds year-fixed effects to the estimations. Unfortunately, data on *Rule of law* is not available for all 17 years of our period of study, which drops our observations to 693 in columns 2 and 3. Estimations excluding *Rule of law*, ergo maintaining 880 observations, produce structurally similar results with our coefficients for $Density$ statistically significant, but around 10% lower.

Across the three sets of results, we see that $Density$ has a consistent positive significant effect. This is in line with theory (explained later) and the intuition taken from Figure 2. The $Density$ elasticity in column 3 implies that a 10% increase in the network density of the average country will increase its GDP per capita by roughly $770 PPP international dollars. In the Appendix (Table 4), we show that the same result holds when using only domestic input-output connections. In that case, a 10% increase in domestic connections in the average country is associated with an increase in GDP per capita of roughly $600. Similarly, if we use a different threshold to define an active connection, $\tilde{\omega} = 0.00001$ instead of $\tilde{\omega} = 0.001$, we observe an even larger coefficient (see Table 5 in the Appendix). We also tested estimations excluding Cambodia, which seems as an outlier in our data set presenting low levels of GDP per capita and $Density$ (blue dots in the bottom left section of Figure 2).[13] Results excluding Cambodia, Table 6 in the Appendix, show that $Density$ is still strongly associated with GDP per capita.

Results show that the elasticity of trade is a statistically significant predictor of GDP per capita, reflecting the importance of trade openness in more-modernized and growing economies, as in Frankel and Romer (1999). When accounting only for country fixed effects, service share and education (*Years of schooling*) affects GDP per capita positively, as evidenced in multiple empirical studies (see, for example, Mankiw, Romer, and Weil, 1992) and Moro (2015)).

In line with the 'Resource Curse' hypothesis (Sachs and Warner, 2001), our variable capturing dependence on natural resource extraction shows a negative effect on GDP level of a country. Such a negative effect can come from different 'resource curse' channels, such as Dutch disease effects or temporary loss of learning by doing (Van der Ploeg, 2011; Fleming et al., 2015). *Rule of law* is another statistically significant predictor of higher levels of GDP per capita, pointing to the relevance of institutional strength to economic progress. [14]

5 A Simple Theoretical Rationale

In this section, we propose two theoretical channels in which a denser production network could generate higher output. In this case, the production network is exogenous and we study how different network structures affect the propagation and amplification of sectoral productivity shocks.

The model in this section differs from those of Jones (2011), Bartelme and Gorodnichenko (2015), Fadinger et al. (2015), and McNerney et al. (2022) in the following aspects. First, the model economy displays non-unitary elasticity of substitution between intermediate inputs and labor. Second, as implied by standard CES production technologies, the productivity of the intermediate input bundle depends on how diversi-

[13] We thank a reviewer for noting this point.

[14] Other World Governance Indicators such as *Control of corruption*, *Government effectiveness*, *Voice and accountability*, *Political stability*, and *Regulatory quality* were also considered in our estimations but left out do to issues of high collinearity. In any case, their inclusion do not change our results.

Table 2. Panel Fixed Effects results, with lags, 1995-2011

	(1)	(2)	(3)
	Ln *GDP pc*	Ln *GDP pc*	Ln *GDP pc*
Ln $Density_{t-1}$	2.102***	0.443**	0.370***
	(0.753)	(0.187)	(0.136)
ECI+_{t-1}		0.078	0.095
		(0.089)	(0.070)
Ln $Sectoral\ dominance_{t-1}$		-0.178*	-0.155**
		(0.094)	(0.071)
Ln $Financial\ sector\ share_{t-1}$		0.095	0.088
		(0.078)	(0.070)
Ln $Service\ sector\ share_{t-1}$		0.461**	0.113
		(0.199)	(0.199)
Ln $Natural\ resources\ share_{t-1}$		-0.415***	-0.377***
		(0.031)	(0.051)
Ln $Trade\ to\ GDP_{t-1}$		0.306***	0.212***
		(0.059)	(0.045)
Ln $Years\ of\ schooling_{t-1}$		0.431***	-0.095
		(0.101)	(0.099)
$Rule\ of\ law_{t-1}$		0.077*	0.097***
		(0.044)	(0.036)
Population growth		-0.018*	-0.006
		(0.001)	(0.008)
Ln $Capital\ stock_{t-1}$		0.014	0.044
		(0.042)	(0.043)
Ln $Backward\ I\text{-}O\ linkages_{t-1}$		0.030	0.0006
		(0.027)	(0.024)
Ln $Forward\ I\text{-}O\ linkages_{t-1}$		0.309	0.134
		(0.224)	(0.197)
Constant	10.15***	7.014***	8.117***
	(0.163)	(0.718)	(0.735)
Observations	880	693	693
R-squared	0.900	0.991	0.995
Number of countries	55	55	55
Country FE	Yes	Yes	Yes
Year FE	No	No	Yes

Note: This table presents a panel fixed-effect regression using log GDP per capita as the dependent variable and several time-variant country characteristics as independent variables. Robust standard errors in parentheses, clustered by country. *** Significant at the 1-percent level; ** Significant at the 5-percent level; * Significant at the 10-percent level.

fied the mix of intermediates is. Third, with CES, the sectoral productivity shocks have asymmetric effects, depending on the network diversification and production flexibility. Due to these three elements, absent from the papers cited above, the model in this section delivers a role for production network diversification in shaping aggregate levels of GDP.[15]

Firms

The economy is composed of N sectors. In each sector, there is a continuum of homogeneous firms that behave competitively. The CES production technology of firms in sector j is

$$Q_j = Z_j\Big(a_j^{\frac{1}{\epsilon_Q}} L_j^{\frac{\epsilon_Q-1}{\epsilon_Q}} + (1-a_j)^{\frac{1}{\epsilon_Q}} M_j^{\frac{\epsilon_Q-1}{\epsilon_Q}}\Big)^{\frac{\epsilon_Q}{\epsilon_Q-1}}, \tag{3}$$

in which the intermediate input bundle is

$$M_j = \Big(\sum_{i=1}^{N} \omega_{ij}^{\varrho_M} M_{ij}^{\frac{\epsilon_M-1}{\epsilon_M}}\Big)^{\frac{\epsilon_M}{\epsilon_M-1}}. \tag{4}$$

The gross output of the representative firm in sector j is Q_j. Sectoral total factor productivity is Z_j; labor is L_j; M_j is the intermediate input bundle of sector j; and M_{ij} is the amount of intermediates that sector j purchases from sector i. The parameter a_j represents how important labor is in the total value of production. The element ω_{ij} reflects the importance of sector i as an input supplier to sector j. Hence, the square matrix Ω—of dimension N and typical element ω_{ij}—defines the input-output structure of the economy. The elasticity of substitution between labor and intermediates is denoted by ϵ_Q, and the elasticity of substitution among material varieties is ϵ_M. While we allow these two elasticities to differ, for mathematical and computational simplicity most of the analysis assumes $\epsilon_Q = \epsilon_M$.

The parameter ϱ_M captures the love for diversification in the bundle of intermediates. As we explain later, the elasticities of substitution between inputs and the love-for-diversification parameters are crucial in determining the relationship between the production network structure and GDP.

[15]The model is similar to the one developed in Miranda-Pinto (2021), but that model focuses on the relationship between the production network structure and short-term macroeconomic volatility, while the present study focuses on the relationship between the structure of the production network and long-term GDP levels.

Households

The representative household maximizes utility

$$U(C_1, ..., C_N) = \prod_{j=1}^{N} C_j^{\beta_j}, \tag{5}$$

subject to the budget constraint

$$w\bar{L} + \sum_{j=1}^{N} \pi_j = \sum_{j=1}^{N} P_j C_j, \tag{6}$$

in which C_j is the household's consumption of sector j's output. The parameter β_j represents the importance of sectoral consumption in aggregate consumption expenditure. We have that $\sum_{j=1}^{N} \beta_j = 1$. We assume that labor is supplied inelastically; π_j is profit from firms in sector j; w is the wage rate; and P_j represents the price of sector j's good.

We use bold letters to distinguish between vectors/matrices and scalars. For example, while P_j is the price of sector j, $\mathbf{P} = [P_1 \ldots P_N]$ is the vector of dimension N by 1 that contains all sectoral prices. Throughout the paper $\mathbf{I}$ refers to the identity matrix and $\mathbf{1}$ a vector of dimension N by 1 full of ones.

Proposition 1 provides the solution for the competitive equilibrium of the model.[16]

Proposition 1 *Assume that $\epsilon_Q = \epsilon_M \neq 1$; and a labor endowment $\bar{L} = 1$. Then, log real GDP ($GDP = C$) in this economy is*

$$\log C = \sum_{j=1}^{N} \beta_j \log\left(\frac{\beta_j}{P_j}\right), \tag{7}$$

while the vector of sectoral prices is

$$\mathbf{P}^{1-\epsilon_Q} = [\mathbf{I} - \mathbf{Z}^{\epsilon_Q - 1} \circ (1 - \mathbf{a})\mathbf{1}' \circ \left(\mathbf{\Omega}'\right)^{\varrho_M \epsilon_Q})]^{-1}(\mathbf{Z}^{\epsilon_Q - 1} \circ \mathbf{a}). \tag{8}$$

Proof: See Appendix.

Proposition 1 shows that log GDP depends on sectoral prices, which, in turn, are a function of the production network structure ($\mathbf{\Omega}$, a), sectoral productivity $\mathbf{Z}$, the elasticity of substitution between inputs ϵ_Q, and the love-for-diversification parameter ϱ_M. As we will see next, log GDP is a function of the production network density, and their relationship depends crucially on the value of $\varrho_M \epsilon_Q$. Bartelme and Gorodnichenko (2015)

[16] Competitive equilibrium is defined as follows. Firms and households take prices as given, and given prices maximize their objective functions subject to constraints, such that the goods market and the labor market clear.

and Fadinger et al. (2021) obtain a related result for the relationship between the input-output structure and log GDP. However, the authors explore the case of unitary elasticity of substitution, in which, as we will see below, the relationship between network density and GDP is non-existent.

The previous proposition presents the global non-linear solution of the model, for any sequence of sectoral productivity. However, to better highlight the key mechanisms at play, the next proposition provides a second-order approximation to log real GDP. Here we borrow results from Baqaee and Farhi (2019)'s Proposition 7. We contribute to these papers by studying the first and second-order effects of productivity shocks for networks that differ in their diversification.

Proposition 2 *To a second-order approximation, around the steady-state equilibrium ($Z_j = 1, \forall j$) of real GDP ($\bar{C}$), and a vector of steady-state sales shares to GDP ratios (Domar weights) $\overline{\lambda}$ of size $N \times 1$, the effect of iid sectoral productivity shocks on real GDP, C, is given by*

$$\log C = \log \overline{C} + \sum_{i=1}^{N} \underbrace{\overline{\lambda}_i}_{\text{size}} \log Z_i + \sum_{i=1}^{N} \frac{1}{2} \underbrace{\frac{d \log \lambda_i}{d \log Z_i}}_{\text{resizing}} (\log Z_i)^2, \tag{9}$$

$$\log \overline{C} = \sum_{j=1}^{N} \beta_j \log \left(\frac{\beta_j}{\overline{P}_j} \right), \tag{10}$$

while the steady-state vector of sectoral prices $\overline{\mathbf{P}}$ and Domar weights $\overline{\lambda}$ are

$$\overline{\mathbf{P}}^{1-\epsilon_Q} = [\mathbf{I} - (1 - \mathbf{a})\mathbf{1}' \circ \left(\mathbf{\Omega}' \right)^{\varrho_M \epsilon_Q})]^{-1} \mathbf{a}, \tag{11}$$

$$\overline{\lambda} = [I - \overline{\mathbf{\Gamma}}]^{-1} \beta, \tag{12}$$

where $\overline{\mathbf{\Gamma}}$ is the steady-state matrix of input-output shares. These shares are equilibrium objects that depend on $\mathbf{\Omega}$, $\mathbf{a}$, ϵ_Q, ϵ_M, ϱ_M, and $\mathbf{Z}$. The changes in sectoral Domar weights are

$$\frac{\mathrm{d}\lambda_i}{\mathrm{d}\log Z_i} = \sum_{j=1}^{N} \lambda_j \left(- \sum_{k=1}^{N} \sum_{h=1}^{N} \overline{\gamma}_{kj} \left(\epsilon_M (\overline{\omega}_{hj} - \delta_{hk}) + (\epsilon_Q - 1)(\overline{\gamma}_{hj} - \overline{\omega}_{hj}) \right) \Psi_{ik} \Psi_{ih} \right), \tag{13}$$

where $\delta_{hk} = 1$ if $h = k$, and 0 otherwise. $\overline{\gamma}_{kj} = \frac{P_k M_{kj}}{P_j Q_j}$, $\overline{\omega}_{kj} = \frac{P_k M_{kj}}{\sum_l^N P_l M_{lj}}$, and $\Psi = [I - \overline{\Gamma}]^{-1}$.

The contribution of this paper is to study the role of production network diversification in shaping these channels. In particular, as we show next, all else equal, production network diversification, can affect the steady-state level of real GDP, can affect the Domar weights of the economy, and can shape the strength of the non-linear *resizing* mechanism.

Previous literature has highlighted the role of forward input-output (IO) linkages (Leontief inverse matrix) in the propagation of shocks and the determination of real GDP (Acemoglu et al., 2012; Liu, 2019, e.g.,). In this paper, we do not claim that forward IO linkages are not relevant but instead provide an additional network statistic that is useful to understand cross-country differences in income. As we discussed above, when technologies are Cobb-Douglas, observed IO linkages are a sufficient statistic to understand real GDP. Once we deviate from unitary elasticities of substitution, forward IO linkages become endogenous and production network diversification plays a role by itself, either by shaping the level of forward IO linkages or shaping the way forward IO linkages change in response to shocks. Network density has different effects, depending on the value of production elasticities, and it affects the economy through two channels, the love of diversification channel and the non-linear propagation channel.

5.1 Gains from diversification (first-order effect)

To better examine the role of production network diversification, we study a two sectors economy. At steady state, $Z_j = 1$ for all j, with common a, β and ϵ_Q, ϵ_M ($\epsilon_M = \epsilon_Q$) across sectors and networks, we have that steady-state output is

$$\log \overline{C} = -\frac{\beta}{1-\epsilon_Q}\left[\log\left(a(1-\widetilde{\gamma}_{22}+\widetilde{\gamma}_{12})\right)+\log\left(a(1-\widetilde{\gamma}_{11}+\widetilde{\gamma}_{21})\right)-2\log\left((1-\widetilde{\gamma}_{11})(1-\widetilde{\gamma}_{22})-\widetilde{\gamma}_{12}\widetilde{\gamma}_{21}\right)\right],$$

in which $\widetilde{\gamma}_{ij} = (1-a)\omega_{ij}^{\varrho_M \epsilon_Q}$.

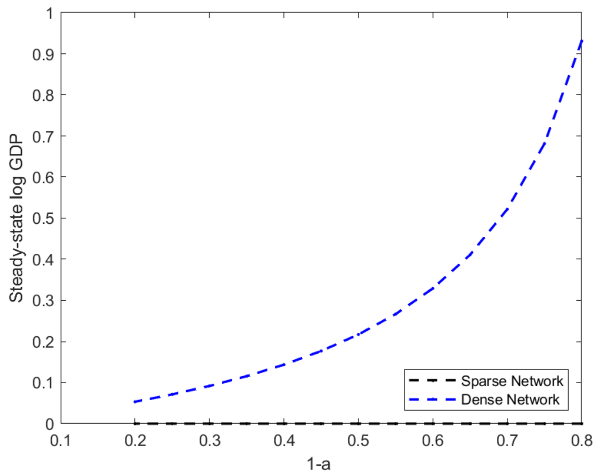
We now study symmetric networks. In this case, the matrix Ω, which represents the input-output network of the economy, displays homogeneous row sums (first-order outdegree). Thus, when a, ϵ_Q, and ϵ_M are common across sectors, sectoral prices are the same across sectors, within the network. Let us define the following symmetric networks

$$\Omega^{sparse} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \text{and} \qquad \Omega^{denser} \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}.$$

$$\log \overline{C}^{dense} - \log \overline{C}^{sparse} = -\frac{2\beta}{1-\epsilon_Q}\left[\log a - \log\left(1-2(1-a)(\frac{1}{2})^{\varrho_M \epsilon_Q}\right)\right].$$

Figure 3 shows $\log \overline{C}^{dense} - \log \overline{C}^{sparse}$ for different values of $(1-a)$, the importance of intermediate inputs in production. We impose $\varrho_M \epsilon_Q < 1$ and $\epsilon_Q > 1$, which we explain

Figure 3. Steady-state log real GDP and production diversification (size)



Note: This figure depicts the difference between log real GDP in the dense network and the sparse network (y-axis). The x-axis is the distribution parameter for intermediate inputs in production. The parameter values assumed are $\beta = 0.5$, $\varrho_M = 0.78$, $\epsilon_Q = 1.2$

next. We observe that in this case, the dense network has larger real GDP than the sparse network. The difference is increasing in the importance of intermediate inputs in production $(1-a)$.

Note that, implied in Dupor (1999), Acemoglu et al. (2012), Bartelme and Gorodnichenko (2015), and Fadinger et al. (2021), when $\epsilon_Q = 1$ or when $\epsilon_Q = \frac{1}{\varrho_M}$, both networks behave the same. In particular, we have $\log \overline{C}^{dense} - \log \overline{C}^{sparse} = 0$. Two symmetric networks, which differ only in terms of their production diversification, display the same sectoral centralities and aggregate GDP, in steady-state. In such a case, the extensive margin of connections does not affect the equilibrium intensity of sectoral connections, which, in the end, determines how shocks propagate along the production chain.

The love for diversification affects the productivity of the intermediate input bundle. The relative price of the intermediate input bundle P_j^M/P_j can be expressed as

$$\frac{P_j^M}{P_j} = \frac{1}{P_j}\left(\sum_{i=1}^{N} \omega_{ij}^{\varrho_M \epsilon_M} P_i^{1-\epsilon_M}\right)^{\frac{1}{1-\epsilon_M}}$$

In symmetric networks, in which sectoral prices P_j are the same across sectors, the intermediate input price index becomes $\frac{P_j^M}{P_j} = \left(\sum_{i=1}^{N} \omega_{ij}^{\varrho_M \epsilon_M}\right)^{\frac{1}{1-\epsilon_M}}$.

In this simple case, having higher production diversification (more intermediates) is associated with a lower relative cost of the intermediate input bundle. When this is the case, if firms have an elasticity of substitution between inputs ϵ_Q above one, they are able to enjoy the love-for-diversification embedded in the intermediate input bundle.

We aim to test this implication using WIOD data on sectoral prices, intermediate input shares, and the number of suppliers with an intermediate input share larger than 1%. Here we are forced to use WIOD data instead of OECD as the latter does not have data on sectoral price indices. In this case, we can only study the relationship between input prices and input diversification for sectors in 40 countries. We note that our evidence here is only suggestive as we are not controlling for unobserved changes in productivity. Table 8 of the Appendix shows that when sectors increase their input diversification by increasing their number of suppliers, the relative price of intermediate to output declines.

In a recent paper, Baqaee et al. (2023) use microdata to support the existence of a love-of-variety in the intermediate input demand of Belgian firms.

5.2 Resilience from diversification and flexibility (second-order effect)

Outside the deterministic steady-state, log real GDP is driven by productivity shocks. As we can see in Proposition 2, these shocks propagate more or less strongly depending on the production network structure. Facing a given sequence of non-negative productivity shocks, an economy with larger Domar weights will experience an increase in real GDP (”*size*”).

If log productivity is, on average, positive, sectoral Domar weights, shaped by the network structure, determine the response of real GDP to changes in productivity (up to first order). In our two sectors example, the vector of steady-state Domar weights is

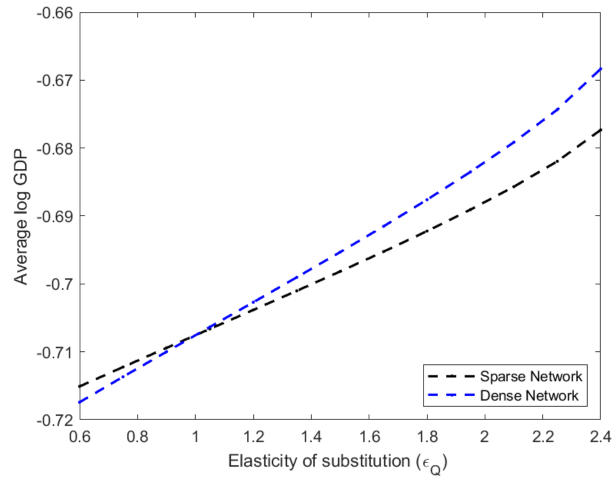
$$\begin{bmatrix} \overline{\lambda}_1 \\ \overline{\lambda}_2 \end{bmatrix} = \begin{bmatrix} \frac{\beta(1-\widetilde{\gamma}_{22}+\widetilde{\gamma}_{12})}{(1-\widetilde{\gamma}_{11})(1-\widetilde{\gamma}_{22})-\widetilde{\gamma}_{12}\widetilde{\gamma}_{21}} \\ \frac{\beta(1-\widetilde{\gamma}_{11}+\widetilde{\gamma}_{21})}{(1-\widetilde{\gamma}_{11})(1-\widetilde{\gamma}_{22})-\widetilde{\gamma}_{12}\widetilde{\gamma}_{21}} \end{bmatrix},$$

where $\widetilde{\gamma}_{ij} = \left((1-a)\omega_{ij}\right)^{\varrho_M \epsilon_Q}$.

Here, $\overline{\lambda}_i = \frac{\beta}{a}$ in the sparse , while $\overline{\lambda}_i = \frac{\beta}{1-\left(\frac{(1-a)}{2}\right)^{\varrho_M \epsilon_Q}}$. When $\varrho_M = \frac{1}{\epsilon_Q}$, both networks display the same Domar weights in steady-state ($\overline{\lambda}_i = \frac{\beta}{a}$). However, when $\varrho_M \epsilon_Q < 1$, the more diversified network has a larger network multiplier. If, in addition, log sectoral productivity has a positive mean, the more diversified economy will enjoy higher GDP.

The last term in equation 9 contains the *resizing* effect in which log productivity, even if its mean is zero, can generate deviations from steady-state real GDP. Extending the analysis in Baqaee and Farhi (2019), here we show that, on average, economies with higher production network diversification also have higher real GDP, when $\epsilon_Q > 1$. As Baqaee and Farhi (2019) demonstrated when inputs are substitutes, positive productivity shocks are amplified, while negative productivity shocks are mitigated. Here, we show

Figure 4. Average log GDP and production network diversification (resizing)



Note: This figure depicts the difference log real GDP in the dense network and the sparse network (y-axis). We assume that sectoral productivity follows a log normal distribution with mean 0 and standard deviation 0.1. Average log real GDP is calculated from 100 simulations of size 1000. The parameter values assumed are $\beta = 0.5$, $\varrho_M = 0.78$, $\epsilon_Q = 1.2$

that this non-linearity is amplified by production network diversification.[17]

Figure 4 plots log real GDP for these two networks using Proposition 2. To isolate the different mechanisms, we assume that $\varrho_M = 1/\epsilon_Q$, meaning there is no love for diversification and $\overline{C}$ and $\bar{\lambda}_i$ are the same in the dense and the sparse network. We can see that the larger the substitutability between inputs, the larger is log real GDP in the dense network compared to the spare network.

5.3 Calibration

In this section, we study the ability of the model to replicate the quantitative relationship between $Density$ and GDP observed in the data. We calibrate the model to match each country's production structure in 1995. We calibrate $1 - a_j$, the importance of intermediate inputs in gross output of sector j, and ω_{ij}, the importance of intermediate input i in all usage of intermediates of sector j, using an iterative process that matches the data and the model's implied intermediate input share.[18] The model is highly non-linear when ϵ_Q and ϵ_M are different from one. The literature estimating production elasticities is large and the results vary substantially depending on the frequency of the data used for estimation, the level of sectoral aggregation, and the econometric approach used, among others. Given that our goal is to understand medium-term to long-term

[17] Miranda-Pinto, Silva, and Young (2023) also study the non-linear effects of network density but their focus is on GDP skewness. In particular, the authors focus on the role of network density in amplifying large negative shocks in the short-run, when firms have a hard time substituting inputs.

[18] More details on the algorithm in Appendix 6.2

relationships, we use lower-frequency elasticities estimated in the literature. Carvalho et al. (2021) ($\epsilon_M = 1.3$, $\epsilon_Q = 0.6$), Huneeus (2018) ($\epsilon_M = 2.8$, $\epsilon_Q = 2.5$), Peter and Ruane (2023) ($\epsilon_M = 3.1$, $\epsilon_Q = 0.6$), Miranda-Pinto (2021) ($\epsilon_M = 1$ and $\epsilon_Q = 3.1$ for service sectors), Nakano and Nishimura (2023) ($\epsilon_M = 1 = \epsilon_Q = 1.5$) estimate production elasticities above one, either for ϵ_Q or ϵ_M. Barrot and Sauvagnat (2016) and Boehm et al. (2019) show that in the shorter term, firms have a much lower ability to substitute among different inputs. Our benchmark calibration uses $\epsilon_Q = \epsilon_M = 1.1$. In our Appendix 6.3, we provide the results for different $\epsilon_Q = \epsilon_M = 1.2$ and $\epsilon_Q = \epsilon_M = 1.3$.

We need to discipline the value of ϱ_M, which drives the love for diversification in our model. Given, the parameter values above, we solve the steady-state of the model for different values of ϱ_M. Then, we obtain the steady-state value of log GDP. For each value of ϱ_M we obtain the OLS coefficient of a cross-sectional regression between log GDP and network density, namely (α_1^{model}). We choose the value of ϱ_M that minimizes $|\alpha_1^{data} - \alpha_1^{model}|$, in which α_1^{data} is the estimated coefficient in column 1 of Table 2, which equals 2.1. Our baseline calibration implies $\varrho_M = 0.746$. We also consider a more conservative calibration in which we match $\alpha_1^{data} = 0.37$ from column 3 of Table 2 ($\varrho_M = 0.86$).

Figure 5 plots the model-implied steady-state log real GDP against the calibrated network $Density$ at the beginning of the period (density in 1995). We observe that the model is able to replicate the observed positive relationship between log GDP and production network density.

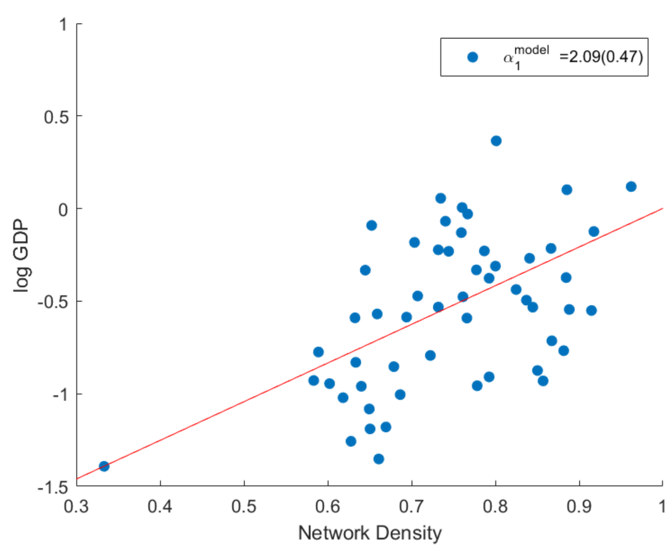


Figure 5. $Density$ and model-implied real GDP

Note: This figure plots, in the y-axis, the model-implied steady-state log real GDP. The x-axis is the value of production network density, calibrated to match countries' network density in 1995. This figure uses $\epsilon_Q = 1.1$ and $\varrho_M = 0.746$

5.4 Counterfactual analysis

We use the model to investigate the role of production network diversification in explaining income differences among countries. To this end, we go beyond the steady-state equilibrium and use data on sectoral productivity dispersion, for a subsample of our countries, to simulate long series ($T = 5000$) of these economies. In particular, we assume that sectoral productivity follows a log-normal distribution with zero mean and standard deviation σ_z. Hence, here we can study the joint role of sectoral productivity and sectoral linkages in driving income differences across countries. Our exercise is the following. We calibrate β, the vector of consumption shares, and a, the vector of labor importance in production, such that the countries are identical in terms of consumption shares and total intermediate input shares. The only differences between countries are: i) the distribution of intermediate inputs embedded in $\mathbf{\Omega}$, and ii) the dispersion of sectoral productivity shocks σ_z, which we calibrate using sectoral TFP data from Fadinger et al. (2021).

We use four countries in our quantitative experiment: Thailand, Indonesia, Denmark, and Australia. Thailand and Indonesia are relatively low-income countries with a GDP per capita in 2011 equal to \$13,535 and \$8,837 USD (PPP adjusted), respectively. These countries also display a relatively low network density of 0.62 and 0.69, respectively, in 1995. On the other hand, Denmark and Australia are relatively high-income countries with a GDP per capita of \$44,403 and \$41,894 USD in 2011, respectively. At the same time, these countries display denser production network structures, with a network density of 0.86 and 84, respectively, in 1995.

In our baseline calibration, $\epsilon_Q = 1.1$, $\varrho_M = 0.746$, our model indicates that if Thailand had Denmark's network diversification, its GDP would be 23.53% larger. In our more conservative calibration, $\epsilon_Q = 1.1$, $\varrho_M = 0.86$, Thailand would experience a 9.54% increase in GDP with Denmark's network structure. On the other hand, if Thailand's dispersion of shocks, $\sigma_z = 0.4$, reduces to Denmark's shocks' volatility, $\sigma_z = 0.21$, the increase in GDP would be 22.63% in the benchmark calibration and 9.35% in the more conservative calibration. When productivity dispersion is the only difference between these countries, income differences are negligible. Hence, productivity dispersion only plays a (small) role through its interaction with production network diversification.[19]

We also compare Indonesia and Australia. In our baseline calibration, $\epsilon_Q = 1.1$, $\varrho_M = 0.746$, if Indonesia had Australia's network diversification, its GDP would increase by 19.23%. In the more conservative calibration, $\epsilon_Q = 1.1$, $\varrho_M = 0.86$, Indonesia would see a 6.8% increase in its. If Indonesia's dispersion of shocks ($\sigma_z = 0.4$) reduces to Australia's

[19]These results do not imply that productivity differences across countries in level, not their volatility, are irrelevant. Indeed, Fadinger et al. (2021) show that sectoral productivity, and its interaction with sectoral network supplier centrality, can play an important role in accounting for income differences.

one ($\sigma_z = 0.15$), the increase in GDP would be reduced to 18.8% in the benchmark calibration and to 5.8% in the more conservative calibration. These results underscore a stronger interaction between sectoral productivity dispersion and network structure.

In our Appendix 6.3 we show that our results are robust to different values of ϵ_Q, and the corresponding re-calibrated values for ϱ_M.

Finally, we find no role for network density when $\varrho_M = 1/\epsilon_Q$ and $Z_j = 1$ for all j, which confirms our results in section 5.1. To understand the role of higher-order effects we can compare the steady-state differences in log GDP (first-order effect) with the differences in average log GDP over the simulated economy solved globally. We find that in our benchmark calibration, the role of first-order effects is large. Indeed, more than 90% of the differences are accounted for first-order effects. One reason is the fact that the elasticities we use are not sufficiently larger (smaller) than one.

6 Conclusion

We show that the details of countries' production network structure—specifically, the number of active links in the production network ($Density$)—are strongly associated with their level of GDP per capita. Even after controlling for key country characteristics that are generally used in the economic growth literature, we show that countries with denser production structures display higher average income. The empirical results also provide strong support for the role of institutions, education, and economic complexity in attaining higher income.

We extend the standard production network model in Acemoglu et al. (2012)— displaying input-output linkages, perfect competition and idiosyncratic sectoral productivity shocks—to rationalize the evidence we document. The model indicates that, in the long run, sectoral productivity levels are amplified in denser production structures, as long as intermediate inputs and labor are easily substitutable, and a more diversified input mix provides higher input productivity.

Our paper contributes to the literature that explores the role of production diversification in economic development by documenting a strong cross-country correlation and also highlighting the theoretical benefits of having a more diversified production network structure. However, we should point out that our evidence and theoretical construct should not be interpreted as the only causal mechanism. In other words, even though we show that higher diversity in the use of intermediary inputs across sectors supports higher levels of a country's GDP per capita, we do not explicitly study the costs of diversifying production systems. Nor do we explore the idea that to reach high diversification, a country might first need to reach high income and minimum levels of other assets, such as educational attainment and institutional quality. The determinants and costs of higher levels of $Density$ are important topics for future research.

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Empirical Appendix

Here we present summary statistics of all variables used in our estimations and alternative regression results considering different $Density$ measures: one measuring the network density across only domestic sectors (i.e., excluding connections to sectors from other countries), and another considering a different minimum threshold to define $\underline{\omega}$ ($\underline{\omega} = 0.00001$). We finish by showing the description of all sectors in the OECD I-O data base used to construct $Density$.

Summary Statistics

Table 3. Summary statistics

Variable	Obs	Mean	Std. Dev.
GDP pc ($US PPP)	935	20,743	13,219
$Density$	935	0.811	0.094
ECI+	935	0.818	0.468
Sectoral Dominance	935	2.504	0.394
Financial sector share	935	0.044	0.022
Trade to GDP	935	0.882	0.622
Service share	935	0.613	0.106
Years of schooling	935	9.569	2.385
Population growth (in %)	880	0.731	0.961
Natural resources share	935	0.318	0.326
Rule of law	747	0.721	0.883
Capital Stock ($US PPP in 000')	935	4,274	8,659
Backward I-O linkages	935	3.110	2.140
Forward I-O linkages	935	2.240	0.240

Domestic Network Density

Table 4. Panel Fixed effects results using Domestic Density, 1995-2011

	(1) Ln *GDP pc*	(2) Ln *GDP pc*	(3) Ln *GDP pc*
Ln *Density Domestic*$_{t-1}$	1.019	0.386***	0.293**
	(0.614)	(0.144)	(0.112)
ECI+$_{t-1}$		0.074	0.101
		(0.087)	(0.069)
Ln *Sectoral dominance*$_{t-1}$		-0.179*	-0.151**
		(0.092)	(0.070)
Ln *Financial sector share*$_{t-1}$		0.092	0.084
		(0.080)	(0.071)
Ln *Service sector share*$_{t-1}$		0.490**	0.142
		(0.201)	(0.201)
Ln *Natural resources share*$_{t-1}$		-0.418***	-0.377***
		(0.031)	(0.052)
Ln *Trade to GDP*$_{t-1}$		0.324***	0.229***
		(0.061)	(0.044)
Ln *Years of schooling*$_{t-1}$		0.425***	-0.102
		(0.102)	(0.102)
Rule of law$_{t-1}$		0.082*	0.100***
		(0.045)	(0.036)
Population growth		-0.018*	-0.007
		(0.010)	(0.008)
Ln *Capital stock*$_{t-1}$		0.010	0.039
		(0.043)	(0.044)
Ln *Backward I-O linkages*$_{t-1}$		0.034	0.004
		(0.028)	(0.026)
Ln *Forward I-O linkages*$_{t-1}$		0.303	0.138
		(0.217)	(0.194)
Constant	9.996***	7.014***	8.125***
	(0.181)	(0.710)	(0.733)
Observations	880	693	693
R-squared	0.895	0.991	0.995
Number of countries	55	55	55
Country FE	Yes	Yes	Yes
Year FE	No	No	Yes

Note: Clustered robust standard errors in parentheses. *** Significant at the 1-percent level; ** Significant at the 5-percent level; * Significant at the 10-percent level.

Network Density considering different thresholds

Table 5. Panel FE Total Density Threshold: 0.00001, 1995-2011

	(1)	(2)	(3)
	Ln *GDP pc*	Ln *GDP pc*	Ln *GDP pc*
Ln $Density_{t-1}$	5.838***	0.654***	0.577***
	(1.270)	(0.239)	(0.214)
ECI+$_{t-1}$		0.068	0.086
		(0.089)	(0.066)
Ln $Sectoral\ dominance_{t-1}$		-0.175*	-0.154**
		(0.096)	(0.073)
Ln $Financial\ sector\ share_{t-1}$		0.085	0.080
		(0.079)	(0.071)
Ln $Service\ sector\ share_{t-1}$		0.503**	0.144
		(0.191)	(0.200)
Ln $Natural\ resources\ share_{t-1}$		-0.410***	-0.372***
		(0.033)	(0.055)
Ln $Trade\ to\ GDP_{t-1}$		0.301***	0.207***
		(0.061)	(0.046)
Ln $Years\ of\ schooling_{t-1}$		0.429***	-0.099
		(0.102)	(0.104)
$Rule\ of\ law_{t-1}$		0.063	0.085**
		(0.049)	(0.038)
Population growth		-0.015	-0.003
		(0.009)	(0.007)
Ln $Capital\ stock_{t-1}$		0.004	0.035
		(0.044)	(0.045)
Ln $Backward\ I\text{-}O\ linkages_{t-1}$		0.038	0.0078
		(0.027)	(0.025)
Ln $Forward\ I\text{-}O\ linakges_{t-1}$		0.385*	0.195
		(0.219)	(0.209)
Constant	9.913***	7.041***	8.172***
	(0.047)	(0.741)	(0.760)
Observations	880	693	693
R-squared	0.919	0.990	0.995
Number of countries	55	55	55
Country FE	Yes	Yes	Yes
Year FE	No	No	Yes

Note: Clustered robust standard errors in parentheses. *** Significant at the 1-percent level; ** Significant at the 5-percent level; * Significant at the 10-percent level.

Network Density without Cambodia

Table 6. Panel FE Total Density (without Cambodia), 1995-2011

	(1) Ln *GDP pc*	(2) Ln *GDP pc*	(3) Ln *GDP pc*
Ln *Density*$_{t-1}$	1.802**	0.443**	0.329**
	(0.798)	(0.205)	(0.137)
ECI+$_{t-1}$		0.078	0.083
		(0.089)	(0.069)
Ln *Sectoral dominance*$_{t-1}$		-0.180*	-0.172**
		(0.100)	(0.073)
Ln *Financial sector share*$_{t-1}$		0.094	0.084
		(0.080)	(0.071)
Ln *Service sector share*$_{t-1}$		0.477**	0.134
		(0.205)	(0.202)
Ln *Natural resources share*$_{t-1}$		-0.416***	-0.383***
		(0.031)	(0.052)
Ln *Trade to GDP*$_{t-1}$		0.307***	0.219***
		(0.061)	(0.046)
Ln *Years of schooling*$_{t-1}$		0.435***	-0.104
		(0.104)	(0.101)
Rule of law$_{t-1}$		0.078*	0.097***
		(0.045)	(0.036)
Population growth		-0.017*	-0.006
		(0.010)	(0.008)
Ln *Capital stock*$_{t-1}$		0.011	0.041
		(0.043)	(0.044)
Ln *Backward I-O linkages*$_{t-1}$		0.030	-0.001
		(0.027)	(0.025)
Ln *Forward I-O linkages*$_{t-1}$		0.307	0.156
		(0.227)	(0.195)
Constant	10.12***	7.064***	8.155***
	(0.168)	(0.720)	(0.748)
Observatios	864	680	680
R-squared	0.882	0.989	0.994
Number of countries	54	54	54
Country FE	Yes	Yes	Yes
Year FE	No	No	Yes

Note: Clustered robust standard errors in parentheses. *** Significant at the 1-percent level; ** Significant at the 5-percent level; * Significant at the 10-percent level.

Sectors Definition

Table 7. Sectors in OECD Database

Sector Number	Sector Name
S1	DOM C01T05: Agriculture, hunting, forestry and fishing
S2	DOM C10T14: Mining and quarrying
S3	DOM C15T16: Food products, beverages and tobacco
S4	DOM C17T19: Textiles, textile products, leather and footwear
S5	DOM C20: Wood and products of wood and cork
S6	DOM C21T22: Pulp, paper, paper products, printing and publishing
S7	DOM C23: Coke, refined petroleum products and nuclear fuel
S8	DOM C24: Chemicals and chemical products
S9	DOM C25: Rubber and plastics products
S10	DOM C26: Other non-metallic mineral products
S11	DOM C27: Basic metals
S12	DOM C28: Fabricated metal products
S13	DOM C29: Machinery and equipment, nec
S14	DOM C30T33X: Computer, Electronic and optical equipment
S15	DOM C31: Electrical machinery and apparatus, nec
S16	DOM C34: Motor vehicles, trailers and semi-trailers
S17	DOM C35: Other transport equipment
S18	DOM C36T37: Manufacturing nec; recycling
S19	DOM C40T41: Electricity, gas and water supply
S20	DOM C45: Construction
S21	DOM C50T52: Wholesale and retail trade; repairs
S22	DOM C55: Hotels and restaurants
S23	DOM C60T63: Transport and storage
S24	DOM C64: Post and telecommunications
S25	DOM C65T67: Financial intermediation
S26	DOM C70: Real estate activities
S27	DOM C71: Renting of machinery and equipment
S28	DOM C72: Computer and related activities
S29	DOM C73T74: R&D and other business activities
S30	DOM C75: Public administration and defence; compulsory social security
S31	DOM C80: Education
S32	DOM C85: Health and social work
S33	DOM C90T93: Other community, social and personal services

Table 8. Relative intermediate input cost and indegrees

VARIABLES	(1) $\log(P^M_j/P_j)$	(2) $\log(P^M_j/P_j)$
$\log$(supplier)	-0.014***	-0.010**
	(0.005)	(0.004)
Observations	9,212	9,211
Adjusted R-squared	0.612	0.677
Country-Sector FE	Yes	Yes
Year FE	Yes	Yes
Country-Year FE	No	Yes
Sector-Year FE	No	Yes

Note: This regression uses data on input and output prices from WIOD database for the period 1996-2009. The dependent variable is the logarithm of the price of the intermediate input bundle relative to the output price. The independent variables are the logarithm of the number of suppliers, the intermediate input share, the weighted average of intermediate input shares, and a set of fixed effects listed in the table.

Theoretical Appendix - Proofs

Proof Proposition 1

To ease notation define $\rho_Q = \frac{\epsilon_Q - 1}{\epsilon_Q}$, $\epsilon_Q = \frac{1}{1-\rho_Q}$, $\rho_M = \frac{\epsilon_M - 1}{\epsilon_M}$, $\epsilon_M = \frac{1}{1-\rho_M}$. Also, let $b_j = 1 - a_j$ the importance of materials in production. The firms first order conditions with respect to inputs are:

$$L_j : P_j Z^{\rho_Q} \Big(\frac{a_j Q_j}{L_j}\Big)^{1-\rho_Q} = w$$

$$M_j : P_j Z^{\rho_Q} \left(\frac{b_j Q_j}{M_j}\right)^{1-\rho_Q} = P^M_j$$

Similarly, firms minimize the cost of the bundle of intermediates $\sum_{i=1}^N P_i M_{ij}$ subject to $M_j = \big(\sum_{i=1}^N \omega_{ij}^{\varrho_M} M_{ij}^{\rho_M}\big)^{1/\rho_M}$. In competitive markets, we obtain:

$$P^M_j = \left(\sum_{i=1}^N \omega_{ij}^{\varrho_M \epsilon_M} P_i^{1-\epsilon_M}\right)^{\frac{1}{1-\epsilon_M}}.$$

Assuming common sectoral elasticities, the production function of firms in sector j can be expressed as:

$$Z_j^{-\rho_Q} = a_j^{1-\rho_Q}\left(\frac{L_j}{Q_j}\right)^{\rho_Q} + b_j^{1-\rho_Q}\left(\frac{M_j}{Q_j}\right)^{\rho_Q},$$

which combined with the FONC gives:

$$P_j^{1-\epsilon_Q} = Z_j^{\epsilon_Q - 1} a_j w^{1-\epsilon_Q} + Z_j^{\epsilon_Q - 1} b \Big(\sum_{i=1}^N \omega_{ij}^{\varrho_M \epsilon_M} P_i^{1-\epsilon_M}\Big)^{\frac{1-\epsilon_Q}{1-\epsilon_M}},$$

assuming $\epsilon_Q = \epsilon_M$and that the salario is the numeraire ($w = 1$) we have, in matrices, the solution for prices:

$$P^{1-\epsilon_Q} = [I - Z^{\epsilon_Q - 1} \circ \big((1-a)1' \circ \Omega'^{\varrho_M \epsilon_Q}\big)]^{-1}(Z^{\epsilon_Q - 1} \circ a) \tag{14}$$

To obtain GDP we use the household budget constraint and first order conditions. From the budget constraint, and assuming labor is the numeraire good, we have $P_c C = 1$, implying that $\log GDP = -\log P_c$. We then minimize the consumption expenditure we obtain

$$P_c = \prod_{j=1}^{N} \left(\frac{P_j}{\beta_j}\right)^{\beta_j}.$$

Thus, as we already solved for prices, we have

$$\log GDP = \sum_{j=1}^{N} \beta_j \log\left(\frac{\beta_j}{P_j}\right).$$

Proposition 2

Proposition 2 mainly follows Baqaee and Farhi (2019). Here we solve for sectoral sale shares $\lambda_j = \frac{P_j Q_j}{GDP}$ (Domar Weights). We multiply sectoral market clearing condition in sector j by sectoral price P_j. we obtain

$$P_j Q_j = P_j C_j + \sum_{i=1}^{N} P_j M_{ji},$$

where S_j is sectoral sales. Let's use the household optimal consumption share for each good (with $\epsilon_D = 1$ we have $P_j C_j = \beta_j P_c C$). We multiply the last term by $\frac{P_i Q_i}{P_i Q_i}$ and obtain

$$P_j Q_j = P_j C_j + \sum_{i=1}^{N} \frac{P_j M_{ji}}{P_i Q_i} P_i Q_i,$$

where $\gamma_{ji} = \frac{P_j M_{ji}}{P_i Q_i}$ is the observed intermediate input share (from sector i using input j). We divide both sides of the equation by nominal $GDP = \sum_{i=1}^{N} P_i C_i$ and obtain

$$\lambda_j = \beta_j + \sum_{i=1}^{N} \widetilde{\gamma}_{ji} \lambda_i,$$

in which λ_i is the Domar Weight of sector i. In matrix form we have

$$\lambda = (I - \widetilde{\Gamma})^{-1} \beta.$$

6.1 Details on the Calibration

From the profit-maximizing condition we have (assuming $\epsilon_Q = \epsilon_M$)

$$M_j : P_j Z^{\rho_Q}\Big(\frac{(1-a_j)Q_j}{M_j}\Big)^{1-\rho_Q} = P_j^M,$$

which after some manipulations imply

$$\frac{P_j^M M_j}{P_j Q_j} = (1-a_j)\Big(\frac{P_j^M}{P_j Z_j}\Big)^{1-\epsilon_Q} \tag{15}$$

We solve the cost-minimizing problem of the firms:

$$\mathcal{L} = \sum_{i=1}^{N} P_i M_{ij} + \lambda\Big((M_j - \big(\sum_{i=1}^{N} \omega_{ij}^{\varrho_M} M_{ij}^{\rho_M}\big)^{1/\rho_M}\Big).$$

In competitive markets, the marginal cost of the bundle (λ) equals the price of the bundle

$$\frac{\partial \mathcal{L}}{\partial M_{ij}} = P_i - P_j^M\Big(\sum_{i=1}^{N} \omega_{ij} M_{ij}^{\rho_M}\Big)^{\frac{1-\rho_M}{\rho_M}} \omega_{ij}^{\varrho_M} M_{ij}^{\rho_M - 1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial M_{ij}} = P_i - P_j^M M_j^{1-\rho_M} \omega_{ij}^{\varrho_M} M_{ij}^{\rho_M - 1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial M_{ij}} = P_i = P_j^M\Big(\frac{M_j}{M_{ij}}\Big)^{1-\rho_M} \omega_{ij}^{\varrho_M}.$$

Manipulating the previous equation we obtain the model-implied intermediate input share (as a fraction of total intermediate expenses)

$$\frac{P_i M_{ij}}{P_j^M M_j} = \Big(\frac{P_j^M}{P_i}\Big)^{\epsilon_M - 1} \omega_{ij}^{\varrho_M \epsilon_M}. \tag{16}$$

Replacing the previous equation into the intermediate bundle technology $M_j = \big(\sum_{i=1}^{N} \omega_{ij} M_{ij}^{\rho_M}\big)^{1/\rho_M}$ yields

$$P_j^M = \Big(\sum_{i=1}^{N} \omega_{ij}^{\varrho_M \epsilon_M} P_i^{1-\epsilon_M}\Big)^{\frac{1}{1-\epsilon_M}}. \tag{17}$$

Our goal is to calibrate $(1-a_i)$ and ω_{ji}, given ϵ_Q and ϱ_M, such that the model-implied intermediate input shares $\frac{P_i^M M_i}{P_i Q_i}$ and $\frac{P_j M_{ji}}{P_i^M M_i}$ equal the data counterparts. We then choose different values from the literature on ϵ_Q, while we choose ϱ_M to match the observed relationship between network density and GDP.

6.2 Calibration Algorithm for Ω and $\mathbf{a}$

We use the following algorithm. Take a guess on the elements of the vector a^0 and the matrix Ω^0. This gives us a total of $N(N+1)$ parameters to calibrate. Given this guess, solve for the model implied intermediate input shares, which requires to solve for prices, to obtain:

Equation 15 and Equation (16)

$$\left(\frac{P_j^M M_j}{P_j Q_j}\right)^0 = (1-a_j^0)\left(\left(\frac{P_j^M}{P_j Z_j}\right)^0\right)^{1-\epsilon_Q}$$

$$\left(\frac{P_j M_{ji}}{P_i^M M_i}\right)^0 = \left(\left(\frac{P_j}{P_i^M}\right)^0\right)^{1-\epsilon_Q} (\omega_{ji}^0)^{\varrho_M \epsilon_Q}$$

The next step involves using the observed shares we want to match to back out the next iteration of parameters in Ω^1 and a^1. Essentially, we invert the system to get

$$1-a_j^1 = \left(\frac{P_j^M M_j}{P_j Q_j}\right)^{obs} \left(\left(\frac{P_j^M}{P_j Z_j}\right)^0\right)^{\epsilon_Q - 1}$$

$$\omega_{ji}^1 = \left(\left(\frac{P_j M_{ji}}{P_i^M M_i}\right)^{data}\left(\left(\frac{P_j}{P_i^M}\right)^0\right)^{\epsilon_Q-1}\right)^{\frac{1}{\varrho_M \epsilon_Q}}.$$

Here, the same steps repeat. We invert the system to find the model implied shares and prices. Using those equilibrium objects, and the observed shares, we obtain the next parameter a^2. We iterate until the distance between the model implied shares and the observed shares is small enough.

Indeed, in symmetric networks, sectoral prices P_j are the same across sectors, the relative price of the intermediate input price index becomes $\frac{P_i^M}{P_j} = \left(\sum_{j=1}^N \omega_{ji}^{\varrho_M \epsilon_M}\right)^{\frac{1}{1-\epsilon_M}}$ which we plug it into the above equation, under the assumption of $\epsilon_Q = \epsilon_M$, to get

$$\frac{P_j M_{ji}}{P_i^M M_i} = \frac{\omega_{ji}^{\varrho_M \epsilon_Q}}{\sum_{j=1}^N \omega_{ji}^{\varrho_M \epsilon_M}},$$

which has the following property

$$\sum_{j=1}^N \frac{P_j M_{ji}}{P_i^M M_i} = \sum_{j=1}^N \frac{\omega_{ji}^{\varrho_M \epsilon_Q}}{\sum_{j=1}^N \omega_{ji}^{\varrho_M \epsilon_M}} = 1.$$

Let us come back to the algorithm. Given an initial value ω_{ji}^0 and values for ϱ_M and ϵ_Q, we can solve for prices $\left(\frac{P_j}{P_i^M}\right)^0$ to obtain the model implied $\left(\frac{P_j M_{ji}}{P_i^M M_i}\right)^0$. Using the observed input share $\left(\frac{P_j M_{ji}}{P_i^M M_i}\right)^{data}$ we can solve for our next value for ω_{ji}^1 as follows

$$\omega_{ji}^1 = \left(\left(\frac{P_j M_{ji}}{P_i^M M_i}\right)^{data}\left(\left(\frac{P_j}{P_i^M}\right)^0\right)^{\epsilon_Q-1}\right)^{\frac{1}{\varrho_M \epsilon_Q}}.$$

Suppose that ω_{ji}^0 implies that $\left(\frac{P_j M_{ji}}{P_i^M M_i}\right)^0 < \left(\frac{P_j M_{ji}}{P_i^M M_i}\right)^{data}$, then, given that $\varrho_M \epsilon_Q > 0$, our new guess $\omega_{ji}^1 > \omega_{ji}^0$ to bring model-implied shares closer to the data. Therefore, the algorithm continues until the maximal deviation $\mid \left(\frac{P_j M_{ji}}{P_i^M M_i}\right)^0 - \left(\frac{P_j M_{ji}}{P_i^M M_i}\right)^{data} \mid$ (for all ij combinations) is not larger than $< \varepsilon = 10e^{-}7$.

6.3 Additional quantitative exercises

Here we provide two alternative calibrations of ϵ_Q and ϱ_M. We focus on the conservative scenario that matches an empirical correlation between log GDP per capita and network density of 0.37 (column 3 of Table 2). When $\epsilon_Q = 1.3$, $\varrho_M = 0.5923$, we find that Thailand's GDP would increase by 9.06% if it had Denmark's network diversification. If $\epsilon_Q = 1.2$, then $\varrho_M = 0.726$, in which case Thailand's GDP would increase by 9.21% if it had Denmark's intermediate input share diversification.