Flexibility and Frictions in Multisector Models

By Jorge Miranda-Pinto and Eric R. Young*

We show that during the Great Recession, more-flexible sectors paid lower sectoral bond spreads. We rationalize this fact with a model with input-output linkages, heterogeneous elasticities, and binding working capital constraints in the use of intermediates. We show that the difference in flexibility between upstream and downstream sectors is key for determining the role of input-output linkages in amplifying or mitigating distortions. Calibrating the model to the US economy, we find that our sectoral elasticity estimates amplify distortions by a factor of 1.7 *compared the Cobb-Douglas case, and that, imply an input-output multiplier* 1.2 *times larger than the homogeneous elasticity case.*

JEL: E32, E23, E44

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The standard narrative of the Great Recession is that financial frictions and interconnected sectors translated a small shock to a relatively-unimportant sector—often argued to be an unexpectedly-large number of subprime mortgage defaults—into a large economy-wide decline in economic activity. Recent theoretical work has shown how productivity and financial shocks can be amplified and propagated

* Miranda-Pinto: University of Queensland, Level 6, Colin Clark Building (39), Blair Drive, St. Lucia, QLD 4072, Australia (e-mail: j.mirandapinto@uq.edu.au). Young: Department of Economics, University of Virginia, Charlottesville, VA 22904 (e-mail: ey2d@virginia.edu); Department of Economics, Zhejiang University, China; Research Department, Federal Reserve Bank of Cleveland, Cleveland OH 44114. We thank the editor, Simon Gilchrist, and two anonymous referees for very helpful comments and suggestions that greatly improved the paper. We also thank Enghin Atalay, Efrem Castelnuovo, Begoña Dominguez, Chris Edmond, Georgios Georgiadis, James Harrigan, Guillermo Hausmann-Guil, Allen Head, Beverly Lapham, Diego Legal, James Morley, Sophie Osotimehin, Adrian Pagan, Pierre-Daniel Sarte, Felipe Schwartzmann, Alvaro Silva, and Axel Wieneke for comments, along with seminar participants at the Kansas City Fed, the Cleveland Fed, the Dallas Fed, UCSB, University of Colorado Boulder, University of Tasmania, University of Melbourne, Monash University, SWUFE, UQ Brown-bag, CUFE, and Universidad de Los Andes (Chile). We also thank Egon Zakrajšek and Ernesto Pasten for providing us with the spread data and the price frequency adjustment data, respectively.

by input-output connections.[1] Despite the increased interest in more disaggregated models of the macroeconomy, the literature has overlooked the potential importance of sectoral heterogeneity in production elasticities (which we call flexibility) for understanding i) the extent to which some sectors are more vulnerable to distortions; and ii) the propagation and amplification of sectoral distortions by means of input-output connections.[2]

In this paper, we document the empirical relationship among sectoral elasticities of substitution in production, the elasticity between labor-capital and intermediate inputs, and sectoral credit spreads during the Great Recession. To explain our findings we develop a model with input-output linkages, heterogeneous elasticities, and working capital constraints. Our model illustrates that financing constraints in the use of intermediate inputs were an important contributor to the severity of the Great Recession.

We then study the macroeconomic implications of our model. In a tractable version of the general model–two sectors and two different network structures (Star supplier and Island economies)–we show that distortions in the use of inputs can be amplified or mitigated by input-output linkages, depending on the average flexibility and depending on the difference in flexibility between the upstream and downstream sectors. We also show that while input-output linkages play no role in amplifying aggregate distortions if the elasticities are homogeneous, linkages do matter for aggregate distortions if elasticities are heterogeneous. In a calibrated general production network model we show that our estimated sectoral elasticities amplify distortions by a factor of 1.7 compared to the Cobb-Douglas unitary elasticity case. In addition, we show that our sectoral elasticity estimates imply an input-output multiplier that is 1.2 times larger than the homogeneous elasticity case.

[1]See Horvath (2000), Foerster, Sarte and Watson (2011), Atalay (2017), Miranda-Pinto (2020), Bigio and La'O (2020), Jones (2011), Baqaee and Farhi (2020), Luo (2020), and Osotimehin and Popov (2020) for some important examples.

[2]vom Lehn and Winberry (2020) extend the work in Foerster, Sarte and Watson (2011) and Atalay (2017) to emphasize the importance sectoral heterogeneity in the supply of investment goods, and the implied asymmetry of the capital flows matrix, in accounting for business cycle regularities post-1980.

To estimate sectoral elasticities, using US data on sectoral input shares and relative input prices, we extend the instrumental variables approach of Atalay (2017). We note that, in addition to the estimation bias attributable to unobserved sectoral productivity, time-varying credit frictions generate a bias in the estimation of production elasticities. We correct for this endogeneity in relative input prices using two complementary approaches. First, we use military spending or military news shocks as instruments that act as demand shifters and affect sectoral prices. Second, we consider the estimated elasticities for the whole sample and for the sample that excludes the Great Recession, a period with especially tight credit. We select the *best* sectoral elasticity estimates, based on strength of the instruments and economic theory.[3] To improve the precision of our estimates, we sort industries into 13 groups. We follow Miranda-Pinto (2020) and separate service sectors from non-service sectors.[4] We then group sectors within services and non-services based on the sectoral ranking of residual spread growth during the Great Recession.[5] In this way, we ask whether sectors that saw larger increases in spreads display lower production flexibility.

We then show that sectoral elasticities of substitution between labor-capital and intermediate inputs (ϵ_Q) are systematically correlated with sectoral bond spreads during the Great Recession. We identify the relationship between flexibility (ϵ_Q) and spreads by interacting sectoral elasticities with two time-varying controls: i) a dummy variable that is 1 for the Great Recession period and 0 otherwise; and ii) the excess bond premium (EBP) index developed by Gilchrist and Zakrajsek (2012).[6] We observe that sectors with high production flexibility–elasticity above the median–saw an increase in spread that was 1.66 percentage points smaller during the Great Recession. To further investigate the relationship between flex-

[3]In particular, we choose the estimates that display strong instruments and non-negative elasticity estimate.

[4]Miranda-Pinto (2020) shows that the elasticity of substitution between intermediates and labor-capital is significantly larger (> 1) in service sectors than in non-service sectors (≈ 1).

[5]Residual spread growth is the residual from a regression with sectoral spread growth as the dependent variable and debt to asset ratio, sales, value of plant, and inventories as independent variables.

[6]Unlike sectoral bond spreads, which account for sectoral credit demand and credit supply forces driving spreads, the EBP index accounts for the aggregate credit supply conditions in the economy.

ibility and credit frictions, we use COMPUSTAT firm-level data and show that more-flexible firms also experienced higher short-term liquidity–as measured by working capital to sales ratio–during the Great Recession.

We interpret these facts through the lens of a multisector model with sectoral linkages through intermediates, heterogeneous production elasticities, and working capital constraints. These constraints require input costs to be partially financed in advance using within-period loans collateralized by end-of-period sales.[7] The Lagrange multiplier on these constraints can be interpreted as a spread. In a simple vertical two-sector model, we are able to analytically characterize the relationship between ϵ_Q and the severity with which sectoral constraints bind.

The relationship between ϵ_Q and the Lagrange multiplier of the constraint depends on a multiplicative wedge' between the cost of value-added and intermediate inputs. This wedge depends on three factors: (i) the fraction of costs that must be paid in advance; (ii) the relative importance of intermediates to value-added; and (iii) the fraction of sales that can be pledged as collateral. If the wedge exceeds one for a particular input, then that input is more costly in periods when the constraint is binding. To facilitate analytical results, we assume that the only value-added input is labor and that inputs either face the working capital constraint fully or not at all (the fraction that must be financed in advance is either one or zero). If constraints on intermediates are binding, the model can generate the observed relationship between spreads and frictions, provided that credit conditions became tighter during the Great Recession. The model also shows that input-output connections amplify distortions on intermediates, while they mitigate distortions on labor.[8]

[7]Formally, this arrangement is quite similar to sudden stop models with flow constraints, such as Bianchi (2011), Benigno et al. (2013), or Melcangi (2020). The assumption of sales being collateral for loans instead of the value of physical assets is consistent with the results in Li (2019), who finds that a model with heterogeneous firms and financial frictions matches firm dynamics facts of Japanese firms best if firms can pledge current and future revenue.

[8]This result is consistent with research highlighting the importance of trade credit during the Great Recession (Altinoglu (2020), Luo (2020), Miranda-Pinto and Zhang (2020) and Reischer (2020)), and also implies that distortions in the use of labor typically used in the sudden stops literature, (Bianchi (2011) or Benigno et al. (2013)), have small aggregate effects in production network models.

We then use an extended two-sector model to study the macroeconomic effects of heterogeneous elasticities. Unlike the previous section, which studied how endogenous wedges vary with shocks and technology, in this section we study how exogenous wedges are amplified or mitigated if elasticities are heterogeneous.[9] Our first result indicates that input-output linkages can amplify or mitigate sectoral distortions, depending on the average elasticity and depending on the difference in flexibility between the upstream sector (the intermediate input supplier) and the downstream sector (the intermediate input user). Following a shock to the Star supplier, the Star supplier economy with a relatively less flexible upstream sector experiences a larger decline in real GDP compared to an economy with an Island input-output structure in which firms use intermediate inputs only from their own sector.

In general, lower flexibility implies less quantity adjustment and more price adjustment of the affected sector. Therefore, the larger increase in intermediate input prices in the economy with a less flexible upstream sector also generates a larger increase in the downstream sector price, which, in turn, decreases the real wage of the household. However, the more-flexible downstream sector cuts production more strongly (or increases output less strongly) when faced with a higher intermediate input price, generating a smaller increase in the rents generated by distortions. Hence, the Star supplier input-output structure experiences a sharper decline in the real wage and a smaller increase in rents from distortions, compared to the Island economy.

While homogeneous elasticities imply no role for the structure of input-output linkages in amplifying aggregate distortions, heterogeneous elasticities do generate an important role for input-output linkages. If elasticities are homogeneous, increasing the distortion in both sectors by the same amount scales down the economy in the same way in both the Star supplier and the Island economy. In con-

[9]In the simpler model we assume that both inputs, labor and intermediates, are subject to the same wedge. In the quantitative model, we show that wedges on both inputs are isomorphic to wedges on intermediates.

trast, if elasticities are heterogeneous, the asymmetric structure of input-output linkages in the Star supplier economy does play a role in amplifying distortions.

We then perform a quantitative exercise to assess the roles of sectoral flexibility, sectoral frictions, and input-output connections in amplifying the Great Recession in the US. We calibrate our model using our estimated sectoral elasticities as well as the observed input-output shares and consumption shares in 2007. We follow Gilchrist, Sim and Zakrajšek (2013) and proxy for financial shocks using sectoral bond spreads.[10] The model calibrated to our sectoral elasticities amplifies distortions by a factor of 1.7 compared to the homogeneous unitary elasticity model. This amplification is due to the fact that our average estimated elasticity is substantially larger than one; Osotimehin and Popov (2020) show that a larger average elasticity amplifies the effects from distortions.

We then show that the exact structure of input-output linkages has very limited role under homogeneous elasticities. The log GDP change of the economy calibrated to the observed U.S economy is similar to the log GDP change of the counterfactual Island input-output economy, in which firms source intermediate inputs from the same sector.[11] This result is consistent with the prediction of our simple model. During the Great Recession, the median off-diagonal pairwise correlation between sectoral spreads is 0.94. Therefore, the limited cross-sectoral dispersion in our measure of distortions renders input-output linkages relatively unimportant.

On the other hand, when we calibrate the model using our estimated heterogeneous elasticities, we find that input-output linkages amplify the aggregate effects of distortions by a factor of 1.2 compared to the homogeneous elasticity case. This result is consistent with the predictions of our tractable model, as our elasticity estimates are relatively larger for more-downstream sectors relative to

[10]Gilchrist, Sim and Zakrajšek (2013) show that the extent of input misallocation can be inferred from cross-industry data on the dispersion of industry spreads.

[11]Note that we study the role of the structure of input-output linkages rather than the role of intermediate inputs. Bigio and La'O (2020), Osotimehin and Popov (2020), and Baqaee and Farhi (2020) investigate how the existence of intermediate inputs amplifies the effect of sectoral distortions.

more-upstream sectors. Indeed, on the one hand, the average elasticity of the sectors with a downstreamness measure above the median is 2.48, compared to an average elasticity of 1.77 for sectors with a downstreamness measure below median. On the other hand, the average elasticity of sectors with an upstreamness measure below the median is 2.41, compared to an average elasticity of 1.76 for sectors with an upstreamness measure above the median.

Our paper contributes to a number of distinct literatures. First, we provide new estimates of sectoral production functions suitable for use in multisector business cycle models. In particular, we are the first to note that sectors have different production technologies, and this fact turns out to matter for a number of questions beyond the ones we address here. For example, Miranda-Pinto (2020) shows that heterogeneous production elasticities are crucial for replicating the cross-country correlations between GDP volatility and input-output linkages.

Second, our paper points out the importance of modeling the macroeconomy with sectoral heterogeneity in flexibility and sectoral financial distortions. Distinct from Bigio and La'O (2020), Liu (2019), Baqaee and Farhi (2020), Osotimehin and Popov (2020), and Peter and Ruane (2020), who study the role of sectoral distortions in production network models with homogeneous elasticities, we i) empirically emphasize the role of heterogeneous sectoral elasticities by providing sector-level and firm-level facts that validate the existence of sectoral distortions in the use of intermediate inputs during the Great Recession; ii) explain the connection between elasticities and distortions in a model of endogenous wedges; and iii) highlight the theoretical and quantitative importance of the heterogeneity in flexibility of upstream and downstream firms in determining the role of input-output connections at amplifying or mitigating distortions. While the aforementioned papers study the role of the presence of intermediates, we study the role of the structure of input-output linkages.

We also contribute to the work of Altinoglu (2020), Luo (2020), Miranda-Pinto and Zhang (2020), and Reischer (2020), who study the role of financing constraints in the use of intermediates in the form of trade credit, by allowing for non-unitary and heterogeneous elasticities and by emphasizing the importance of constraints to intermediates over constraints to the labor input. Our paper also relates to the study in Pasten, Schoenle and Weber (2019) of the importance of sectoral heterogeneity at amplifying aggregate monetary policy shocks. Different from that paper, which emphasizes the role of sectoral price stickiness in amplifying monetary policy shocks, we highlight the role of the sectoral elasticity in amplifying or mitigating aggregate distortions.

Finally, our model has implications for sectoral policies. Sectoral distortions in the use of inputs and sectoral linkages imply the existence of significant pecuniary externalities due to the presence of prices in the collateral constraints. Miranda-Pinto (2018) and Liu (2019) study the policy implications of related models.[12] Our paper also highlights the importance of considering input-output linkages and heterogeneous elasticities in the design of input or sales taxes.[13]

I. Spreads and Elasticities

In this section, we analyze the empirical relationship between sectoral bond spread growth during the Great Recession and sectoral heterogeneity in production elasticities. We use sectoral bond spread data from Gilchrist and Zakrajsek (2012). For each non-financial firm, the GZ credit spread measures the arithmetic average of the difference between the firm i bond yield and a hypothetical Treasury security of the same maturity, for all the unsecured bonds issued by firm i at quarter t. The average maturity of the corporate bonds in Gilchrist and Zakrajsek (2012) is 13 years. However, because of the cash flows generated by coupon payments, the average duration of these bonds is considerably shorter. The sectoral bond spread is defined as the median spread of all firms in sector j

[12] Liu (2019) studies which sectors should be subsidized in order to reduce input misallocation the most, while Miranda-Pinto (2018) studies different combinations of input subsidies that are able to fully undo sectoral distortions by taking advantage of sectoral connections.

[13] While we motivate our paper with financial frictions during the Great Recession, sectoral distortions could also represent markups or sectoral input/sales taxes.

at time t.

Figure 1 shows significant heterogeneity in residual credit spread growth (compared to same quarter in the previous year) across sectors during the Great Recession in the U.S. While the hospital and electric equipment sectors experienced a small increase in spreads ($\leq 30\%$ increase), sectors such as publishing industries and performing arts and housing experienced a large increase in bond spreads ($\geq 200\%$ increase).

Table 1 reports the descriptive statistics of sectoral bond spreads, at the 3-digit NAICS classification, for 2007q1 and 2009q1. We observe that during the first quarter of 2009, the median sectoral spread was 4.5 times larger than the median spread in the first quarter of 2007 (6.4% compared to 1.4%). Not only the median, but also the cross-sectoral dispersion of spreads, were substantially higher. The standard deviation of sectoral spreads in the first quarter of 2009 was eight times larger than it was in the first quarter of 2007 (5.7% compared to 0.7%). Similarly, the interquartile range of spreads increased by 3.8 times, from 1.15% to 4.37%.

What can account for the large cross-sectoral heterogeneity in sectoral spreads during the Great Recession? Sectoral leverage is obviously one potential source. Table 1, rows 3 and 4, reports the relevant statistics for sectoral leverage, as measured by the corporate debt to assets ratio. While we observe an increase in the median leverage ratio (from 0.29 to 0.32), we do not observe an increase in the cross-sectoral dispersion of leverage: the standard deviation of sectoral leverage stays at 0.13, while the interquartile range barely declines from 0.17 to 0.16, so clearly something else is driving this heterogeneity.

A. *Framework for estimation of elasticities*

We now turn to estimating sectoral elasticities, so that we can explore the connection between production flexibility and sectoral bond spreads. Suppose that sectoral production uses an aggregate of capital and labor (value added V_j) and an aggregate of intermediates (material input M_j) to produce a final good

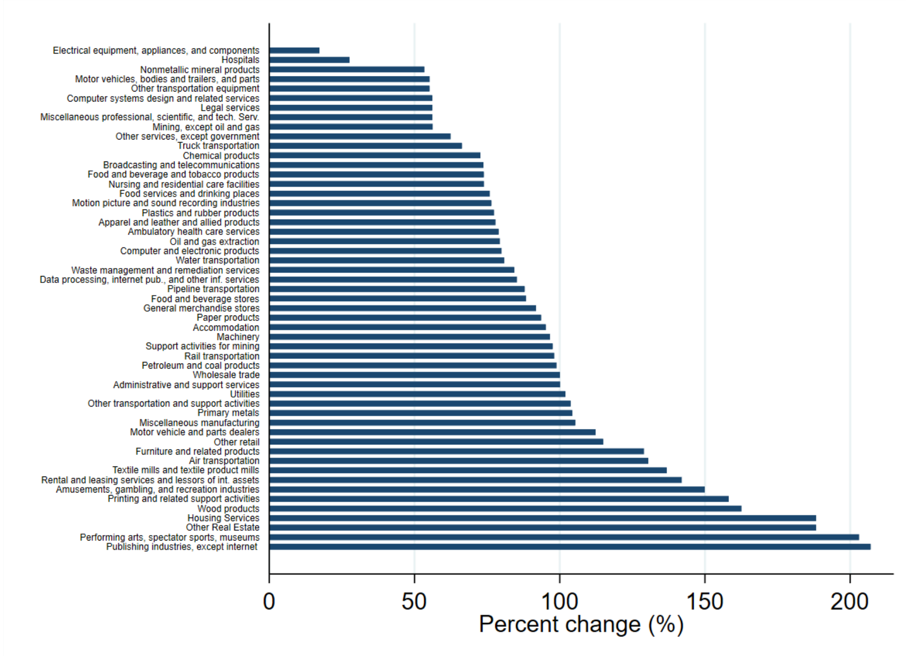

Figure 1. Residual spread growth during The Great Recession

Note: This figure shows sectoral residual spread growth during the Great Recession. The residual spread is the residual from a regression with sectoral spread growth as the dependent variable and the debt to asset ratio, sales, the value of plants, and inventories as independent variables.
Source: Spread data from Gilchrist and Zakrajsek (2012) and Compustat.

Q_j:

$$Q_j = Z_j \left(a_j^{\frac{1}{\epsilon_{Q_j}}} V_j^{\frac{\epsilon_{Q_j}-1}{\epsilon_{Q_j}}} + (1-a_j)^{\frac{1}{\epsilon_{Q_j}}} M_j^{\frac{\epsilon_{Q_j}-1}{\epsilon_{Q_j}}} \right)^{\frac{\epsilon_{Q_j}}{\epsilon_{Q_j}-1}}, \tag{1}$$

where $\epsilon_{Q,j}$ is the elasticity of substitution and is sector-specific. The sectoral total factor productivity is Z_j. The importance of valued-added in gross production is a_j. The material input bundle M_j is constructed using intermediates from all

TABLE 1—SECTORAL SPREADS AND LEVERAGE (2007Q1 AND 2009Q1)

	median	sd	min	p25	p75	max
Bond spreads 2007Q1	1.39%	0.71%	0.59%	1.01%	2.16%	3.85%
Bond spreads 2009Q1	6.29%	5.73%	2.32%	4.42%	8.79%	25.92%
Debt to assets 2007Q1	0.29	0.13	0.09	0.23	0.40	0.76
Debt to assets 2009Q1	0.32	0.13	0.13	0.27	0.43	0.75

Note: In this table, we report descriptive statistics of corporate bond spreads at the 3-digit NAICS industry classification. Source: Gilchrist and Zakrajsek (2012) and Compustat.

sectors:

$$M_j = \left(\sum_{i=1}^{N} \omega_{ij}^{\frac{1}{\epsilon_{M_j}}} M_{ij}^{\frac{\epsilon_{M_j}-1}{\epsilon_{M_j}}} \right)^{\frac{\epsilon_{M_j}}{\epsilon_{M_j}-1}}, \tag{2}$$

where ϵ_{M_j} is the elasticity of substitution between different material inputs, and ω_{ij} represents the importance of intermediate inputs from sector i in the total cost of intermediates of sector j.

In addition, firms are constrained in the financing of inputs. The working capital constraints are

$$\theta_j^v P_j^v V_j + \theta_j^m \sum_{i=1}^{N} P_i M_{ij} \leq \eta_j P_j Q_j, \tag{3}$$

where θ_j^v and θ_j^m are the fraction of the value-added cost (such as office space rent and the wage bill) and the intermediate input (M_j) cost, respectively, that must be paid in advance. Firms are constrained in obtaining external funds by a limited commitment problem. In particular, firms in sector j can borrow only up to a fraction η_j of total revenue $P_j Q_j$.[14]

[14] A microfoundation for this constraint is detailed in Bigio and La'O (2020). Before production takes place, firms borrow the amount of input expenses needed to produce from a competitive financial intermediary. There is a limited commitment problem since, after sales, firms can default on their debt without repaying the intermediary. Therefore, firms are required to pledge a fraction of sales as collateral. If a firm does not repay, the financial intermediary seizes a fraction η_j of total sales. In an equilibrium without default, the incentive compatibility constraint implies that firms can externally borrow up to a fraction η_j of total sales.

The cost minimization conditions imply that

$$\text{(4)}\quad \Delta\log\left(\frac{P_{it}M_{ijt}}{P_{jt}Q_{jt}}\right)=(\epsilon_{M_j}-1)\Delta\log\left(\frac{P^M_{jt}}{P_{it}}\right)+(\epsilon_{Q_j}-1)\Delta\log\left(\frac{P_{jt}}{P^M_{jt}}\right)+\nu_{jt},$$

in which P_{jt} is the price of output produced in sector j and P^M_{jt} is the price index for the bundle of intermediates used as inputs by sector j (see Appendix A.A1 for more details). The error term ν_{jt} is a function of sectoral unobserved productivity and credit wedges. In particular, $\nu_{jt}=(\epsilon_{Q_j}-1)\Delta\log(Z_{jt})+\epsilon_{Q_j}\Delta\log\left(\overline{\vartheta}_{jt}\right)$, where $\overline{\vartheta}_{jt}$ is the sectoral wedge. The credit wedge, which we explain in more detail in the next section, is a function of the Lagrange multiplier of the constraint (μ_{jt}) and the importance of working capital constraints on intermediates θ^m_j and value-added inputs θ^v_j.[15] If firms are unconstrained, $\mu_j=0$ and $\vartheta_j=1$, while $\mu_j>0$ and $0<\vartheta_j<1$ indicate that firms in sector j are constrained.

Time variation in productivities and wedges biases the OLS estimators of the elasticities. The literature typically emphasizes the role of unobserved productivity in this bias (such as Atalay (2017)). Here, we note that the estimation of sectoral elasticities is further biased by the presence of time-varying sectoral frictions in the use of inputs (ϑ_{jt}). We follow Atalay (2017) and use military spending or military news shocks from Ramey and Zubairy (2018) as instruments that act as exogenous demand shifters. Alternatively, for any set of instruments, we estimate IV regressions for the whole sample and for the sample before the Great Recession. We do expect weaker instruments with the whole sample, given the fact that time variation in sectoral wedges during the Great Recession can induce important bias in the estimation of elasticities. We will select the best elasticity estimate, sample and instrument, based on the strength of the first stage and on economic theory (meaning that we require non-negative elasticities).

[15] We define the wedge, from the binding collateral constraint, in the cost minimization problem as $\overline{\vartheta}_{jt}$, while we define the wedge from the profit maximization problem as ϑ_{jt}.

B. IV estimates

We first consider the instrument used in Acemoglu, Akcigit and Kerr (2015) and Atalay (2017), sectoral military spending.[16] Higher military spending in sector j, or in sectors that use sector j output intensively, increases the demand for sector j's output and, therefore, increases the price. The assumptions implicit here are that military spending is orthogonal to changes in sectoral productivity and that spending affects input shares only through changes in the relative cost of inputs.[17]

Following Atalay (2017), we construct instruments for the output price of sector j (P_{jt}), the price of the intermediate input bundle of sector j (P_{jt}^M), and the price of the intermediate input from sector i (P_{it}) that is used in the production of sector j. To formally define the instrument, define S_{ji} as the share of sector j's output that is purchased by sector i. Our instruments are, then,

$$Military_{p_j,t} = \sum_i (I-S)^{-1}_{ji}\, S_{i,military} \cdot \Delta \log(MilitarySpending_t),$$

$$Military_{p_i,t} = \sum_j (I-S)^{-1}_{ij}\, S_{j,military} \cdot \Delta \log(MilitarySpending_t)$$

$$Military_{p_j^m,t} = \sum_i \frac{P_{ijt}M_{ijt}}{P_{jt}^M M_{jt}} \cdot Military_{p_i,t}.$$

The term $(I-S)^{-1}$ measures the sum of direct and indirect changes that occur due to network connections.[18] Changes in military spending on sector i's output can have important indirect effects on sector j's output demand i) if military industries purchase a large fraction of sector i's output (large $S_{i,military}$); or (ii)

[16] Acemoglu, Akcigit and Kerr (2015) do not precisely use military spending as an instrument but rather as a demand shock. Obviously the two interpretations are closely related.

[17] Peter and Ruane (2020) estimates ϵ_M using firm level data from Indian firms. To correct for the endogeneity in the estimation of elasticities, the authors use changes in tariffs as an instrument. Unlike Atalay (2017), Peter and Ruane (2020) finds that ϵ_M is substantially above one, which is likely a 'long run' elasticity; similar large long-run results can be found in the literature on the elasticity of substitution between capital and labor.

[18] Note that, unlike the well-known Leontief inverse matrix, this matrix does not account for indirect upstream links–sectoral supplier importance–but rather captures only indirect downstream links. That is, it captures how important other sectors are for the demand of a given sector's output.

sector i, directly or indirectly, purchases a large fraction of sector j's output (large $(I-S)^{-1}_{ji}$).

We also consider military spending news shocks from Ramey and Zubairy (2018) as an alternative instrument. Using the input-output matrix and the approach described above, we construct $News_{p_j,t}$, $News_{p_i,t}$, and $News_{p^m_j,t}$. The advantage of using military news is that we account for anticipation issues that may contaminate the connection between military spending and sectoral outcomes. The disadvantage of using military news is that it contains less time variation over the sample we consider, weakening its strength as an instrument.

Grouping sectors. — We allow for the elasticities to vary across sectors, but we will be limited in how much heterogeneity we can accommodate due to data limitations and weak instrument problems. We use the Bureau of Economic Analysis (BEA) annual Input-Output data for the period 1997-2007(2018). To start, there are 71 sectors of the economy (of which 66 are non-government sectors); we have credit spread data for 53 sectors.[19] To improve the precision of our IV estimates we cluster industries into 13 groups (roughly four sectors per group). We use two criteria for grouping the sectors. First, we use a technological criterion based on Miranda-Pinto (2020). The author shows that services sectors have a different average elasticity of substitution from non-service sectors. Therefore, we begin by splitting sectors into service and non-service sectors. Second, within the service and non-service sectors, we group sectors based on the observed sectoral residual spread growth during the Great Recession (Figure 1). We then ask whether the heterogeneity in spread growth during the Great Recession is related to sectoral production elasticities.

[19]For each sector we keep the top 25 intermediate goods' supplier sectors. The results are similar using the top 20 or 30 suppliers.

The empirical counterpart of Equation (4) is

$$\Delta \log \left(\frac{P_{it} M_{ijt}}{P_{jt} Q_{jt}} \right) = \phi_t + \alpha_j \Delta \log \left(\frac{P^M_{jt}}{P_{it}} \right) + \beta_j \Delta \log \left(\frac{P_{jt}}{P^M_{jt}} \right) + \tilde{\nu}_{ijt}, \tag{5}$$

where ϕ_t are year fixed-effects that control for aggregate shocks. The error term is denoted by $\tilde{\nu}_{ijt}$. We can obtain the elasticities as

$$\epsilon_{Q_j} = 1 + \beta_j$$
$$\epsilon_{M_j} = 1 + \alpha_j.$$

In our IV approach, we separately estimate α_j and β_j for each group of sectors. Our OLS panel fixed-effect estimates for α_j and β_j take advantage of the panel structure of our data and are obtained from interacting sectoral prices with sectoral group dummies.

Table 2 reports the elasticity estimates for ϵ_Q, the confidence intervals, and the F-test for weak instruments.[20] We report the point estimate of statistically significant coefficients (at the 90 percent confidence level), while non-statistically significant coefficients display unitary elasticity. We also report the 90 percent confidence interval of the point estimates.

The IV estimates are chosen from the four sets of estimates we have (two types of instruments and two samples). We choose the estimates based on the strength of the first stage and on economic theory ($\epsilon_Q \geq 0$). We report the Sanderson-Windmeijer (SW) first-stage F statistics for weak identification. We choose the IV estimates for which the hypothesis for weak instruments is rejected (F test larger than 9.08). There is only one group in which no instruments are strong,

[20]Note that we estimate elasticities for the sample of sectors for which we have spread data available. We have spread data for most sectors at the 3-digit industry classification. We do not have spread data for the following sectors: farms; forestry, fishing, and related activities; construction; fabricated metal products; warehousing and storage; Federal Reserve banks, credit interm., and rel. act.; securities, commodity contracts, and investments; insurance carriers and related activities; Funds, trusts, and other financial vehicles; Management of companies and enterprises; Educational services

Sector	Group	$\epsilon_Q^{FE^*}$	$\epsilon_Q^{FE_{P10}}$	$\epsilon_Q^{FE_{P90}}$	$\epsilon_Q^{IV^*}$	F test (SW)	$\epsilon_Q^{IV_{P10}}$	$\epsilon_Q^{IV_{P90}}$	Instrument
Oil and gas extraction	3	1.70	0.56	2.35	2.32	17.26	1.53	3.12	Military 97-07
Mining, except oil and gas	2	0.32	-0.41	1.04	0.30	44.09	-0.08	0.68	Military 97-18
Support activities for mining	4	0.86	0.04	1.68	3.76	18.13	2.83	4.70	Military 97-18
Utilities	11	0.76	0.06	1.45	1.00	22.44	-5.33	5.81	Military 97-07
Wood products	5	0.99	0.48	1.49	1.00	10.97	-0.06	8.48	Military 97-07
Nonmetallic mineral products	1	1.70	1.26	2.13	4.08	35.69	2.03	6.13	Military 97-18
Primary metals	4	0.86	0.04	1.68	3.76	18.13	2.83	4.70	Military 97-18
Machinery	4	0.86	0.04	1.68	3.76	18.13	2.83	4.70	Military 97-18
Computer and electronic products	3	1.70	0.56	2.35	2.32	17.26	1.53	3.12	Military 97-07
Electrical equipment, appliances, and components	1	1.70	1.26	2.13	4.08	35.69	2.03	6.13	Military 97-18
Motor vehicles, bodies and trailers, and parts	1	1.70	1.26	2.13	4.08	35.69	2.03	6.13	Military 97-18
Other transportation equipment	1	1.70	1.26	2.13	4.08	35.69	2.03	6.13	Military 97-18
Furniture and related products	5	0.99	0.48	1.49	1.00	10.97	-0.06	8.48	Military 97-07
Miscellaneous manufacturing	5	0.99	0.48	1.49	1.00	10.97	-0.06	8.48	Military 97-07
Food and beverage and tobacco products	2	0.32	-0.41	1.04	0.30	44.09	-0.08	0.68	Military 97-18
Textile mills and textile product mills	5	0.99	0.48	1.49	1.00	10.97	-0.06	8.48	Military 97-07
Apparel and leather and allied products	3	1.70	0.56	2.35	2.32	17.26	1.53	3.12	Military 97-07
Paper products	3	1.70	0.56	2.35	2.32	17.26	1.53	3.12	Military 97-07
Printing and related support activities	5	0.99	0.48	1.49	1.00	10.97	-0.06	8.48	Military 97-07
Petroleum and coal products	4	0.86	0.04	1.68	3.76	18.13	2.83	4.70	Military 97-18
Chemical products	2	0.32	-0.41	1.04	0.30	44.09	-0.08	0.68	Military 97-18
Plastics and rubber products	2	0.32	-0.41	1.04	0.30	44.09	-0.08	0.68	Military 97-18
Wholesale trade	10	0.42	-0.28	1.11	1.00	24.63	-1.60	1.20	Military 97-07
Motor vehicle and parts dealers	11	0.76	0.06	1.45	1.00	22.44	-5.33	5.81	Military 97-07
Food and beverage stores	9	-0.38	-1.01	0.25	1.00	79.14	0.83	5.67	News 97-07
General merchandise stores	10	0.42	-0.28	1.11	1.00	24.63	-1.60	1.20	Military 97-07
Other retail	12	0.55	-0.14	1.25	-	-	-	-	OLS (weak instruments)
Air transportation	12	0.55	-0.14	1.25	-	-	-	-	OLS (weak instruments)
Rail transportation	10	0.42	-0.28	1.11	1.00	24.63	-1.60	1.20	Military 97-07
Water transportation	8	1.16	0.35	1.98	6.86	19.95	4.54	9.18	Military 97-07
Truck transportation	7	1.08	0.44	1.72	1.00	29.59	-2.74	4.38	Military 97-18
Pipeline transportation	9	-0.38	-1.01	0.25	1.00	79.14	0.83	5.67	News 97-07
Other transportation and support activities	11	0.76	0.06	1.45	1.00	22.44	-5.33	5.81	Military 97-07
Publishing industries, except internet	13	0.78	0.04	1.51	1.00	31.45	-22.92	9.89	Military 97-07
Motion picture and sound recording industries	8	1.16	0.35	1.98	6.86	19.95	4.54	9.18	Military 97-07
Broadcasting and telecommunications	7	1.08	0.44	1.72	1.00	29.59	-2.74	4.38	Military 97-18
Data processing, internet pub., and other inf. services	9	-0.38	-1.01	0.25	1.00	79.14	0.83	5.67	News 97-07
Housing Services	13	0.78	0.04	1.51	1.00	31.45	-22.92	9.89	Military 97-07
Other Real Estate	13	0.78	0.04	1.51	1.00	31.45	-22.92	9.89	Military 97-07
Rental and leasing services and lessors of int. assets	12	0.55	-0.14	1.25	-	-	-	-	OLS (weak instruments)
Legal services	6	1.05	0.39	1.71	2.69	25.05	1.20	4.18	Military 97-07
Computer systems design and related services	6	1.05	0.39	1.71	2.69	25.05	1.20	4.18	Military 97-07
Miscellaneous professional, scientific, and tech. Serv.	6	1.05	0.39	1.71	2.69	25.05	1.20	4.18	Military 97-07
Administrative and support services	11	0.76	0.06	1.45	1.00	22.44	-5.33	5.81	Military 97-07
Waste management and remediation services	9	-0.38	-1.01	0.25	1.00	79.14	0.83	5.67	News 97-07
Ambulatory health care services	8	1.16	0.35	1.98	6.86	19.95	4.54	9.18	Military 97-07
Hospitals	6	1.05	0.39	1.71	2.69	25.05	1.20	4.18	Military 97-07
Nursing and residential care facilities	7	1.08	0.44	1.72	1.00	29.59	-2.74	4.38	Military 97-18
Performing arts, spectator sports, museums	13	0.78	0.04	1.51	1.00	31.45	-22.92	9.89	Military 97-07
Amusements, gambling, and recreation industries	12	0.55	-0.14	1.25	-	-	-	-	OLS (weak instruments)
Accommodation	10	0.42	-0.28	1.11	1.00	24.63	-1.60	1.20	Military 97-07
Food services and drinking places	8	1.16	0.35	1.98	6.86	19.95	4.54	9.18	Military 97-07
Other services, except government	7	1.08	0.44	1.72	1.00	29.59	-2.74	4.38	Military 97-18
Average		0.85			2.14				
Standard Deviation		0.54			1.84				

TABLE 2—OLS AND IV ϵ_Q ESTIMATES

Note: This table presents the OLS and IV ϵ_Q estimates (significant at the 90 percent level) along with the 90 percent confidence intervals (constructed using standard errors clustered at the sector level). We report the Sanderson-Windmeijer (SW) first-stage F statistics (which is either the the Cragg-Donald Wald statistic (if i.i.d.) or the Kleibergen-Paap rk Wald statistic for multiple endogenous regressors) for weak identification. The F test critical value for a 10 percent maximal IV relative bias from Stock-Yogo weak instruments is 9.08.

and we use the OLS estimated elasticity in this case.[21]

Two important messages arise from Table 2. First, the (unweighted) average elasticity between value-added input and intermediates is substantially larger than

[21] As in Atalay (2017) and Miranda-Pinto (2020), the first stage regressions are consistent with a demand shifter instrument. $\frac{P_j}{P_j^M}$ is negatively related to $Military_{p_j}$ and $\frac{P_j^M}{P_i}$ is positively related to $Military_{p_j^m}$.

one (2.14).[22] This contrasts with the estimated unitary elasticity in Atalay (2017) and Miranda-Pinto (2020) when imposing homogeneous elasticities. The reason is that a common elasticity estimation weights sectors equally, but several large sectors appear to have large elasticities. Second, there is substantial heterogeneity in sectoral flexibility. The standard deviation of our IV sectoral estimates is 1.84. In the next section, we investigate whether this sectoral dispersion in flexibility is related to sectoral bond spreads during the Great Recession.

C. Spread and elasticities during the Great Recession

We now estimate the relationship between sectoral spread and elasticities. To focus on the Great Recession, we restrict our sample to the period 2002q1-2015q4. Our sectoral classification is at the 3-digit NAICS. To control for other firm-level covariates, unconnected to sectoral flexibility, that might cause a firm or sector to pay a higher premium at a given point in time, we use COMPUSTAT data on sectoral sales, the value of property and plants, inventories, and leverage (total debt divided by assets).

Given that sectoral elasticities are assumed to be constant over time, our identification relies on interacting the elasticities with time-varying variables. In this case, we are interested in how the change in sectoral spreads differs in recessions for firms with different elasticities of substitution. Thus, we interact the elasticities with the Great Recession dummy, which equals one for the period 2007q4-2009q2 and zero otherwise. We also interact the sectoral elasticity with the aggregate Excess Bond Premium (EBP) developed by Gilchrist and Zakrajsek (2012) to identify periods of overall tight credit conditions.

The results in Table 3 show strong support for the negative correlation between sectoral elasticities and the change in credit spreads, both during the Great Recession and more generally during periods of tight credit conditions. The results

[22] Note that the OLS estimates are, in general, smaller than the IV estimates, which is consistent with the downward bias generated by unobserved productivities and frictions.

in column 1 indicate that a sector with an elasticity of 2.69 (the hospital sector) experienced an increase in credit spreads during the Great Recession that was 0.65 percentage points lower compared to a sector with unitary elasticity (housing services). Similarly, we find that at the peak of the Great Recession (EBP ≈ 3 percent), a sector with an elasticity of 2.69 displayed an increase in spreads that was 0.91 percentage points lower than that in a sector with unitary elasticity.[23]

Columns 3 and 4 of Table 3 further group sectors into high and low elasticity sectors, in which a *high elasticity sector* is one with an elasticity higher than the median.[24] With this approach, we mitigate the concern that our elasticities are estimated with error. While the results in Table 3 use elasticity estimates that correct for statistical significance, it could still be the case that the confidence intervals of statistically significant coefficients overlap.[25] The results in column 3 indicate that sectors with high production flexibility experienced an increase in bond spreads during the Great Recession that was 1.66 percentage points smaller than in low-flexibility sectors. Similarly, in column 4 we observe that, at the peak of the Great Recession (EBP 3 percent), sectors with high production flexibility displayed an increase in bond spreads that was 2.7 percentage points lower than in low-flexibility sectors. A similar relationship between spreads and flexibility holds if we use the OLS biased elasticity estimates instead (see Table B2 in our Appendix.)

Additional robustness checks. — Here, we show that our correlation is robust to controlling for demand-side channels that depend on sectoral characteristics. Nakamura and Steinsson (2010) show that sectoral heterogeneity in price sticki-

[23]In our Appendix, Table B1, we show that the same results hold when considering statistically significant elasticities at the 95% confidence instead of 90%. Also, the same results hold when we use the average sectoral spread rather than the median sectoral spread (Table B3).

[24]This exercise is related to Barrot and Sauvagnat (2016), who study how input specificity affects the upstream propagation of shocks through production networks. The authors use different indexes to measure the specificity of intermediate inputs and classify suppliers as specific if they display an index above the sample median.

[25]A previous version of this paper used bootstrap techniques to account for the generated regressors problem.

Table 3—Spreads and Flexibility

VARIABLES	(1) Δ Spread	(2) Δ Spread	(3) Δ Spread	(4) Δ Spread
$\epsilon_Q^{IV} \cdot GR$	-0.386***			
	(0.128)			
$\epsilon_Q^{IV} \cdot EBP$		-0.182**		
		(0.074)		
High $\epsilon_Q^{IV} \cdot GR$			-1.664***	
			(0.482)	
High $\epsilon_Q^{IV} \cdot EBP$				-0.915***
				(0.248)
Observations	2,917	2,917	2,917	2,917
Adjusted R-squared	0.436	0.439	0.439	0.448
Time FE	Yes	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes	Yes

Note: This table presents an OLS regression using the four-quarter change in median sectoral credit spreads as the dependent variable. The independent variables are average sectoral sales, the average value of property and plants, average inventories, average leverage (total debt divided by assets), the excess bond premium (EBP), time fixed-effects, sector fixed-effects, the estimates sectoral elasticity of substitution, the interaction between the elasticity and a Great Recession dummy, and the interaction between the elasticity and the EBP. ϵ_Q^{IV} are the IV estimates of sectoral elasticity in Table 2. High ϵ_Q^{IV} is a dummy that takes the value of 1 for sectors with an elasticity above median and the value of 0 otherwise. Standard errors presented in parentheses are clustered at the sector level. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

ness can generate heterogeneous responses of spreads to monetary policy shocks, and a large literature documents the sluggish response of durable consumption in the face of aggregate shocks (e.g., Berger and Vavra (2015)). To show that our results are not confounded with these mechanisms, we use sectoral price frequency adjustment data from Pasten, Schoenle and Weber (2019) and a dummy for consumption durable goods sectors.[26] Table 4 shows that the correlation between flexibility and spreads is robust to adding these additional controls.

[26]We are deeply grateful to Pasten, Schoenle and Weber (2019) for sharing their data on sectoral frequency price adjustment. Our durable consumption dummy takes the value of 1 for construction; wood products; machinery; computer and electronic products; electrical equipment, appliances, and components; motor vehicles, bodies and trailers, and parts; other transportation equipment; furniture and related products; textile mills and textile product mills; apparel and leather and allied products; motor vehicle and parts dealers; and housing services.

Table 4—Spreads and Flexibility (Robustness)

VARIABLES	(1) Δ Spread	(2) Δ Spread	(3) Δ Spread	(4) Δ Spread
$\epsilon_Q^{IV} \cdot GR$	-0.281*		-0.392***	
	(0.151)		(0.133)	
High $\epsilon_Q^{IV} \cdot GR$		-1.254**		-1.767***
		(0.498)		(0.502)
Frequency $\cdot GR$	0.364	0.015		
	(2.645)	(2.655)		
Durables $\cdot GR$			0.787	0.998
			(0.894)	(0.850)
Observations	2,254	2,254	2,917	2,917
Adjusted R-squared	0.438	0.441	0.438	0.442
Time FE	Yes	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes	Yes

Note: This table presents an OLS regression using sectoral credit spread growth as the dependent variable. The independent variables are sectoral sales, the value of property and plants, inventories, leverage (total debt divided by assets), the excess bond premium (EBP), time fixed-effects, sector fixed-effects, the high elasticity dummies, the median frequency of price adjustment from Pasten, Schoenle and Weber (2019), a durable consumption dummy, the interaction between the high elasticity dummy and a Great Recession dummy, the interaction between the frequency of price adjustment and a Great Recession dummy, and the interaction between the durable consumption dummy and a Great Recession dummy. ϵ_Q^{IV} are the IV estimates of sectoral elasticity in Table 2. High ϵ_Q^{IV} is a dummy that takes the value of 1 for sectors with an elasticity above median and the value of 0 otherwise. Standard errors presented in parentheses are clustered at the sector level. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Flexibility and short-term liquidity. — In Appendix Table B4, we provide additional support for our mechanism using firm-level data on short term liquidity. We find that more-flexible firms also experienced larger working capital-to-sales ratio growth during the Great Recession and during periods of tight credit conditions.

II. Understanding the role of flexibility

In this section, we develop a simple model that can illuminate our empirical results. Compared to Bigio and La'O (2020) and Baqaee and Farhi (2020), who also study multisector models with linkages and frictions, our framework allows for heterogeneity in sectoral elasticities and frictions. Unlike the papers cited above, which take frictions as given and study the macroeconomic effects of distortions, we study how endogenous wedges are affected by tighter credit conditions and how these wedges depend on sectoral flexibility.

A. A simple model

There are two sectors; the first sector produces using only labor, and the second sector produces using labor and intermediates from both sectors:

$$Q_1 = Z_1 L_1$$
$$Q_2 = Z_2 \left(a_2^{\frac{1}{\epsilon_Q}} L_2^{\frac{\epsilon_Q - 1}{\epsilon_Q}} + (1-a_2)^{\frac{1}{\epsilon_Q}} M_{12}^{\frac{\epsilon_Q - 1}{\epsilon_Q}} \right)^{\frac{\epsilon_Q}{\epsilon_Q - 1}}.$$

Each sector faces a collateral constraint on working capital:

$$\theta_1^w w L_1 \leq \eta_1 P_1 Q_1 \tag{6}$$
$$\theta_2^w w L_2 + \theta_{12}^m P_1 M_{12} \leq \eta_2 P_2 Q_2. \tag{7}$$

Firms in sector j need to externally finance a fraction θ_j^w of the wage bill wL_j and a fraction θ_{ij}^m of the cost of intermediates purchased from sector i $P_i M_{ij}$. However, firms are limited in the amount of borrowing they can obtain. Different sectors can pledge a different fraction η_j of total sales as collateral. The variable μ_j denotes the Lagrange multiplier for the sectoral borrowing constraint in Equation (6), which represents the firms' shadow cost of debt: μ_j represents how much firms in sector j value a marginal increase in external funds that would allow them to produce closer to the optimal scale.[27]

[27]See Footnotes 7 and 14 for a discussion of this constraint.

The representative household maximizes

$$U(C) = \log C,$$

subject to the budget constraint

$$w\bar{L} + \Pi \geq P_2 C;$$

labor supply is inelastic, which is not essential but simplifies the algebra. We assume that firms' profits $\Pi = \Pi_1 + \Pi_2$ are rebated back to the household.

In equilibrium, labor market clearing requires that

$$\bar{L} = L_1 + L_2,$$

and goods market clearing requires that

$$\begin{aligned} M_{12} &= Q_1 \\ C &= Q_2. \end{aligned}$$

Note that, for simplicity of the resulting algebra, the output of sector one is not consumed. Adding capital, either as a fixed input or a rented input supplied elastically, will not change our results if value-added is produced using a Cobb-Douglas aggregate of capital and labor, so, again, for ease of presentation we simply ignore it. We normalize both wages w and total labor endowment $\bar{L}$ to 1.

We vary the values of the elasticities and examine the relationship between the Lagrange multiplier μ_2 on the collateral constraint for sector 2 and the elasticity of interest.

B. Flexibility and frictions

We first examine the extent to which a constrained sector is constrained, which corresponds to the quantitative size of the Lagrange multiplier of the collateral constraint μ_j, provided $\mu_j > 0$. The model identifies three quantities that matter for this calculation: i) the fraction of each input to be paid in advance (θ_j^m and θ_j^w); ii) the importance of intermediates in production $(1 - a_2)$; and iii) the borrowing capacity of a sector (η).

While we interpret μ_j ($j = 1, 2$ in this case) as a spread, we can also think of the spread being related to sectoral wedges ϑ_j. Indeed, in the quantitative section, we proxy sectoral wedges using spread data.[28] In the model, the wedge ϑ_j is negatively correlated to μ_j and, therefore, is positively related to η_j. To see this, we use the FONC for intermediates. The sectoral wedge in the use of intermediates ϑ_j^m is defined by

$$P_j Z_j^{\rho_Q} \Big(\frac{(1 - a_j) Q_j}{M_j}\Big)^{1-\rho_Q} = P_j^M \underbrace{\frac{(1 + \mu_j \theta_j^m)}{(1 + \mu_j \eta_j)}}_{(\vartheta_j^m)^{-1}} = \frac{P_j^M}{\vartheta_j^m}, \tag{8}$$

in which $\rho_Q = \frac{\epsilon_Q - 1}{\epsilon_Q}$. If firms are unconstrained ($\mu_j = 0$), the sectoral wedge equals one, and firms are able to choose their desired input combination. However, if the constraint binds, the sectoral wedge is smaller than one, implying that the actual relative price of intermediate inputs ($\frac{P_j^M}{\vartheta_j^m}$) rises. In other words, there is a wedge between the marginal product of intermediates–the left-hand side of the equation–and the marginal cost of intermediates P_j^M. We can see from Equation (8) that, if intermediates need to be paid fully in advance ($\theta_j^m = 1$), the wedge ($\vartheta_j^m = \frac{1+\mu_j \eta_j}{1+\mu_j}$) is inversely related to μ_j but positively related to η_j. A decline in η_j (a tightening of the constraint) increases the denominator $(1 + \mu_j)$ more than

[28] We follow Bigio and La'O (2020) and assume that sectoral wedges equal the inverse of the gross spread $\vartheta_j = 1/(1 + r_j)$

the numerator $(1+\mu_j\eta_j)$, which then decreases the wedge ϑ_j^m.

The next propositions describe the intensive margin of sectoral frictions. We focus on the case in which sectors are constrained in the use of intermediates and use μ_j as our proxy for sectoral spreads. Our first proposition studies the intensity of the constraint for sectors that differ only in their elasticity. Our second proposition studies how the change in μ_j, due to tightening credit conditions, depends on the elasticity.

PROPOSITION 1: *Suppose that sectors 1 and 2 are constrained ($\mu_1 > 0$ and $\mu_2 > 0$), and that sector 2 needs to externally finance only intermediate input expenses ($\theta_{12}^m = 1$ and $\theta_2^w = 0$). When $\frac{1}{(\eta_1 Z_1)^{\epsilon_Q}} \geq 1$, we have that*

- *a higher elasticity ϵ_Q in sector 2 relaxes the constraint, $\frac{\partial \mu_2}{\partial \epsilon_Q} < 0$;*
- *and the premium for flexibility ($\frac{\partial \mu_2}{\partial \epsilon_Q} < 0$) is increasing, $\frac{\partial\left(\partial \mu_2/\partial \epsilon_Q\right)}{\partial \phi_m} < 0$, in the friction-adjusted price of intermediates*

$$\phi_m = \frac{(1-\eta_2)(1-a_2)}{Z_1\eta_1\eta_2 a_2} = \frac{(1-\eta_2)(1-a_2)}{\eta_2 a_2} P_1.$$

Proof: see Appendix A.A2.

If sector 1 is constrained ($\eta_1 < 1$) and sectoral productivity Z_1 is not too high, we have $\frac{1}{(\eta_1 Z_1)^{\epsilon_Q}} \geq 1$, implying that more-flexible firms are less constrained ($\frac{\partial \mu_2}{\partial \epsilon_Q} < 0$).[29] In addition, the negative relationship between the elasticity and the Lagrange multiplier is more negative when the friction-adjusted price of intermediates ϕ_m is large. Indeed, the condition for the constraint to be binding is that $\phi_m \geq \frac{1}{(\eta_1 Z_1)^{\epsilon_Q}}$, and the condition for $\frac{\partial \mu_2}{\partial \epsilon_Q} < 0$ is that $\phi_m > 1$. The friction-adjusted relative price of intermediates ϕ_m describes the extra (shadow) price of

[29]The condition $\frac{1}{(\eta_1 Z_1)^{\epsilon_Q}} \geq 1$ is always met if the constraint binds for $\eta_j < 1$ at the steady state value of productivity $Z_j = 1$. If productivity is sufficiently high ($Z_j > 1$) and the constraint is loose ($\eta_j \approx 1$), intermediates are relatively cheaper than the labor input. In that case, more flexible firms would be more constrained instead as they would prefer to use more of the constrained input (intermediates).

intermediate inputs, via the output loss generated by the binding constraint. We can see this relationship in the Lagrange multiplier equation

$$\mu_2 = \text{Max}\{\phi_m^{\frac{1}{\epsilon_Q}} \eta_1 Z_1 - 1, 0\},$$

where a higher ϕ_m–due to low collateral constraint parameters η_1 and η_2, low productivity Z_1, or high intermediate input weight $(1 - a_2)$–increases the shadow value of working capital.

If intermediates are relatively more expensive, high-flexibility downstream firms are able to dampen the effect of the constraint by using more of the unconstrained input (labor). Low-flexibility firms must keep using the more expensive intermediate inputs, which then tightens the credit constraint even further. Moreover, the premium for production flexibility increases with ϕ_m. The larger the value of ϕ_m, the tighter is the constraint, and the smaller is the output response by sectors that can easily substitute the (unconstrained) labor input for intermediates.

Proposition 1 also shows the importance of intermediate inputs in the transmission of distortions. We can see that collateral constraint shocks to sector 2 are amplified when the upstream sector is more constrained. The friction-adjusted relative price of input ϕ_m is inversely related to the collateral constraint parameter η_2

$$\frac{\partial \phi_m}{\partial \eta_2} = \frac{-(1 - a_2)}{\eta_1 Z_1 \eta_2 a_2},$$

and this relationship is more negative the lower is η_1 (or Z_1)–that is, the more constrained (or less productive) sector 1 is. Similarly, the effect of η_2 on ϕ_m is amplified if intermediate inputs are more important in the production process (larger $(1 - a_2)$).

We now study how the elasticity affects the change in the Lagrange multiplier due to a financial shock in sector 1 or 2.Specifically, we analyze when $\frac{\partial(\partial \mu_2 / \partial \eta_j)}{\partial \epsilon_Q} > 0$, implying that more-flexible firms experience a smaller increase in the Lagrange multiplier following a reduction in either η_1 or η_2.

PROPOSITION 2: *Suppose that sectors 1 and 2 are constrained ($\mu_1 > 0$ and $\mu_2 > 0$), and that sector 2 needs to externally finance only intermediate input expenses ($\theta_{12}^m = 1$ and $\theta_2^w = 0$). Then, if $\frac{1}{(\eta_1 Z_1)^{\epsilon_Q}} \geq 1$ we have that*

- *a higher elasticity ϵ_Q mitigates the increase in the Lagrange multiplier μ_2 followed by a reduction in η_2, $\frac{\partial(\partial\mu_2/\partial\eta_2)}{\partial\epsilon_Q}$; and*
- *a higher elasticity ϵ_Q mitigates the increase (or amplifies the decline) in the Lagrange multiplier μ_2 followed by a reduction in η_1, $\frac{\partial(\partial\mu_2/\partial\eta_1)}{\partial\epsilon_Q} > 0$, if $1 - \frac{(\epsilon_Q - 1)}{\epsilon_Q} \ln \phi_m > 0$.*

Proof: see Appendix A.A2.

The first part of Proposition 2 implies that, if $\frac{1}{(\eta_1 Z_1)^{\epsilon_Q}} \geq 1$, more-flexible firms have smaller increases in μ_2 due to tightening credit constraints ($\frac{\partial(\partial\mu_2/\partial\eta_2)}{\partial\epsilon_Q} > 0$) in sector 2. The second part of Proposition 2 indicates that, if labor and intermediates are complements, we have that $1 - \frac{(\epsilon_Q - 1)}{\epsilon_Q} \ln \phi_m > 0$, which implies that more-flexible firms in sector 2 experience a smaller increase in μ_2 when sector 1's constraint is tightened ($\frac{\partial(\partial\mu_2/\partial\eta_1)}{\partial\epsilon_Q} > 0$). If labor and intermediates are substitutes, more-flexible firms in sector 2 have larger declines in spreads when distortions to sector 1 are tightened ($\frac{\partial(\partial\mu_2/\partial\eta_1)}{\partial\epsilon_Q} > 0$). In any case, if distortions affect both sectors, a more-flexible downstream sector experiences smaller increases in μ_2.

Proposition 2 also emphasizes the role of a non-unitary elasticity of substitution in amplifying distortions through intermediate input linkages. The next equation describes how the Lagrange multiplier of sector 2 changes with a financial shock to sector 1:

$$\frac{\partial\mu_2}{\partial\eta_1} = \phi_m^{1-\rho_Q} Z_1 \frac{(\epsilon_Q - 1)}{\epsilon_Q}.$$

This equation shows that declines in η_1 increase (decrease) the shadow cost of working capital when $\epsilon_Q < 1$ ($\epsilon_Q > 1$).[30] With Cobb-Douglas technologies, tightening credit conditions for sector 1 has no effect on sector 2's shadow cost of debt;

[30]When $\epsilon_Q > 1$ and labor is unconstrained, a negative financial shock to sector 1 generates an increase in the labor-to-intermediates ratio, which, in turn, mitigates the constraint.

however, CES technologies render sectoral wedges endogenous and dependent on other sectors' constraints via input-output linkages.[31]

Constraints on labor. — We have focused our attention on the role of flexibility if firms are constrained in the use of intermediates. In Appendix A.A3, we derive the implications for distortions in the use of labor ($\theta_j^w = 1$ and $\theta_j^m = 0$ for all j.). The results indicate that, on the one hand, distortions in the use of labor are not able to deliver the facts, and, on the other hand, input-output linkages mitigate, rather than amplify the effects of distortions. The model with constraints on labor delivers the observed negative relationship between elasticities and spreads only if the intermediate input supplier sector is unconstrained (or weakly constrained) and displays high productivity (roughly speaking, during boom times). If sectors experience tight credit conditions (low η_j), the output price (the value of one unit of collateral) is relatively large compared to the labor cost, and, therefore, the constraints are not binding.

The take-aways from this section are: i) frictions in the use of intermediates and heterogeneous non-unitary elasticities of substitution allow us to rationalize the cross-sectional correlations between spreads and flexibility observed during the Great Recession; and ii) frictions in the use of intermediates, as opposed to frictions in the use of labor, are amplified due to the existence of input-output linkages.

III. Aggregate effects of heterogeneous elasticities

Here, we explain the aggregate importance of heterogeneous elasticities. We extend the model in the previous section and study, under heterogeneous elasticities,

[31]The same statement applies for $\frac{\partial \mu_2}{\partial Z_1}$, which is zero only under Cobb-Douglas technologies. If the elasticity is not one, sectoral productivity shocks can also affect wedges through these linkages. As a result, identifying sectoral productivity and financial shocks separately is a challenge. This result is an important difference with respect to Altinoglu (2020) who assumes Cobb-Douglas technologies and is able to identify financial shocks and productivity shocks based on $\frac{\partial \mu_2}{\partial Z_1} = 0$.

the roles of elasticities and of input-output linkages in amplifying distortions.[32] We follow Bigio and La'O (2020) and Baqaee and Farhi (2020) and study how exogenous sectoral wedges are amplified by means of input-output connections. A key difference between this section and the previous section is that, while in the previous section we studied the connection between endogenous wedges and elasticities, here we study the macroeconomic implications of heterogeneous elasticities, taking sectoral wedges as given.

A. The Model

The general production network model entails N sectors. In each sector, there is a representative firm that maximizes profits π_j subject to technology (1) and (2) and subject to the working capital constraint in (3).

The representative household supplies labor inelastically and maximizes aggregate consumption

$$C = \prod_{j=1}^{N} C_j^{\beta_j},$$

subject to the budget constraint

$$w\bar{L} + \Pi \geq P_c C,$$

in which β_j equals, from cost-minimizing conditions, the share of sector j's output in total consumption expenditures. The consumer price index is P_c, and w is the wage, which we use as the numeraire. For tractability purposes, we assume that the sectoral elasticity of substitution between intermediates inputs ϵ_{M_j} is equal to the sectoral elasticity of substitution between value-added and intermediates ϵ_{Q_j}. We will show later that the key elasticity in our analysis is ϵ_Q; this result

[32]Osotimehin and Popov (2020) study the role of homogeneous elasticities in amplifying distortions. The authors show that complementarities in production mitigate the aggregate effects of sectoral distortions. The notion that complementarities amplify distortions applies only to distortions that are not rebated back to the households. In this case, as Bigio and La'O (2020) show, wasted distortions are isomorphic to productivity shocks and are amplified by complementarities (Atalay (2017)).

arises because we assume distortions on intermediate inputs as a whole rather than intermediate input-specific distortions. The elasticity of substitution between intermediates ϵ_M takes on more importance if sectors face heterogeneous distortions across intermediates input varieties, as in Peter and Ruane (2020) and Osotimehin and Popov (2020). The solution of the model is described in the following proposition.

PROPOSITION 3: *Given sectoral productivities Z_j and wedges ϑ_j^m, ϑ_j^w, and assuming that $\epsilon_{M_j} = \epsilon_{Q_j}$, real GDP in this economy is*

$$\log C = \sum_{j=1}^{N} \beta_j \log\big(\frac{\beta_j}{P_j}\big) + \log(1+\Pi),$$

in which the vector of sectoral prices is the solution to

$$P^{1-\epsilon_Q} = a \circ (Z \circ \vartheta^w)^{\circ(\epsilon_Q - 1)} + \big((1-a) \circ (Z \circ \vartheta^m)^{\circ(\epsilon_Q-1)} 1'\big) \circ (\Omega \circ (P1')^{\circ((1-\epsilon_Q)1')'})' 1,$$

profits are the solution to

$$(1+\Pi) = \frac{1}{\sum_{j=1}^{N} a_j (\vartheta_j^w)^{\epsilon_{Q_j}} Z_j^{\epsilon_{Q_j}-1} P_j^{\epsilon_{Q_j}-1} s_j},$$

and the vector of sectoral sales shares are the solution to

$$s = [I - \big((P1')^{\circ((1-\epsilon_Q)1')'}\big) \circ \big((\vartheta^m)^{\circ \epsilon_Q} \circ (Z \circ P)^{\circ(\epsilon_Q - 1)} 1'\big)' \circ ((1-a)1')' \circ \Omega]^{-1} \beta,$$

in which $(A \circ B)$ is an element-by-element multiplication of two matrices, A and B, of the same dimension. On the other hand, $A^{\circ B}$ represents an element-by-element exponent. This is, the element A_{ij} is raised to the power of B_{ij}, for all ij.

B. *Tractable networks*

We explain the main mechanisms by comparing two simple network structures– the Star supplier economy (Ω^S) and the Island economy (Ω^I):

$$\Omega^S = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \qquad \text{and} \qquad \Omega^I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

These networks are starkly different and are fairly easy to analyze; in the first, one sector is upstream and one is downstream, whereas in the second the two sectors are completely separated from each other with respect to intermediates (they compete in the market for labor). In this economy, the only relevant elasticity is ϵ_{Q_j}, the elasticity between labor and intermediates. Thus, to simplify notation, we use ϵ_j to describe flexibility. To understand the aggregate effects of sectoral distortions consider the following decomposition in the Star supplier economy

$$\frac{\partial \log C^S}{\partial \vartheta_1} = \underbrace{-\beta \frac{\partial \log P_1^S}{\partial \vartheta_1}}_{\text{Direct real wage channel}} - \underbrace{(1-\beta)\frac{\partial \log P_2^S}{\partial \vartheta_1}}_{\text{Indirect real wage channel}}$$
$$- \underbrace{(1+\Pi^S)a\Big[\epsilon_1 s_1^S \vartheta_1^{\epsilon_1 - 1}(P_1^S)^{\epsilon_1 - 1} + s_1^S \vartheta_1^{\epsilon_1} \frac{\partial (P_1^S)^{\epsilon_1 - 1}}{\partial \vartheta_1}}_{\text{Direct rent channel}} + \underbrace{\vartheta_1^{\epsilon_1}(P_1^S)^{\epsilon_1 - 1}\frac{\partial s_1^S}{\partial \vartheta_1} + s_2^S \vartheta_2^{\epsilon_2}\frac{\partial (P_2^S)^{\epsilon_2 - 1}}{\partial \vartheta_1}\Big]}_{\text{Indirect rent channel}}.$$

The first two terms capture the change in the real wage due both to direct effects and indirect network effects, and the last four terms capture the change in the rents from distortions (composition effect and relocation effect). The key difference between the Island economy and the Star supplier are the indirect effects, which are not present in the Island economy. Indeed, the indirect effects are the ones affected by the heterogeneity in elasticities.

Before presenting the propositions that highlight the role of production elasticities, we define the input-output multiplier as $IOM = \frac{\partial lnGDP^S}{\partial \vartheta_1} - \frac{\partial lnGDP^I}{\partial \vartheta_1}$. A positive value of IOM implies that the Star supplier economy (GDP^S) amplifies distortions compared to the Island economy (GDP^I), while a negative IOM implies that the Star supplier network mitigates distortions compared to the Island network structure.

PROPOSITION 4 (Sectoral distortion): *Suppose that there are two sectors in the economy ($N = 2$), $a_1 = a_2 = a$, $\beta_1 = \beta$, $\beta_2 = 1 - \beta$, we have that*

- *for homogeneous elasticities $\epsilon_1 = \epsilon_2 = \epsilon$*

$$IOM \approx \hat{\psi}_1(\epsilon) - \hat{\psi}_2(\epsilon) - \hat{\psi}_3(\epsilon) + \epsilon\big(\hat{\psi}_4(\epsilon) - \hat{\psi}_5(\epsilon) - \hat{\psi}_2(\epsilon)\big)$$

  where $\hat{\psi}_1, \hat{\psi}_2, \hat{\psi}_4, \hat{\psi}_5$ are positive and non-linear functions of ϵ, while $\hat{\psi}_3$ is a non-linear function of ϵ that can take positive or negative values.

- *for heterogeneous elasticities $\epsilon_1 \neq \epsilon_2$*

$$IOM \approx \tilde{\psi}_1(\epsilon_1,\epsilon_2) - \tilde{\psi}_2(\epsilon_1,\epsilon_2) + \epsilon_1(\tilde{\psi}_3(\epsilon_1,\epsilon_2) - \tilde{\psi}_4(\epsilon_1,\epsilon_2)) + \epsilon_2\big(\tilde{\psi}_5(\epsilon_1,\epsilon_2) - \tilde{\psi}_6(\epsilon_1,\epsilon_2)\big)$$

  where $\tilde{\psi}_1, \tilde{\psi}_2, \tilde{\psi}_3, \tilde{\psi}_4, \tilde{\psi}_5, \tilde{\psi}_6$ are positive and non-linear functions of ϵ_1 and ϵ_2.

The general message from Proposition 4 is that the asymmetric input-output structure in the Star supplier economy can amplify ($IOM > 0$) or mitigate ($IOM < 0$) distortions, compared to the Island economy, depending on the average elasticity and the difference between the upstream and downstream elasticities. As highlighted by Osotimehin and Popov (2020) in a general production network economy with homogeneous elasticities, “the role of the elasticity is complex and non-monotonic”. This complexity is amplified in the case of heterogeneous elasticities.

We study the quantitative implications of Proposition 4 in Figure 2. The results in Figure 2 echo the quantitative results of our general production network economy in the next section. The main takeaways are: i) the higher the average elasticity the larger the input-output multiplier and ii) the higher the relative flexibility of the downstream sector, compared to the upstream sector, the larger the input-output multiplier compared to the homogeneous elasticity case.

The panels in Figure 2 show the change in log GDP for a given sequence of distortions to sector 1. The left panel shows the results assuming homogeneous elasticities. In one case, we assume $\epsilon_1 = \epsilon_2 = 0.1$, while in the other case we assume $\epsilon_1 = \epsilon_2 = 1.5$. In this case, as Osotimehin and Popov (2020) show, if the elasticity is close to zero, input-output linkages in the Star supplier economy generate very limited aggregate effects. While the reduction in real wage is large—more price adjustment and less quantity adjustment—the increase in rents is also large (while price increases substantially, the optimal production plan barely changes). These two effects exactly offset when labor and intermediates are perfect complements. On the other hand, when the value of the elasticity is higher the rents' effect dominates the real wage effect and the Star supplier economy displays a sharper reduction in GDP compared to the Island economy.

The right panel in Figure 2 shows that an economy with a relatively less flexible upstream sector displays a larger reduction in real GDP, compared to the Island economy, from a shock to sector 1. To illustrate that this amplification is driven by the heterogeneity in flexibility, rather than the average elasticity, we also plot the change in log GDP assuming that the elasticity is homogeneous and equals the size-weighted average elasticity of the Star supplier economy. Intuitively, when the upstream sector has relatively low flexibility, the distortion generates a larger increase in intermediate input price P_1—lower flexibility implies less quantity and more price adjustment in the affected sector. The larger increase in P_1 affects the production cost of sector 2, which increases P_2, as well. These two effects reduce the real wage of the household. On the other hand, the rents from distortions depend on sectoral sales ($\pi_j = (1-\vartheta_j)P_jQ_j$). An economy with a more flexible downstream sector will display a smaller increase in rents from sector 2 (P_2Q_2 increases less or declines more), which, in turn generates less aggregate consumption, compared to the Island economy. In the homogeneous elasticity case, these two effects tend to mute each other.

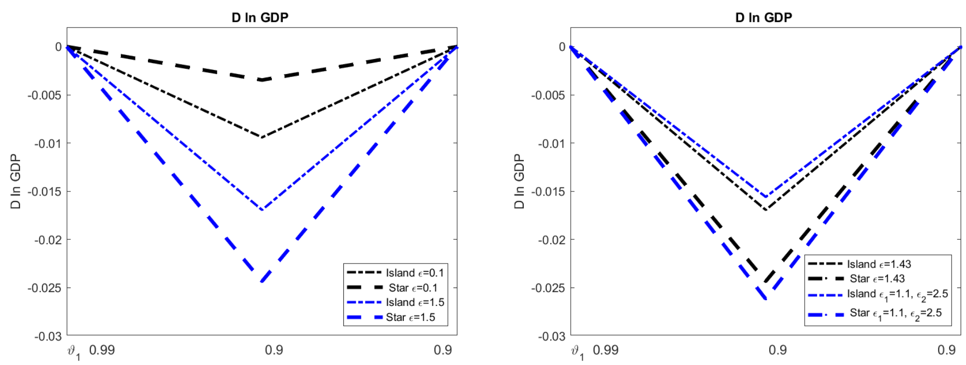

FIGURE 2. TWO-SECTOR STAR SUPPLIER VERSUS ISLAND ECONOMY, HETEROGENEOUS ELASTICITIES, AND DISTORTION TO SECTOR 1.

Note: The simulation assumes $a = 0.4$, $\beta_1 = 0.5$, $\vartheta_2 = 0.99$, and $\vartheta_1 = [0.99, 0.9, 0.9]$.

The next proposition specifies an irrelevance result for the role of input-output linkages under homogeneous elasticities and perfectly correlated sectoral distortions.

PROPOSITION 5 (Aggregate shock): *Suppose that there are two sectors in the economy ($N = 2$), $a_1 = a_2 = a$, $\beta_1 = \beta$, $\beta_2 = 1 - \beta$. Then,*

- *when sectoral elasticities are homogeneous, $\epsilon_1 = \epsilon_2$,*

$$IOM = 0.$$

- *when sectoral elasticities are heterogeneous $\epsilon_1 \neq \epsilon_2$,*

$$IOM \approx \bar{\psi}_1(\epsilon_1, \epsilon_2) - \bar{\psi}_2(\epsilon_1, \epsilon_2) + \epsilon_1\big(\bar{\psi}_3(\epsilon_1, \epsilon_2) - \bar{\psi}_4(\epsilon_1, \epsilon_2)\big) + \epsilon_2\big(\bar{\psi}_5(\epsilon_1, \epsilon_2) - \bar{\psi}_6(\epsilon_1, \epsilon_2) - \bar{\psi}_7(\epsilon_1, \epsilon_2)\big)$$

where $\bar{\psi}_1, \bar{\psi}_2, \bar{\psi}_3, \bar{\psi}_4, \bar{\psi}_5, \bar{\psi}_6$ are positive and non-linear functions of ϵ_1 and

ϵ_2, while $\bar{\psi}_7$ is a non-linear function of ϵ_1 and ϵ_2 that can take positive or negative values.

Proposition 5 shows that aggregate distortions simply change the scale the economy, independent of the details of sectoral connections. However, if sectors are heterogeneous in terms of flexibility, aggregate distortions have differing effects depending on the details of the input-output network.

Figure 3 depicts the implications of Proposition 5.[33] We observe that the input-output structure plays no role in amplifying aggregate distortions when the value of the elasticity is common across sectors (size-weighted average elasticity). On the other hand, as in Proposition 4, the economy with a relatively more flexible downstream sector amplifies aggregate distortions, compared to the Island economy.

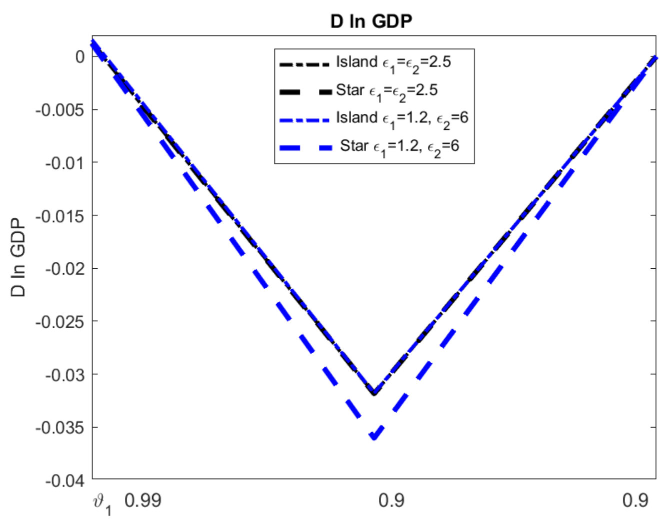

FIGURE 3. TWO-SECTOR STAR SUPPLIER VERSUS ISLAND ECONOMY, HETEROGENEOUS ELASTICITIES, AND DISTORTION TO SECTOR 1 AND 2.

Note: The simulation assumes $a = 0.4$, $\beta_1 = 0.5$, $\vartheta_2 = 0.99$, and $\vartheta_1 = [0.99, 0.9, 0.9]$.

[33]This result is related to Pasten, Schoenle and Weber (2019), which details the role of sectoral heterogeneity in price flexibility for amplification of monetary policy shocks.

In this section, we have shown two results: i) if there is heterogeneity in input-output linkages and in elasticities (as in the data), those elasticities matter for the amplification of distortions (sectoral and aggregate); and ii) if sectors are interconnected (Star supplier economy), but there is no heterogeneity in elasticities, the value of the elasticity is irrelevant for aggregate distortions.

IV. Quantitative exploration: the Great Recession in the U.S

In this section, we solve the general model and study the macroeconomic implications of heterogeneous elasticities in the US during the Great Recession. We will show that the key implications of Propositions 4 and 5 also hold in the general calibrated model.

CALIBRATIONWe calibrate our model economy to year 2007. To facilitate calibration we assume that in 2007, sectoral constraints were not binding. This way, the distribution parameter a_j and the input-output weights ω_{ij} correspond to the observed cost shares of labor (and capital) to total sales and the cost share of intermediate input as a fraction of total intermediate expenses, respectively. We calibrate sectoral elasticities using our set of sectoral estimates in Table 2.

We calibrate sectoral wedges–wedges in the use of intermediates, labor, or both–using sectoral spreads as our measure of distortions:

$$\vartheta_{jt} = \frac{1}{1 + r_{jt}},$$

in which r_{jt} is the sectoral spread.[34] We focus on the role of distortions and assume that sectoral productivity $Z_j = 1$ for all j and t. We choose this approach rather than structurally estimating productivity and financial shocks due to the challenge of separately identify them when elasticities of substitution are non-

[34] We assume that sectors for which we have no spread data face the average sectoral spread in each period and have Cobb-Douglas production technologies.

unitary. We leave for future research a full estimation of the model that identifies the role of productivity and financial shocks in driving aggregate fluctuations.[35]

A. The role of linkages and aggregate distortions

In this section, we show that, as in Propositions 4 and 5, homogeneous elasticities generate a limited role for the input-output structure in amplifying sectoral distortions and no role in amplifying aggregate distortions. Heterogeneous elasticities instead imply a large role for IO connections in amplifying distortions, due to the fact that more-downstream sectors are more flexible than more upstream sectors.

To investigate the role of the structure of input-output linkages, we follow the approach in Foerster, Sarte and Watson (2011) and vom Lehn and Winberry (2020). We compare the propagation and amplification of shocks in an economy with the observed asymmetric structure of US input-output connections (see Acemoglu et al. (2012)) with a hypothetical economy with a diagonal input-output structure (Island economy).[36]

Figure 4 shows the model's implied evolution of log GDP change for different assumptions on the elasticities and input-output connections. The left panel shows that under homogeneous unitary elasticities, as estimated by Atalay (2017) and Miranda-Pinto (2020), the US input-output structure does not amplify distortions (as measured by sectoral spreads) during the Great Recession. Indeed, compared to a diagonal input-output structure, the Cobb-Douglas implies that the US input-output structure mitigates distortions. Once we use our estimated sectoral elasticities, we observe two results that are consistent with Proposition 4. On the one hand, the decline in GDP growth is 1.7 larger. On the other hand,

[35]In our model, sectoral productivity and financial shocks tighten sectors' constraints, either directly or indirectly through input-output linkages, which complicates econometric identification. In Altinoglu (2020), the restriction to Cobb-Douglas technologies implies that only financial shocks tighten sectoral wedges, directly or indirectly via trade credit linkages, which enables the identification of productivity and financial shocks separately. However, our estimates do not support Cobb-Douglas technologies.

[36]Bigio and La'O (2020), Baqaee and Farhi (2020), and Osotimehin and Popov (2020) perform a different counterfactual experiment. Those authors compare an economy with the observed input-output structure with a hypothetical economy without intermediates inputs.

the input-output structure generates a decline in GDP growth, compared to the diagonal input-output economy, that is 9 percent larger.

The right panel shows that, while the size-weighted average elasticity case ($\epsilon_Q = 1.79$) generates a similar decline in GDP growth, compared to the heterogeneous elasticity case, it implies a very different input-output multiplier. While the homogeneous elasticity predicts that the US input-output structure mitigated the Great Recession, our heterogeneous elasticity model predicts that the asymmetric structure of input-output linkages amplified the Great Recession. Indeed, the input-output multiplier is 20 percent larger in the heterogeneous elasticity case, compared to the homogeneous elasticity case.

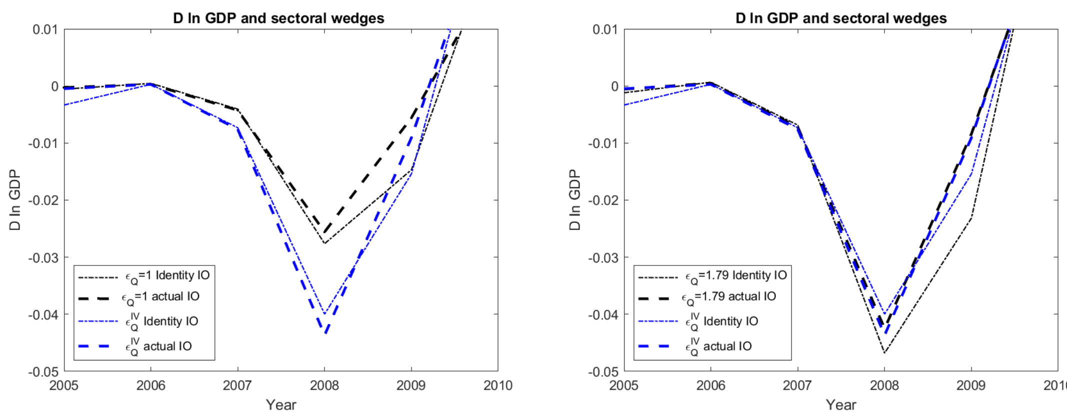

Figure 4. Great Recession, Sectoral Spreads, and IO linkages

As our simple model predicts, this result is due to the fact that more-upstream sectors have very different production flexibility compared to less-upstream sectors. To measure sectoral upstreamness we use the vector of Leontief inverse elements

$$Upstreamness = \mathbf{1}'(\mathbf{I} - \mathbf{\Gamma})^{-\mathbf{1}},$$

where Γ is the observed matrix of input-output shares as a fraction of total gross output (e.g., see Liu (2019)). The Leontief inverse elements capture how important are sectors as suppliers of intermediates inputs, directly and indirectly. Figure 5 shows that there is substantial heterogeneity in flexibility across different degrees of upstreamness. Furthermore, star supplier sectors are more likely to be less flexible.

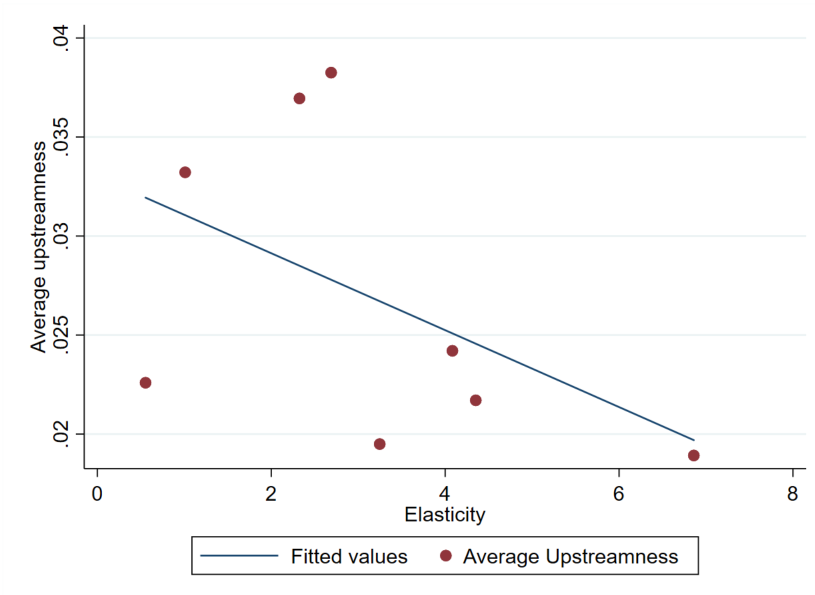

Figure 5. Flexibility and Upstreamness

Note: This figure shows the relationship between sectoral elasticities and upstreamness. The upstreamness measure is the average upstreamness within elasticity groups.
Source: Authors calculations using BEA data.

Correlated distortions. — The results in Figure 4 echo the implications of Proposition 5 regarding the role of input-output linkages in amplifying aggregate distortions. The limited role played by input-output connections in amplifying distortions during the Great recession under homogeneous elasticities is a consequence of the high correlation of sectoral spreads. Figure 6 depicts the distribution of sectoral spreads' pairwise correlations. The left panel shows the distribution of pairwise correlations for the whole sample, while the right panel focuses on the period 2007-2009. There is a clear shift in the distribution during the Great Recession. The median off-diagonal pairwise correlation for the whole sample is 0.68 (the mean is 0.63), while it rises to 0.94 (with a mean of 0.88) for the period 2007-2009.

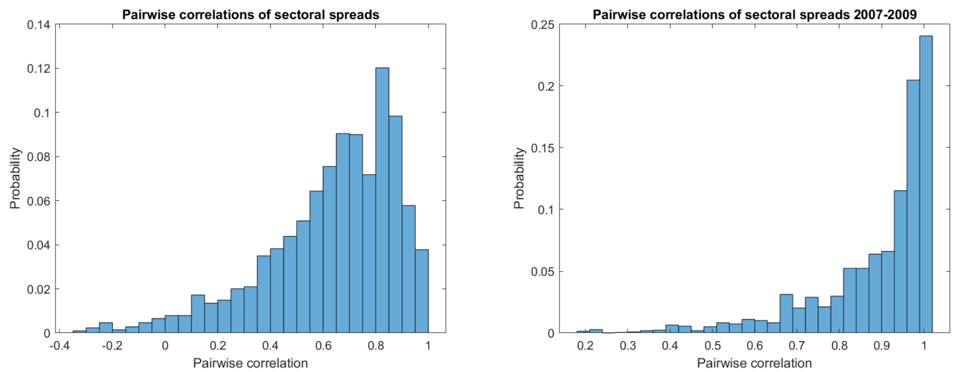

Figure 6. Pairwise correlation sectoral spreads Gilchrist and Zakrajsek (2012), three-digit industry classification

Figure 7 confirms the predictions in Proposition 5. We assume that each sector is hit by the same average wedge and observe that, under homogeneous elasticities, input-output linkages play a very limited role in amplifying aggregate distortions. The aggregate wedge scales down the economy in a similar way in the diagonal input-output economy (Island economy) and in the economy calibrated to match the observed input-output structure in the US in 2007. In this case, the heterogeneous elasticity case amplifies aggregate distortions by a factor of 1.72 compared to the Cobb-Douglas case, and it generates and input-output multiplier of 1.11, which is 1.06 times the average homogeneous elasticity case.

Distortions on intermediates only. — In this final section we show that, as predicted by Proposition 1, distortions only in the use of intermediates amplify distortions significantly.[37] Figure 8 shows that, assuming that $\vartheta_j^m = \frac{1}{1+r_t}$ and

[37]Note that the result in Proposition 1 states that input-output linkages amplify the effect of distortions on intermediates, while they mitigate the effects of distortions on labor. Proposition 1 does not analyze

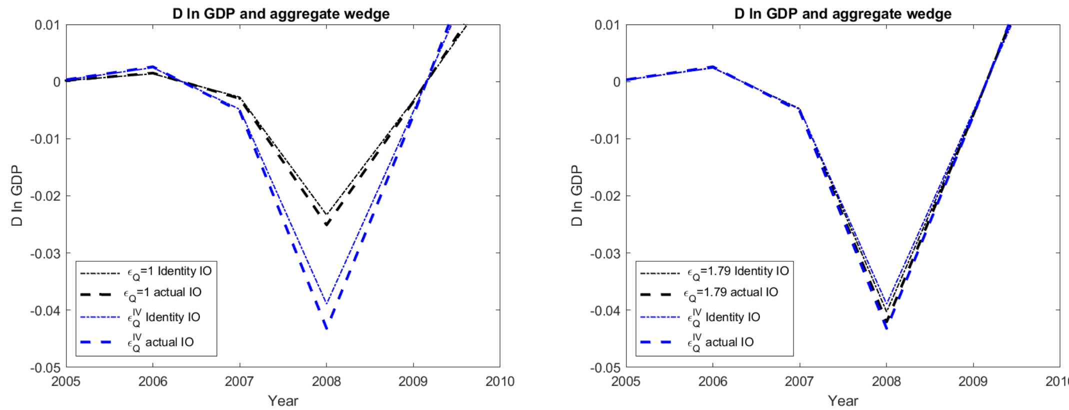

Figure 7. Great Recession, Aggregate Spreads, and IO linkages

$\vartheta_j^w = 1$ for all j generate a very similar path for the model-implied log GDP change, compared to Figure 4. Under homogeneous elasticities, the input-output structure plays a very limited role in amplifying distortions to the use of intermediate inputs, and the effects of distortions are amplified by the value of the elasticity. However, the heterogeneous elasticity model amplifies distortions by a factor of 1.54, compared to the Cobb-Douglas case, and implies an input-output multiplier 1.13 larger than the homogeneous elasticity case.[38]

V. Conclusion

In this paper, we have shown that the heterogeneity in sectoral production elasticities is important for matching sectoral facts during the Great Recession in the US, as well as to understanding the amplification mechanisms of distortions in multisector economies with input-output linkages. Empirically, our results indicate that during the Great Recession, firms with higher substitutability in production paid lower spreads on corporate bonds and held more working capital.

the input-output multiplier (IOM) as defined in Proposition 4.

[38]Our results do not imply that distortions on labor are irrelevant, but instead that distortions on labor are not amplified by input-output connections. Melcangi (2020) shows that financing constraints in the use of labor input are important to match the firm-level employment response to credit shocks in the UK.

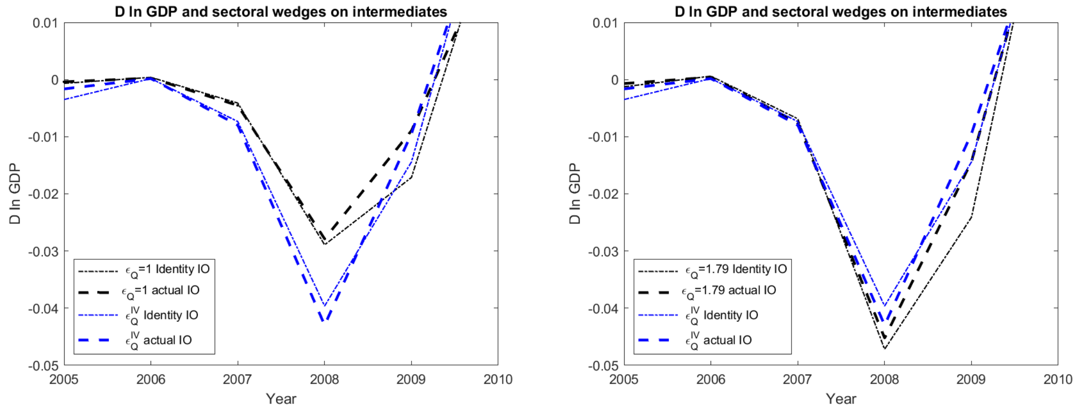

FIGURE 8. GREAT RECESSION, SECTORAL SPREADS, AND IO LINKAGES. DISTORTIONS ON INTERMEDIATES ONLY.

We use this evidence to build a multisector model with heterogeneous elasticities and working capital constraints in the use of inputs. We then study the macroeconomic implications of our model. Our results indicate that not accounting for heterogeneous elasticities leads to misleading results regarding the role of input-output linkages in amplifying distortions during the Great Recession. Our sectoral elasticity estimates generate an amplification of distortions that is 1.70 times the Cobb-Douglas case. We also show that, while under homogeneous elasticities the input-output structure can mitigate or slightly amplify the effect of distortions, our heterogeneous elasticity estimates imply an input-output amplification that is 20 percent larger than the average homogeneous elasticity case.

We believe that our elasticity estimates will be useful for researchers trying to understand questions regarding the sources of business cycles (sectoral vs. aggregate) and the causes of comovement (both in output and input usage) between sectors. Moreover, our model economy with sectoral linkages and distortions has implications for the design of sectoral policies. Sectoral distortions can be important during macroeconomic downturns or in developing economies with underdeveloped financial sectors. The existing literature (Miranda-Pinto (2018) and Liu (2019)) does not take into account the heterogeneity of sectoral flexibility, which we see as a natural next step.

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Mathematical Appendix

A1. Model's implied regression to estimate elasticities

Let's start by defining $\rho_{Q_j} = \frac{\epsilon_{Q_j}-1}{\epsilon_{Q_j}}$. To derive the Equation (4) we solve the cost minimization problem for firms in sector j, subject to the working capital constraint in the use of value-added and intermediates $\theta_j^v P_j^v V_j + \theta_j^m P_j^M M_j \leq \eta_j P_j Q_j$. The Lagrangian of this problem is (max - (cost))

$$\begin{aligned}
\mathcal{L} \quad = \quad & -P_j^v V_j - P_j^M M_j - \lambda_j^1 \left(Q_j - Z_j \left[a_j^{\frac{1}{\epsilon_{Q_j}}} V_j^{\rho_{Q_j}} + (1-a_j)^{\frac{1}{\epsilon_{Q_j}}} M_j^{\rho_{Q_j}} \right]^{\frac{1}{\rho_{Q_j}}} \right) \\
& -\mu_j^C \left(\theta_j^v P_j^v V_j + \theta_j^m P_j^M M_j - \eta_j P_j Q_j \right).
\end{aligned}$$

The first-order necessary and sufficient conditions for M_j is

$$-P_j^M + \lambda_j^1 \frac{\partial Q_j}{\partial M_j} + \mu_j^C \eta_j P_j \frac{\partial Q_j}{\partial M_j} - \mu_j^C \theta_j^m P_j^M = 0.$$

Rearranging, using the fact that $\frac{\partial Q_j}{\partial M_j} = Z_j^{\rho_{Q_j}} \left(\frac{a_j Q_j}{M_j} \right)^{\frac{1}{\epsilon_{Q_j}}}$ and that in competitive markets the marginal cost of production in sector j (λ_j^1) is the price of good P_j, we have

$$P_j^M = Z_j^{\rho_{Q_j}} \left(\frac{a_j Q_j}{M_j} \right)^{\frac{1}{\epsilon_{Q_j}}} P_j \overline{\vartheta}_j, \tag{A1}$$

where $0 \leq \overline{\vartheta}_j = \frac{1+\mu_j^C \eta_j}{1+\mu_j^C \theta_j^m} \leq 1$ is the wedge that reduces the value of the marginal product of intermediates. Raising the previous equation to the power of ϵ_{Q_j}, taking logs, and rearranging we obtain

(A2)
$$\log\left(\frac{P^M_{jt}M_{jt}}{P_{jt}Q_{jt}}\right)=\log(a_j)+(1-\epsilon_{Q_j})\log\left(\frac{P^M_{jt}}{P_{jt}}\right)+(\epsilon_{Q_j}-1)\log Z_{jt}+\epsilon_{Q_j}\log\overline{\vartheta}_{jt}.$$

Now, we minimize the cost of the intermediate input bundle $\sum_{i=1}^N P_i M_{ij}$ subject to $M_j=\big(\sum_{i=1}^N \omega_{ij}^{\frac{1}{\epsilon_{M_j}}} M_{ij}^{\rho_{M_j}}\big)^{\frac{1}{\rho_{M_j}}}$. The Lagrangian for this problem is

$$\mathcal{L} \;=\; -\sum_{i=1}^N P_i M_{ij}-\lambda_j^2\left(M_j-\big(\sum_{i=1}^N \omega_{ij}^{\frac{1}{\epsilon_{M_j}}} M_{ij}^{\rho_{M_j}}\big)^{\frac{1}{\rho_{M_j}}}\right).$$

Taking first order conditions with respect to M_{ij}, using the fact that in competitive markets $\lambda_j^2=P_j^M$, and rearranging yields

$$\Delta\log\left(\frac{P_{it}M_{ijt}}{P^M_{jt}M_{jt}}\right)=(1-\epsilon_{M_j})\Delta\log\left(\frac{P_{it}}{P^M_{jt}}\right). \tag{A3}$$

Combining Equations (A2) and (A3) yields Equation (4).

A2. Two-sector model solutions

We proceed to find an analytical expression for sector's 2 Lagrange multiplier μ_2. To this end, we need to solve for sectoral prices and input demand, using input optimality conditions, binding working capital constraints, and market clearing conditions.

Assume the wage rate is the numeraire ($w=1$). From the production function of sector 1 ($Q_1=Z_1L_1$) and from the binding constraint in sector 1 ($L_1=\eta_1P_1Q_1$), we obtain

$$P_1=\frac{1}{\eta_1 Z_1}.$$

Using the market clearing condition for the consumption good ($Q_2 = C$), the market clearing condition for (inelastic) labor ($\bar{L} = L_1 + L_2 = 1$), and the household budget constraint $\bar{L} + \Pi = P_2 C$, we obtain

$$P_2 = \frac{1 + \Pi}{Q_2}.$$

The binding constraint of sector 2 and the market clearing condition for sector 1's goods ($Q_1 = M_{12}$) imply

$$\theta_2^w L_2 + \theta_{12}^m P_1 Q_1 = \eta_2 P_2 Q_2,$$

$$\theta_2^w L_2 + \theta_{12}^m \frac{1 - L_2}{\eta_1} = \eta_2 (1 + \Pi),$$

and that

$$L_2 = \frac{\eta_1 \eta_2 (1 + \Pi) - \theta_{12}^m}{\eta_1 \theta_2^w - \theta_{12}^m} = \frac{\eta_1 \eta_2 (1 + \Pi) - \theta_{12}^m}{\phi_1},$$

implying

$$L_1 = 1 - \big(\frac{\eta_1 \eta_2 (1 + \Pi) - \theta_{12}^m}{\eta_1 \theta_2^w - \theta_{12}^m}\big) = \frac{\eta_1 \big(\theta_2^w - \eta_2 (1 + \Pi)\big)}{\eta_1 \theta_2^w - \theta_{12}^m} = \frac{\eta_1 \big(\theta_2^w - \eta_2 (1 + \Pi)\big)}{\phi_1},$$

in which $\phi_1 = \eta_1 \theta_2^w - \theta_{12}^m$. We solve for profit and the Lagrange multiplier below.

Having solved for L_1, L_2 we obtain

$$Q_1 = M_{12} = Z_1 L_1$$

and

$$Q_2 = Z_2 \big(a^{1-\rho_Q} L_2^{\rho_Q} + (1 - a)^{1-\rho_Q} M_{12}^{\rho_Q}\big)^{\frac{1}{\rho_Q}},$$

where $\rho_Q = (\epsilon_Q - 1) / \epsilon_Q$. Finally, using first order and necessary condition

(FONC) in the use of labor or intermediates for firms in sector 2:

$$P_2 Z_2^{\rho_Q} (\frac{aQ_2}{L_2})^{1-\rho_Q} - \frac{(1+\mu_2\theta_2^w)}{(1+\mu_2\eta_2)} = 0,$$

$$P_2 Z_2^{\rho_Q} (\frac{(1-a)Q_2}{M_{12}})^{1-\rho_Q} - P_1 \frac{(1+\mu_2\theta_{12}^m)}{(1+\mu_2\eta_2)} = 0,$$

we can solve for μ_2.

PROOF OF PROPOSITION 1:

Constraint on intermediates: set $\theta_2^w = 0$ and $\theta_{12}^m = 1$, which implies $L_2 = 1 - \eta_1\eta_2(1+\Pi)$ and $Q_1 = Z_1\eta_1\eta_2(1+\Pi)$. From the FONC for L_2, and from the fact that $P_2 = \frac{1+\Pi}{Q_2}$, we obtain

$$(\frac{Q_2}{Z_2})^{\rho_Q} = (1+\mu_2\eta_2)(\frac{a_2}{L_2})^{1-\rho_Q}(1+\Pi).$$

Similarly, using the production function for sector 2 we obtain

$$(\frac{Q_2}{Z_2})^{\rho_Q} = a_2^{1-\rho_Q} L_2^{\rho_Q} + (1-a_2)^{1-\rho_Q} Q_1^{\rho_Q},$$

implying

$$(1+\mu_2\eta_2)(\frac{a_2}{L_2})^{1-\rho_Q}(1+\Pi) = a_2^{1-\rho_Q} L_2^{\rho_Q} + (1-a_2)^{1-\rho_Q} Q_1^{\rho_Q},$$

$$(1+\mu_2\eta_2)(\frac{a_2}{(1-\eta_1\eta_2(1+\Pi))})^{1-\rho_Q}(1+\Pi) = a_2^{1-\rho_Q}(1-\eta_1\eta_2(1+\Pi))^{\rho_Q} + (1-a_2)^{1-\rho_Q}(Z_1\eta_1\eta_2(1+\Pi))^{\rho_Q},$$

and

$$\mu_2 = \left(\frac{(1-\eta_1\eta_2(1+\Pi))\,(1-a_2)}{a_2\eta_2(1+\Pi)}\right)^{1-\rho_Q} (\eta_1 Z_1)^{\rho_Q} + \frac{1}{\eta_2(1+\Pi)} - \eta_1 - \frac{1}{\eta_2}.$$

To solve for profits Π we divide the FONCs for L_2 with the FONCs for M_{12}

$$\mu_2 = \left(\frac{(1-\eta_1\eta_2(1+\Pi))(1-a_2)}{a_2\eta_2(1+\Pi)}\right)^{1-\rho_Q}(\eta_1 Z_1)^{\rho_Q} - 1,$$

$$\Pi = \frac{(1-\eta_1)\eta_2}{1-(1-\eta_1)\eta_2} = \bar{\eta},$$

implying

$$\mu_2 = \left(\frac{(1-\eta_2)(1-a_2)}{Z_1\eta_1\eta_2 a_2}\right)^{1-\rho_Q}\eta_1 Z_1 - 1,$$

Therefore,

$$\frac{\partial\mu_2}{\partial\epsilon_Q} = -\frac{1}{\epsilon_Q^2}(\eta_1 Z_1)^{\rho_Q}\phi_m^{1-\rho_Q}\ln\phi_m$$

where $\phi_m = \frac{(1-\eta_2)(1-a_2)}{Z_1\eta_1\eta_2 a_2}$. If $\phi_m > 1$ the derivative is negative, otherwise it is positive. From the binding constraint we have that

$$\mu_2 = (\phi_m)^{1-\rho_Q}\eta_1 Z_1 - 1 > 0,$$

implying that $\phi_m > \frac{1}{(\eta_1 Z_1)^{\epsilon_Q}}$. Hence, evaluated at $Z_1 = 1$ (steady state productivity value), it is always the case that, as long as firms in sector 1 and sector 2 are constrained ($\eta_1 < 1$ and $\mu_2 > 0$), $\phi_m > 1$. Therefore, more flexible firms are less constrained $\frac{\partial\mu_2}{\partial\epsilon_Q} < 0$. The premium for production flexibility is larger when ϕ_m is larger (due to lower collateral constraint parameters η_1, η_2 , or lower productivity Z_1, or larger intermediate input share $(1-a_2)$)

$$\frac{\partial\mu_2}{\partial\phi_m} = \frac{1}{\epsilon_Q}\phi_m^{-\rho_Q}\eta_1 Z_1 > 0,$$

PROOF OF PROPOSITION 2:

Her we study how the Lagrange multiplier μ_2 changes with financial shocks to sector 1 and 2, and then how the elasticity affects the change in the Lagrange multiplier. Following from Proposition 1, we have

$$\frac{\partial \mu_2}{\partial \eta_2} = (1-\rho_Q)\phi_m^{-\rho_Q}\eta_1 Z_1 \frac{\partial \phi_m}{\partial \eta_2}$$

$$\frac{\partial \mu_2}{\partial \eta_2} = (1-\rho_Q)\phi_m^{-\rho_Q}\eta_1 Z_1 \frac{(a_2-1)}{Z_1\eta_1\eta_2^2 a_2} = \frac{1}{\epsilon_Q}\phi_m^{-\rho_Q}\frac{(a_2-1)}{\eta_2^2 a_2} < 0.$$

We then have that

$$\frac{\partial(\partial \mu_2/\partial \eta_2)}{\partial \epsilon_Q} = \frac{\phi_m^{-\rho_Q}(1-a_2)}{\epsilon_Q^2 a_2 \eta_2^2}\big(1+\frac{1}{\epsilon_Q}\ln \phi_m\big),$$

which is positive as long as $1+\frac{1}{\epsilon_Q}\ln\phi_m > 0$. As long as $\phi_m > 1$, the condition for $\partial\mu_2/\partial\epsilon_Q < 0$, it then holds that $\frac{\partial(\partial \mu_2/\partial \eta_2)}{\partial \epsilon_Q} > 0$, which implies that a more flexible sector displays smaller increases in μ_2 due to tightening credit constraints.

We now study how the Lagrange multiplier changes with a financial shock to sector 1

$$\frac{\partial \mu_2}{\partial \eta_1} = \phi_m^{1-\rho_Q} Z_1 \frac{\epsilon_Q - 1}{\epsilon_Q},$$

which implies that declines in η_1 increase (decrease) the shadow cost of working capital when $\epsilon_Q < 1$ ($\epsilon_Q > 1$). Note that for Cobb-Douglas technologies, tightening credit conditions for sector 1 have no effect on sector 2's shadow cost of debt. If $\frac{\partial(\partial \mu_2/\partial \eta_1)}{\partial \epsilon_Q} > 0$, more flexible firms would experience a larger decline or a smaller increase in the Lagrange multiplier followed by a credit tightening in sector 1. We have that

$$\frac{\partial(\partial \mu_2/\partial \eta_1)}{\partial \epsilon_Q} = \frac{\phi_m^{1-\rho_Q} Z_1}{\epsilon_Q^2}\big(1-\frac{(\epsilon_Q-1)}{\epsilon_Q}\ln\phi_m\big),$$

which is positive as long as $\big(1-\frac{(\epsilon_Q-1)}{\epsilon_Q}\ln\phi_m > 0\big)$. When labor and intermediates are substitutes, $\frac{\partial(\partial \mu_2/\partial \eta_1)}{\partial \epsilon_Q} > 0$ is positive.

A3. Constraint on labor

PROOF:

Set $\theta_2^w = 1$ and $\theta_{12}^m = 0$, which implies $L_2 = \eta_2(1+\Pi)$ and $Q_1 = Z_1\left(1-\eta_2(1+\Pi)\right)$. From the FONC for M_{12}, and from the fact that $P_2 = \frac{1+\Pi}{Q_2}$ and $P_1 = \frac{1}{Z_1\eta_1}$, we obtain

$$(\frac{Q_2}{Z_2})^{\rho_Q} = Z_1\eta_1(1+\mu_2\eta_2)(\frac{(1-a_2)}{M_{12}})^{1-\rho_Q}(1+\Pi).$$

Again using the production function we obtain

$$(\frac{Q_2}{Z_2})^{\rho_Q} = a_2^{1-\rho_Q}L_2^{\rho_Q} + (1-a_2)^{1-\rho_Q}M_{12}^{\rho_Q},$$

which implies

$$(1+\mu_2\eta_2)(\frac{(1-a_2)}{M_{12}})^{1-\rho_Q}Z_1\eta_1(1+\Pi) = a_2^{1-\rho_Q}L_2^{\rho_Q} + (1-a_2)^{1-\rho_Q}M_{12}^{\rho_Q},$$

$$(1+\mu_2\eta_2)(\frac{(1-a_2)}{Z_1\big(1-\eta_2(1+\Pi)\big)})^{1-\rho_Q}Z_1\eta_1(1+\Pi) = a_2^{1-\rho_Q}\big(\eta_2(1+\Pi)\big)^{\rho_Q} + (1-a_2)^{1-\rho_Q}Z_1^{\rho_Q}\big(1-\eta_2(1+\Pi)\big)^{\rho_Q},$$

and

$$\mu_2 = \frac{1}{Z_1\eta_1}\left(\frac{\big(1-\eta_2(1+\Pi)\big)a_2Z_1}{(1-a_2)\,\eta_2(1+\Pi)}\right)^{1-\rho_Q} + \frac{\big(1-(1+\Pi)(\eta_1+\eta_2)\big)}{\eta_1\eta_2(1+\Pi)}.$$

To solve for profits Π we divide the FONCs for L_2 with the FONCs for M_{12}

$$\mu_2 = \frac{1}{Z_1\eta_1}\left(\frac{\big(1-\eta_2(1+\Pi)\big)a_2Z_1}{(1-a_2)\,\eta_2(1+\Pi)}\right)^{1-\rho_Q} - 1,$$

implying

$$\Pi = \frac{1}{\eta_1+\eta_2-\eta_1\eta_2}$$

and

$$\mu_2 = \frac{1}{Z_1\eta_1}\left(\frac{\eta_1(1-\eta_2)a_2 Z_1}{(1-a_2)\eta_2}\right)^{1-\rho_Q} - 1.$$

Therefore,

$$\frac{\partial \mu_2}{\partial \epsilon_Q} = -\frac{1}{\epsilon_Q^2}\frac{1}{Z_1\eta_1}\phi_w^{1-\rho_Q}\ln\left(\phi_w\right),$$

where $\phi_w = \frac{\eta_1(1-\eta_2)a_2Z_1}{(1-a_2)\eta_2}$. If $\phi_w > 1$ the derivative is negative, otherwise it is positive. For the constraint to be binding, we require $\mu_2 > 0$, implying

$$\phi_w > (Z_1\eta_1)^{\epsilon_Q}.$$

Therefore, only for high values of Z_1 and η_1, the model can replicate the negative relationship between elasticities and the shadow cost of debt.

Let us see how sector 1's constraint affects sector 2's wedge, when sector 2's constraint tightens. We have that

$$\frac{\partial \phi_w}{\partial \eta_2} = -\frac{\eta_1 Z_1 a_2(1-a_2)}{\left((1-a_2)\eta_2\right)^2}$$

a tightening of sector 2's constraint raises the cost of labor (constrained input). On the other hand, we have that

$$\frac{\partial(\partial\phi_w/\partial\eta_2)}{\partial\eta_1} < 0$$

implying that a tighter constraint in sector 1 mitigates the increase in ϕ_w due to a tightening in η_2 (it makes $\frac{\partial\phi_w}{\partial\eta_2}$ less negative).

PROOF OF PROPOSITION 3:

Let us define $\rho_{Q_j} = \frac{\epsilon_{Q_j}-1}{\epsilon_{Q_j}}$ and assume $\epsilon_{M_j} = \epsilon_{Q_j}$ for all j. To obtain real GDP in this economy, use the cost minimizing problem

$$Min \quad \sum_{j=1}^{N} P_j C_j,$$

subject to

$$C = \prod_{j=1}^{N} C_j^{\beta_j},$$

which yields

$$P_j C_j = \beta_j \sum_{j=1}^{N} P_j C_j.$$

Combining the previous condition with the household budget constraint

$$\sum_{j=1}^{N} P_j C_j = WL + \Pi,$$

gives

$$P_j C_j = \beta_j (WL + \Pi),$$

and the fact that labor is inelastically supplied $L = 1$ and the wage rate is the numeraire

$$C_j = \frac{\beta_j(1+\Pi)}{P_j}$$

$$C = \prod_{j=1}^{N} C_j^{\beta_j} = \prod_{j=1}^{N} (\frac{\beta_j(1+\Pi)}{P_j})^{\beta_j}$$

$$\log C = \sum_{j=1}^{N} \beta_j \log\big(\frac{\beta_j(1+\Pi)}{P_j}\big)$$

$$\log C = \sum_{j=1}^{N} \beta_j \log\big(\frac{\beta_j}{P_j}\big) + \sum_{j=1}^{N} \beta_j \log(1+\Pi),$$

using the fact that $\sum_{j=1}^{N} \beta_j = 1$ we have that real GDP in this economy is

$$\log C = \sum_{j=1}^{N} \beta_j \log\big(\frac{\beta_j}{P_j}\big) + \log(1+\Pi).$$

We need to solve for sectoral prices. We first modify the production function

$$Z_j^{-\rho_{Q_j}} = a_j^{1-\rho_{Q_j}}\big(\frac{L_j}{Q_j}\big)^{\rho_{Q_j}} + (1-a_j)^{1-\rho_{Q_j}}\big(\frac{M_j}{Q_j}\big)^{\rho_{Q_j}},$$

define wedges as follows

$$\vartheta_j^m = \frac{(1+\mu_j\eta_j)}{(1+\mu_j\theta_j^m)},$$

$$\vartheta_j^w = \frac{(1+\mu_j\eta_j)}{(1+\mu_j\theta_j^w)},$$

and use the first order conditions for labor and intermediates

$$P_j Z_j^{\rho_{Q_j}}\big(\frac{a_j Q_j}{L_j}\big)^{1-\rho_{Q_j}} = \frac{(1+\mu_j\theta_j^w)}{(1+\mu_j\eta_j)} = (\vartheta_j^w)^{-1},$$

$$P_j Z_j^{\rho_{Q_j}}\big(\frac{(1-a_j)Q_j}{M_j}\big)^{1-\rho_{Q_j}} = P_j^M \frac{(1+\mu_j\theta_j^m)}{(1+\mu_j\eta_j)} = P_j^M(\vartheta_j^m)^{-1}.$$

This definition of wedge implies that a decline in η_j decreases the wedge ϑ_j. A decline in η_j increases μ_j. Therefore, the denominator increases more than the numerator. A decline in η_j corresponds to tighter credit, which is isomorphic to an increase in sectoral spreads (or EBP to be more precise). Thus, increases in sectoral spread decrease ϑ_j.

To solve for real GDP we first need to solve for sectoral prices. We use sectoral first order conditions

$$\big(\frac{L_j}{Q_j}\big)^{\rho_{Q_j}} = P_j^{\epsilon_{Q_j}-1} Z_j^{\frac{(\epsilon_{Q_j}-1)^2}{\epsilon_{Q_j}}} a_j^{\frac{(\epsilon_{Q_j}-1)}{\epsilon_{Q_j}}} (\vartheta_j^w)^{\epsilon_{Q_j}-1},$$

$$(\frac{M_j}{Q_j})^{\rho_{Q_j}} = (\frac{P_j}{P_j^M})^{\epsilon_{Q_j}-1} Z_j^{\frac{(\epsilon_{Q_j}-1)^2}{\epsilon_{Q_j}}} (1-a_j)^{\frac{(\epsilon_{Q_j}-1)}{\epsilon_{Q_j}}} (\vartheta_j^m)^{\epsilon_{Q_j}-1},$$

implying (now allowing for heterogeneous elasticities)

$$P_j^{1-\epsilon_{Q_j}} = a_j Z_j^{\epsilon_{Q_j}-1} (\vartheta_j^w)^{\epsilon_{Q_j}-1} + (1-a_j) Z_j^{\epsilon_{Q_j}-1} (\vartheta_j^m)^{\epsilon_{Q_j}-1} (P_j^M)^{1-\epsilon_{Q_j}},$$

$$P_j^{1-\epsilon_{Q_j}} = a_j Z_j^{\epsilon_{Q_j}-1} (\vartheta_j^w)^{\epsilon_{Q_j}-1} + (1-a_j) Z_j^{\epsilon_{Q_j}-1} (\vartheta_j^m)^{\epsilon_{Q_j}-1} \Big(\sum_{i=1}^N \omega_{ij} P_i^{1-\epsilon_{M_j}}\Big)^{\frac{1-\epsilon_{Q_j}}{1-\epsilon_{M_j}}}.$$

Now assume that $\epsilon_{Q_j} = \epsilon_{M_j}$ for all j implies

$$P_j^{1-\epsilon_{Q_j}} = a_j Z_j^{\epsilon_{Q_j}-1} (\vartheta_j^w)^{\epsilon_{Q_j}-1} + (1-a_j) Z_j^{\epsilon_{Q_j}-1} (\vartheta_j^m)^{\epsilon_{Q_j}-1} \sum_{i=1}^N \omega_{ij} P_i^{1-\epsilon_{Q_j}},$$

and in matrix form

$$P^{1-\epsilon_Q} = a \circ (Z \circ \vartheta^w)^{\circ \epsilon_Q - 1} + \big((1-a) \circ (Z \circ \vartheta^m)^{\circ \epsilon_Q - 1} 1'\big) \circ (\Omega \circ (P1')^{\circ((1-\epsilon_{Q_j})1')'})'1.$$

Note here that the term $\sum_{i=1}^N \omega_{ij} P_i^{1-\epsilon_{Q_j}}$ has all sectoral prices and intermediates shares, from i to N, raised to the power of sector's j elasticity. With common elasticity expressing these terms in matrix form is trivial: $\Omega' P^{1-\epsilon_Q}$. Nevertheless, the matrix form with heterogeneous elasticities is $\Omega \circ (P1')^{\circ((1-\epsilon_{Q_j})1')'})'1$.

We now solve for sectoral sale shares. We multiply sectoral market clearing condition for sector j by sectoral price P_j we obtain

$$S_j = P_j C_j + \sum_{i=1}^N P_j M_{ji},$$

where S_j is sectoral sales. Let's use the household optimal consumption share for each good (with $\epsilon_D = 1$ we have $P_j C_j = \beta_j P_c C$) and rearrange the firm optimality condition for M_{ji}

$$P_j M_{ji}^{1-\rho_{Q_i}} = \vartheta_i^m Z_i^{\rho_{Q_i}} \big((1-a_i)\omega_{ji}\big)^{1-\rho_{Q_i}} M_i^{\rho_{Q_i}-\rho_{M_i}} P_i Q_i^{1-\rho_{Q_i}},$$

which combined with the FONC for M_i

$$M_i = (\vartheta_i^m)^{\epsilon_{Q_i}} Z_i^{\epsilon_{Q_i}-1} \frac{P_i^{\epsilon_{Q_i}}}{(P_i^M)^{\epsilon_{Q_i}}}(1-a_i)Q_i,$$

yields

$$P_j M_{ji}^{1-\rho_{Q_i}} = \vartheta_i^m Z_i^{\rho_{Q_i}} \big((1-a_i)\omega_{ji}\big)^{1-\rho_{Q_i}} \Big((\vartheta_i^m)^{\epsilon_{Q_i}} Z_i^{\epsilon_{Q_i}-1} \frac{P_i^{\epsilon_{Q_i}}}{(P_i^M)^{\epsilon_{Q_i}}}(1-a_i)Q_i\Big)^{\rho_{Q_i}-\rho_{M_i}} P_i Q_i^{1-\rho_{Q_i}},$$

Note that unlike the case $\epsilon_{Q_j} = \epsilon_{M_j}$, when $\epsilon_{Q_j} \neq \epsilon_{M_j}$ there is no linear closed-form solution for sales shares (given prices).

Assuming that $\epsilon_{Q_j} = \epsilon_{M_j}$

$$P_j M_{ji}^{1-\rho_{Q_i}} = \vartheta_i^m Z_i^{\rho_{Q_i}} \big((1-a_i)\omega_{ji}\big)^{1-\rho_{Q_i}} P_i Q_i^{1-\rho_{Q_i}},$$

$$P_j M_{ji} = \big(\frac{P_i}{P_j}\big)^{\epsilon_{Q_i}-1} (\vartheta_i^m)^{\epsilon_{Q_i}} Z_i^{\epsilon_{Q_i}-1}(1-a_i)\omega_{ji} P_i Q_i,$$

to get

$$S_j = \beta_j P_c C + \sum_{i=1}^N P_j^{1-\epsilon_{Q_i}} P_i^{\epsilon_{Q_i}-1} (\vartheta_i^m)^{\epsilon_{Q_i}} Z_i^{\epsilon_{Q_i}-1}(1-a_i)\omega_{ji} S_i.$$

$$\frac{S_j}{P_c C} = \beta_j + \sum_{i=1}^N P_j^{1-\epsilon_{Q_i}} P_i^{\epsilon_{Q_i}-1} (\vartheta_i^m)^{\epsilon_{Q_i}} Z_i^{\epsilon_{Q_i}-1}(1-a_i)\omega_{ji} \frac{S_i}{P_c C},$$

$$s = [I - \big((P1')^{\circ((1-\epsilon_Q)1')')}\big) \circ \big((\vartheta^m)^{\circ\epsilon_Q} \circ (Z \circ P)^{\circ(\epsilon_Q - 1)} 1'\big)' \circ ((1-a)1')' \circ \Omega]^{-1}\beta,$$

in which $s = \frac{S_j}{P_c C} = \frac{S_j}{1+\Pi}$. Note that with common elasticity the matrix form solution simplifies to

$$s = [I - (P^{\circ(1-\epsilon_Q)}1') \circ \left((\vartheta^m)^{\circ\epsilon_Q} \circ (Z \circ P)^{\circ(\epsilon_Q - 1)}1'\right)' \circ ((1-a)1')' \circ \Omega]^{-1}\beta,$$

Having solved for prices and sales shares we can solve for profits. Combining the firms FONCs for input we have

$$P_j Z_j^{\rho_{Q_j}} \left(\frac{(1-a_j)Q_j}{M_j}\right)^{1-\rho_{Q_j}} = \frac{P_j^M}{\vartheta_j^m},$$

$$M_j^{1-\rho_{Q_j}} = \vartheta_j^m \frac{P_j}{P_j^M} Z_j^{\rho_{Q_j}} \left((1-a_j)Q_j\right)^{1-\rho_{Q_j}},$$

$$M_j^{1-\rho_{Q_j}} = \vartheta_j^m \frac{P_j^{\rho_{Q_j}}}{P_j^M} Z_j^{\rho_{Q_j}} \left((1-a_j)\right)^{1-\rho_{Q_j}} (P_j Q_j)^{1-\rho_{Q_j}},$$

$$M_j = (\vartheta_j^m)^{\epsilon_{Q_j}} Z_j^{\epsilon_{Q_j}-1} \frac{P_j^{\epsilon_{Q_j}-1}}{(P_j^M)^{\epsilon_{Q_j}}} (1-a_j) \frac{P_j Q_j}{P_c C} P_c C,$$

where $P_c C = 1 + \Pi$ and $s_j = \frac{P_j Q_j}{P_c C}$, implying

$$M_j = (\vartheta_j^m)^{\epsilon_{Q_j}} Z_j^{\epsilon_{Q_j}-1} \frac{P_j^{\epsilon_{Q_j}-1}}{(P_j^M)^{\epsilon_{Q_j}}} (1-a_j) s_j (1+\Pi),$$

which combined with the ratio between the labor and intermediates first order condition

$$L_j = \left(\frac{P_j^M \vartheta_j^w}{\vartheta_j^m}\right)^{\epsilon_{Q_j}} \frac{a_j M_j}{(1-a_j)},$$

yields

$$L_j = \left(\frac{P_j^M \vartheta_j^w}{\vartheta_j^m}\right)^{\epsilon_{Q_j}} \frac{a_j}{(1-a_j)} (\vartheta_j^m)^{\epsilon_{Q_j}} Z_j^{\epsilon_{Q_j}-1} \frac{P_j^{\epsilon_{Q_j}-1}}{(P_j^M)^{\epsilon_{Q_j}}} (1-a_j) s_j (1+\Pi).$$

$$L_j = (\vartheta_j^w)^{\epsilon_{Q_j}} a_j Z_j^{\epsilon_{Q_j}-1} P_j^{\epsilon_{Q_j}-1} s_j (1+\Pi).$$

We then use the labor market clearing condition, the solution for prices, and the solution for sale shares, to solve for profits

$$(1+\Pi) \sum_{j=1}^{N} a_j (\vartheta_j^w)^{\epsilon_{Q_j}} Z_j^{\epsilon_{Q_j}-1} P_j^{\epsilon_{Q_j}-1} s_j = 1.$$

$$(1+\Pi) = \frac{1}{\sum_{j=1}^{N} a_j (\vartheta_j^w)^{\epsilon_{Q_j}} Z_j^{\epsilon_{Q_j}-1} P_j^{\epsilon_{Q_j}-1} s_j}.$$

Solution two-sector model with heterogeneous CES

In the Island economy (suppose $Z_j = 1$ for all j and $a_j = a$ for all j), the solution for prices, sales shares, and profits is

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} & \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} & \vartheta_2^{\epsilon_2-1} \end{bmatrix} \circ \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \circ \begin{bmatrix} P_1^{1-\epsilon_1} & P_1^{1-\epsilon_2} \\ P_2^{1-\epsilon_1} & P_2^{1-\epsilon_2} \end{bmatrix} \right)' \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} & \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} & \vartheta_2^{\epsilon_2-1} \end{bmatrix} \circ \begin{bmatrix} P_1^{1-\epsilon_1} & 0 \\ 0 & P_2^{1-\epsilon_2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} P_1^{1-\epsilon_1} & 0 \\ 0 & \vartheta_2^{\epsilon_2-1} P_2^{1-\epsilon_2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} P_1^{1-\epsilon_1} \\ \vartheta_2^{\epsilon_2-1} P_2^{1-\epsilon_2} \end{bmatrix},$$

implying

$$
\begin{aligned}
P_1^{1-\epsilon_{Q_1}} &= \frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)}, \\
P_2^{1-\epsilon_{Q_2}} &= \frac{a}{\vartheta_2^{1-\epsilon_{Q_2}} - (1-a)}.
\end{aligned}
$$

To obtain sales, we have

$$
= [I - \left(s(P1')^{\circ((1-\epsilon_Q)1')')}\right) \circ \left((\vartheta^m)^{\epsilon_Q} \circ (Z \circ P)^{\epsilon_Q - 1} 1'\right)' \circ ((1-a)1')' \circ \Omega]^{-1}\beta,
$$

$$
\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \Big[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} P_1^{1-\epsilon_1} & P_1^{1-\epsilon_2} \\ P_2^{1-\epsilon_1} & P_2^{1-\epsilon_2} \end{bmatrix} \circ \begin{bmatrix} \vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 1} & \vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 1} \\ \vartheta_2^{\epsilon_2} P_2^{\epsilon_2 - 1} & \vartheta_2^{\epsilon_2} P_2^{\epsilon_2 - 1} \end{bmatrix}' \begin{bmatrix} 1-a & 0 \\ 0 & 1-a \end{bmatrix} \Big]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},
$$

$$
\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \Big[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} P_1^{1-\epsilon_1} & P_1^{1-\epsilon_2} \\ P_2^{1-\epsilon_1} & P_2^{1-\epsilon_1} \end{bmatrix} \circ \begin{bmatrix} \vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 1} & \vartheta_2^{\epsilon_2} P_2^{\epsilon_2 - 1} \\ \vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 1} & \vartheta_2^{\epsilon_2} P_2^{\epsilon_2 - 1} \end{bmatrix} \begin{bmatrix} 1-a & 0 \\ 0 & 1-a \end{bmatrix} \Big]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},
$$

$$
\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \Big[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} (1-a)P_1^{1-\epsilon_1} \vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 1} & 0 \\ 0 & (1-a)P_2^{1-\epsilon_2} \vartheta_2^{\epsilon_2} P_2^{\epsilon_2 - 1} \end{bmatrix} \Big]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},
$$

$$
\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 1-(1-a)\vartheta_1^{\epsilon_1} & 0 \\ 0 & 1-(1-a)\vartheta_2^{\epsilon_2} \end{bmatrix}^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},
$$

$$
\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \frac{1}{(1-(1-a)\vartheta_1^{\epsilon_1})(1-(1-a)\vartheta_2^{\epsilon_2})} \begin{bmatrix} 1-(1-a)\vartheta_2^{\epsilon_2} & 0 \\ 0 & 1-(1-a)\vartheta_1^{\epsilon_1} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},
$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \frac{1}{(1-(1-a)\vartheta_1^{\epsilon_1})(1-(1-a)\vartheta_2^{\epsilon_2})} \begin{bmatrix} \beta_1(1-(1-a)\vartheta_2^{\epsilon_2}) \\ \beta_2(1-(1-a)\vartheta_1^{\epsilon_1}) \end{bmatrix},$$

which yields

$$\begin{aligned} s_1 =& \frac{\beta_1}{1-(1-a)\vartheta_1^{\epsilon_1}}, \\ s_2 =& \frac{\beta_2}{1-(1-a)\vartheta_2^{\epsilon_2}}. \end{aligned}$$

In the Star Supplier Economy (suppose $Z_j = 1$ for all j and $a_j = a$ for all j), the solution for prices, sales shares, and profits is

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} & \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} & \vartheta_2^{\epsilon_2-1} \end{bmatrix} \circ \Big(\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \circ \begin{bmatrix} P_1^{1-\epsilon_1} & P_1^{1-\epsilon_2} \\ P_2^{1-\epsilon_1} & P_2^{1-\epsilon_2} \end{bmatrix} \Big)' \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} & \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} & \vartheta_2^{\epsilon_2-1} \end{bmatrix} \circ \begin{bmatrix} P_1^{1-\epsilon_1} & 0 \\ P_1^{1-\epsilon_2} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} P_1^{1-\epsilon_1} & 0 \\ \vartheta_2^{\epsilon_2-1} P_1^{1-\epsilon_2} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} P_1^{1-\epsilon_1} \\ P_2^{1-\epsilon_2} \end{bmatrix} = a \begin{bmatrix} \vartheta_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2-1} \end{bmatrix} + (1-a) \begin{bmatrix} \vartheta_1^{\epsilon_1-1} P_1^{1-\epsilon_1} \\ \vartheta_2^{\epsilon_2-1} P_1^{1-\epsilon_2} \end{bmatrix},$$

implying

$$
\begin{aligned}
P_1^{1-\epsilon_{Q_1}} =& \frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)},\\
P_2^{1-\epsilon_{Q_2}} =& a\vartheta_2^{\epsilon_{Q_2}-1} + (1-a)\vartheta_2^{\epsilon_{Q_2}-1}\Big(\frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)}\Big)^{\frac{1-\epsilon_{Q_2}}{1-\epsilon_{Q_1}}}
\end{aligned}
$$

To obtain sales, we have

$$
s = [I - \big((P1')^{\circ((1-\epsilon_Q)1')'}\big)) \circ \big((\vartheta^m)^{\epsilon_Q} \circ (Z \circ P)^{\epsilon_Q - 1}1'\big)' \circ ((1-a)1')' \circ \Omega]^{-1}\beta,
$$

$$
\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \Big[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} P_1^{1-\epsilon_1} & P_1^{1-\epsilon_2} \\ P_2^{1-\epsilon_1} & P_2^{1-\epsilon_2} \end{bmatrix} \circ \begin{bmatrix} \vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1} & \vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1} \\ \vartheta_2^{\epsilon_2} P_2^{\epsilon_2-1} & \vartheta_2^{\epsilon_2} P_2^{\epsilon_2-1} \end{bmatrix}' \begin{bmatrix} 1-a & 1-a \\ 0 & 0 \end{bmatrix} \Big]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},
$$

$$
\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \Big[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} P_1^{1-\epsilon_1} & P_1^{1-\epsilon_2} \\ P_2^{1-\epsilon_1} & P_2^{1-\epsilon_2} \end{bmatrix} \circ \begin{bmatrix} \vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1} & \vartheta_2^{\epsilon_2} P_2^{\epsilon_2-1} \\ \vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1} & \vartheta_2^{\epsilon_2} P_2^{\epsilon_2-1} \end{bmatrix} \begin{bmatrix} 1-a & 1-a \\ 0 & 0 \end{bmatrix} \Big]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},
$$

$$
\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \Big[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} (1-a)P_1^{1-\epsilon_1}\vartheta_1^{\epsilon_1}P_1^{\epsilon_1-1} & (1-a)P_1^{1-\epsilon_2}\vartheta_2^{\epsilon_2}P_2^{\epsilon_2-1} \\ 0 & 0 \end{bmatrix} \Big]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},
$$

$$
\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 1-(1-a)\vartheta_1^{\epsilon_1} & -(1-a)P_1^{1-\epsilon_2}\vartheta_2^{\epsilon_2}P_2^{\epsilon_2-1} \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},
$$

$$
\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \frac{1}{1-(1-a)\vartheta_1^{\epsilon_1}} \begin{bmatrix} 1 & (1-a)P_1^{1-\epsilon_2}\vartheta_2^{\epsilon_2}P_2^{\epsilon_2-1} \\ 0 & 1-(1-a)\vartheta_1^{\epsilon_1} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix},
$$

$$
\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \frac{1}{1-(1-a)\vartheta_1^{\epsilon_1}} \begin{bmatrix} \beta_1 + \beta_2(1-a)P_1^{1-\epsilon_2}\vartheta_2^{\epsilon_2}P_2^{\epsilon_2-1} \\ \beta_2(1-(1-a)\vartheta_1^{\epsilon_1}) \end{bmatrix},
$$

which yields the following solutions for the Star supplier economy

$$
\begin{aligned}
(P_1^S)^{1-\epsilon_{Q_1}} =& \frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)}, \\
(P_2^S)^{1-\epsilon_{Q_2}} =& a\vartheta_2^{\epsilon_{Q_2}-1} + (1-a)\vartheta_2^{\epsilon_{Q_2}-1}\Big(\frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)}\Big)^{\frac{1-\epsilon_{Q_2}}{1-\epsilon_{Q_1}}}, \\
s_1^S =& \frac{\beta}{1-(1-a)\vartheta_1^{\epsilon_1}} + \frac{\beta_2 (P_1^S)^{1-\epsilon_2}(P_2^S)^{\epsilon_2-1}\vartheta_2^{\epsilon_2}(1-a)}{1-(1-a)\vartheta_1^{\epsilon_1}} \\
s_2^S =& 1-\beta,
\end{aligned}
$$

and the following solutions for the Island economy

$$
\begin{aligned}
P_1^{1-\epsilon_{Q_1}} =& \frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)}, \\
(P_2^I)^{1-\epsilon_{Q_2}} =& \frac{a}{\vartheta_2^{1-\epsilon_{Q_2}} - (1-a)}, \\
s_1^I =& \frac{\beta}{1-(1-a)\vartheta_1^{\epsilon_1}} \\
s_2^I =& \frac{1-\beta}{1-(1-a)\vartheta_2^{\epsilon_2}}
\end{aligned}
$$

A4. Sectoral shock: heterogeneous elasticities

PROOF PROPOSITION 4:

We start by defining the input-output multiplier (IOM) as

$$IOM = \frac{\partial \log C^S}{\partial \vartheta_1} - \frac{\partial \log C^I}{\partial \vartheta_1},$$

in which C^S and C^I stand for real GDP in the Star supplier and Island economies, respectively.

From the definition of real GDP, we have

$$\frac{\partial \log C^I}{\partial \vartheta_1} = \underbrace{-\beta \frac{\partial \log P_1}{\partial \vartheta_1}}_{\text{Real wage channel}} \underbrace{-(1+\Pi^I)a\Big[\epsilon_1 s_1^I \vartheta_1^{\epsilon_1-1} P_1^{\epsilon_1-1} + s_1^I \vartheta_1^{\epsilon_1} \frac{\partial P_1^{\epsilon_1-1}}{\partial \vartheta_1} + \vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1} \frac{\partial s_1^I}{\partial \vartheta_1}\Big]}_{\text{Rents channel}}$$

$$\frac{\partial \log C^S}{\partial \vartheta_1} = \underbrace{-\beta \frac{\partial \log P_1}{\partial \vartheta_1} - (1-\beta) \frac{\partial \log P_2^S}{\partial \vartheta_1}}_{\text{Real wage channel}} \underbrace{-(1+\Pi^S)a\Big[\epsilon_1 s_1^S \vartheta_1^{\epsilon_1-1} P_1^{\epsilon_1-1} + s_1^S \vartheta_1^{\epsilon_1} \frac{\partial P_1^{\epsilon_1-1}}{\partial \vartheta_1} + \vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1} \frac{\partial s_1^S}{\partial \vartheta_1} + s_2^S \vartheta_2^{\epsilon_2} \frac{\partial (P_2^S)^{\epsilon_2-1}}{\partial \vartheta_1}\Big]}_{\text{Rents channel}},$$

where we differentiate the effects of distortions on the real wage and on the rents rebated to the household. Using the fact that $\frac{\partial \log P_2}{\vartheta_1} = \frac{1}{P_2}\frac{\partial P_2}{\vartheta_1}$, that $\frac{\partial P_2^{\epsilon_2-1}}{\vartheta_1} = (\epsilon_2-1)P_2^{\epsilon_2-2}\frac{\partial P_2}{\vartheta_1}$, and that $s_2^S = 1-\beta$, we reorganize the IOM as follows

$$\begin{aligned} IOM = & \underbrace{-\frac{\partial \log P_2^S}{\partial \vartheta_1}(1-\beta)\Big(1 + a\vartheta_2^{\epsilon_2}(P_2^S)^{\epsilon_2-1}(1+\Pi^S)(\epsilon_2-1)\Big)}_{\text{Term 1}} \\ & \underbrace{-a\epsilon_1 \vartheta_1^{\epsilon_1-1} P_1^{\epsilon_1-1}\big[(1+\Pi^S)s_1^S - (1+\Pi^I)s_1^I\big]}_{\text{Term 2}} \\ & \underbrace{-a\vartheta_1^{\epsilon_1}(\epsilon_1-1)P_1^{\epsilon_1-2}\frac{\partial P_1}{\partial \vartheta_1}\big[(1+\Pi^S)s_1^S - (1+\Pi^I)s_1^I\big]}_{\text{Term 3}} \\ & \underbrace{-a\vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1}\big[(1+\Pi^S)\frac{\partial s_1^S}{\partial \vartheta_1} - (1+\Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1}\big]}_{\text{Term 4}}. \end{aligned}$$

We first analyze Term 1

$$\text{Term 1} = -\frac{\partial \log P_2^S}{\partial \vartheta_1}(1-\beta)\Big(1 + a\vartheta_2^{\epsilon_2} P_2^{\epsilon_2-1}(1+\Pi^S)(\epsilon_2-1)\Big),$$

$$\text{Term 1} = \frac{(1-a)a\vartheta_1^{-\epsilon_1}\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}}{\Big(a + (1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)\big(\vartheta_1^{1-\epsilon_1} - (1-a)\big)^2}(1-\beta)\Big(1+a\vartheta_2^{\epsilon_2} P_2^{\epsilon_2-1}(1+\Pi^S)(\epsilon_2-1)\Big),$$

where $\phi = \frac{a}{\vartheta_1^{1-\epsilon_1}-(1-a)}$ (>0 so prices are positive) and

$$(P_2^S)^{1-\epsilon_2} = \vartheta_2^{\epsilon_2-1}\Big(a + (1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)$$

$$(P_2^S)^{\epsilon_2-1} = \vartheta_2^{1-\epsilon_2}\Big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)^{-1},$$

implying

$$\text{Term 1} = \frac{(1-a)a\vartheta_1^{-\epsilon_1}(1-\beta)\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}}{\Big(a+(1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^2}\Big[1+\frac{a\vartheta_2(1+\Pi^S)(\epsilon_2-1)}{\Big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)}\Big],$$

$$\text{Term 1} = \frac{(1-a)a\vartheta_1^{-\epsilon_1}(1-\beta)\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}}{\Big(a+(1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^2}+\frac{(1-a)a\vartheta_1^{-\epsilon_1}(1-\beta)\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}a\vartheta_2(1+\Pi^S)(\epsilon_2-1)}{\Big(a+(1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)^2\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^2}$$

$$\text{Term 1} = \frac{\psi_1^{t1}(\epsilon_1)\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}}{a+(1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}}+\frac{\psi_2^{t1}(\epsilon_1)(\epsilon_2-1)\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}}{\Big(a+(1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)^2},$$

$$\text{Term 1} = \psi_1^{t1}(\epsilon_1,\epsilon_2)+(\epsilon_2-1)\psi_2^{t1}(\epsilon_1,\epsilon_2),$$

where $\psi_1^{t1}(\epsilon_1,\epsilon_2)$ and $\psi_2^{t1}(\epsilon_1,\epsilon_2)$ are positive and non-linear functions of ϵ_1 and ϵ_2.

Based on the last term, Term 1 is positive and increasing in ϵ_2 (whenever $\epsilon_2 > 1$), implying that larger flexibility of the downstream sector generates a smaller increase in rents (downstream rents), from shrinking production more given the shock to the supplier, compared to the Island economy. It is also the case though that a larger downstream elasticity mitigates the price increase in sector 2, which in turn mitigates the reduction in real wage due to the shock. Nevertheless, the first effect dominates. It could be the case that the last term becomes negative for sufficiently low ϵ_2. To analyze that possibility assume $\epsilon_2 = 0$. In this case, Term 1 becomes

$$\text{Term } 1_{\epsilon_2=0} = \frac{(1-a)a\vartheta_1^{-\epsilon_1}\phi^{\frac{\epsilon_1}{1-\epsilon_1}}}{\Big(a+(1-a)\phi^{\frac{1}{1-\epsilon_1}}\Big)\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^2}(1-\beta)\Big(\frac{\Big(a+(1-a)\phi_1^{\frac{1}{1-\epsilon_1}}\Big)-a\vartheta_2(1+\Pi^S)}{\Big(a+(1-a)\phi_1^{\frac{1}{1-\epsilon_1}}\Big)}\Big),$$

$$\text{Term } 1_{\epsilon_2=0} = \frac{(1-a)a\vartheta_1^{-\epsilon_1}\phi^{\frac{\epsilon_1}{1-\epsilon_1}}}{\left(a+(1-a)\phi^{\frac{1}{1-\epsilon_1}}\right)\left(\vartheta_1^{1-\epsilon_1}-(1-a)\right)^2}(1-\beta)\Big(\frac{a(1-\vartheta_2(1+\Pi^S))+(1-a)\phi_1^{\frac{1}{1-\epsilon_1}}\Big)}{\left(a+(1-a)\phi_1^{\frac{1}{1-\epsilon_1}}\right)}\Big),$$

which is still positive as long as $a(1-\vartheta_2(1+\Pi^S)) > 0$. This is the case whenever sector 2 is reasonably constrained ($\vartheta_2 << 1$).

The key difference with respect to the homogeneous elasticity case is that while the term increases monotonically with ϵ_2 or ϵ, it actually decreases with ϵ_1. Intuitively, a higher ϵ_1 reduces the price increase of sector 1, which then implies a lower increase in the marginal cost of the downstream sector, and then a smaller increase in P_2. With homogeneous elasticities, even when a higher elasticity mitigates shocks to the supplier (less price adjustment and more quantity adjustment), it also amplifies the response of the downstream sector (larger reduction in rents and, therefore, income to the household). The latter effect does not exist when we only change ϵ_1 and keep ϵ_2 fixed. In other words, it is the higher elasticity of the downstream sector, not the upstream sector, that amplifies the aggregate effects from distortions in Term 1. In any case, with a larger common elasticity, it is more likely that, through this term, the $IOM > 0$ and the Star supplier amplifies shocks to sector 1. With heterogeneous elasticities, a high ϵ_2 and a low ϵ_1 imply $IOM > 0$, all else equal.

We now analyze Term 2

$$\text{Term } 2 = -a\epsilon_1\vartheta_1^{\epsilon_1-1}P_1^{\epsilon_1-1}\left[(1+\Pi^S)s_1^S-(1+\Pi^I)s_1^I\right]$$

$$\text{Term } 2 = -\epsilon_1\psi_1^{t2}(\epsilon_1,\epsilon_2),$$

in which $\psi_1^{t2}(\epsilon_1,\epsilon_2)$ is positive and non-linear function of ϵ_1 and ϵ_2. Term 2 is negative as s_1^S is larger than s_1^I, $s_1^S = \underbrace{\frac{\beta}{1-(1-a)\vartheta_1^{\epsilon_1}}}_{s_1^I} + \underbrace{\frac{\beta_2(P_1^S)^{1-\epsilon_2}(P_2^S)^{\epsilon_2-1}\vartheta_2^{\epsilon_2}(1-a)}{1-(1-a)\vartheta_1^{\epsilon_1}}}_{s_1^{S_2}>0}),$

while $\Pi^S \approx \Pi^I$. Through this, when sector 1 is slightly constrained ($\vartheta_1 \approx 1$), a shock to sector 1 is mitigated in the star supplier economy, more so the higher ϵ_1. Intuitively, if $\epsilon_1 = 0$ this term is irrelevant because the distorted sector in both networks is optimally not changing its production plan (M, L). When $\epsilon_1 > 0$ in the Star supplier economy, a larger fraction of the economy is better able to couple with the shock. However, when sector 1 is heavily distorted, this term shrinks when ϵ_1 is larger, and larger than 1. Thus, when the distortion is severe, the composition effect dominates the relocation effect, and the Star supplier economy displays a larger reduction in real GDP, all else equal.

Term 3 is

$$\text{Term 3} = -a(\epsilon_1 - 1)\vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 2} \frac{\partial P_1}{\partial \vartheta_1} \big[(1 + \Pi^S)s_1^S - (1 + \Pi^I)s_1^I\big]$$

$$\text{Term 3} = (\epsilon_1 - 1)\psi_1^{t3}(\epsilon_1, \epsilon_2).$$

where $\psi_1^{t3}(\epsilon_1, \epsilon_2)$ is positive and non-linear function of ϵ_1 and ϵ_2. Term 3 is positive when $\epsilon_1 > 1$ (as $s_1^S > s_1^I$ and $\frac{\partial P_1}{\partial \vartheta_1} < 0$), but negative when $\epsilon_1 < 1$. Here a higher elasticity amplifies further (if $1 < \epsilon_1 < \bar{\epsilon}_1$ and distortion is not too tight). This effect is not the direct effect on P_1, as that is the same for both networks, but it is the effect on sector 1's rents. When the distorted sector is very flexible, it optimally shrinks more, reducing households rents (a function of revenue). However, $\frac{\partial P_1}{\partial \vartheta_1}$ is less negative the larger ϵ_1. When the distortion is initially very tight, or the elasticity very large, a further increase in the elasticity reduces the value Term 3. A larger ϵ_1 also reduces the value of $\frac{\partial P_1}{\partial \vartheta_1}$.

Term 4 is

$$\text{Term 4} = -a\vartheta_1^{\epsilon_1} P_1^{\epsilon_1 - 1} \big[(1 + \Pi^S)\frac{\partial s_1^S}{\partial \vartheta_1} - (1 + \Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1}\big],$$

where we use the fact that $s_1^S = \underbrace{\frac{\beta}{1-(1-a)\vartheta_1^{\epsilon_1}}}_{s_1^I} + \underbrace{\frac{\beta_2 (P_1^S)^{1-\epsilon_2}(P_2^S)^{\epsilon_2-1}\vartheta_2^{\epsilon_2}(1-a)}{1-(1-a)\vartheta_1^{\epsilon_1}}}_{s_1^{S_2}>0}$

to obtain

$$-a\vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1}\big[(1+\Pi^S)\big(\frac{\partial s_1^I}{\partial \vartheta_1} + \frac{\partial s_1^{S_2}}{\partial \vartheta_1}\big) - (1+\Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1}\big]$$
$$-a\vartheta_1^{\epsilon_1} P_1^{\epsilon_1-1}\big[(\Pi^S - \Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1} + (1+\Pi^S)\frac{\partial s_1^{S_2}}{\partial \vartheta_1}\big],$$

where $(\Pi^S - \Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1} = (\Pi^S - \Pi^I)\frac{\epsilon_1(1-a)\beta_1\vartheta^{\epsilon_1-1}}{(1-(1-a)\vartheta^{\epsilon_1})^2} \approx 0$. Regarding the term $\frac{\partial s_1^{S_2}}{\partial \vartheta_1}$, we have

$$s_1^{S_2} = \frac{\beta_2 (P_1^S)^{1-\epsilon_2}(P_2^S)^{\epsilon_2-1}\vartheta_2^{\epsilon_2}(1-a)}{1-(1-a)\vartheta_1^{\epsilon_1}},$$

in which

$$(P_1^S)^{1-\epsilon_{Q_1}} = \frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)} = \phi_1,$$
$$(P_2^S)^{1-\epsilon_{Q_2}} = a\vartheta_2^{\epsilon_{Q_2}-1} + (1-a)\vartheta_2^{\epsilon_{Q_2}-1}\Big(\frac{a}{\vartheta_1^{1-\epsilon_{Q_1}} - (1-a)}\Big)^{\frac{1-\epsilon_{Q_2}}{1-\epsilon_{Q_1}}} = \vartheta_2^{\epsilon_{Q_2}-1}\big(a + (1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big),$$

implying

$$s_1^{S_2} = \frac{\beta_2 \phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\vartheta_2(1-a)}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a + (1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)},$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta_1} =&\epsilon_1 \frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta_1^{\epsilon_1-1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)^2\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\\
&+(\epsilon_2-1)\frac{\vartheta_2(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta_1^{-\epsilon_1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}-1}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\\
&-(\epsilon_2-1)\frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}-1}\beta_2\vartheta_1^{-\epsilon_1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)^2},.\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta_1} =&\epsilon_1 \frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta_1^{\epsilon_1-1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)^2\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\\
&+(\epsilon_2-1)\Big[\frac{\vartheta_2(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta_1^{-\epsilon_1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}-1}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}-\frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}-1}\beta_2\vartheta_1^{-\epsilon_1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)^2}\Big],\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta_1} =&\epsilon_1 \frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta_1^{\epsilon_1-1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)^2\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\\
&+(\epsilon_2-1)\Big[\frac{\vartheta_2(1-a)\beta_2\vartheta_1^{-\epsilon_1}a^{\frac{1-\epsilon_2}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}-1}\Big]\Big[1-\frac{(1-a)a^{-1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\Big],\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta_1} =&\epsilon_1 \frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta_1^{\epsilon_1-1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)^2\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\\
&+(\epsilon_2-1)\Big[\frac{\vartheta_2(1-a)\beta_2\vartheta_1^{-\epsilon_1}a^{\frac{1-\epsilon_2}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}-1}\Big]\Big[1-\frac{(1-a)a^{-1}(a/\phi_1)\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\Big],\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta_1} =&\epsilon_1 \frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}} \beta_2 \vartheta_1^{\epsilon_1-1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)^2\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\\ &+(\epsilon_2-1)\Big[\frac{\vartheta_2(1-a)\beta_2\vartheta_1^{-\epsilon_1}a^{\frac{1-\epsilon_2}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}-1}\Big]\Big[1-\frac{(1-a)\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}-1}}{\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\Big],\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta_1} =&\epsilon_1 \frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}} \beta_2 \vartheta_1^{\epsilon_1-1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)^2\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\\ &+(\epsilon_2-1)\Big[\frac{\vartheta_2(1-a)\beta_2\vartheta_1^{-\epsilon_1}a^{\frac{1-\epsilon_2}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}-1}\Big]\Big[1-\frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}}{\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\Big],\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta_1} =&\epsilon_1 \underbrace{\frac{\vartheta_2(1-a)^2 a^{\frac{1-\epsilon_2}{1-\epsilon_1}} \beta_2 \vartheta_1^{\epsilon_1-1}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)^2\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}}_{>0}\\ &+(\epsilon_2-1)\underbrace{\Big[\frac{\vartheta_2(1-a)\beta_2\vartheta_1^{-\epsilon_1}a^{\frac{1-\epsilon_2}{1-\epsilon_1}}}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^{\frac{\epsilon_1+\epsilon_2-2}{1-\epsilon_1}}\Big]\Big[\frac{a}{\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)}\Big]}_{>0},\end{aligned}$$

Recall that

$$\text{Term 4} \approx -a\vartheta_1^{\epsilon_1}P_1^{\epsilon_1-1}(1+\Pi^S)\frac{\partial s_1^S}{\partial \vartheta_1},$$

implying

$$\text{Term 4} \approx -\epsilon_1\psi_1^{t4}(\epsilon_1,\epsilon_2)-(\epsilon_2-1)\psi_2^{t4}(\epsilon_1,\epsilon_2),$$

where $\psi_1^{t4}(\epsilon_1,\epsilon_2), \psi_2^{t4}(\epsilon_1,\epsilon_2), \psi_3^{t4}(\epsilon_1,\epsilon_2)$ are positive and non-linear functions of ϵ_1 and ϵ_2

We can see that, given $\epsilon_2 > 1$, a larger ϵ_1 makes Term 4 more negative. The distorted sector shrinks more, which is bad for rents but good to relocate activity to the less distorted sector. Given ϵ_1, a larger ϵ_2 also helps mitigating the effect of the distortion as sector 1. This is because sector 2 will demand less intermediates (it shrinks more), making the distorted sector smaller.

We can then rewrite the IOM as

$$IOM \approx \psi_1^{t1}(\epsilon_1,\epsilon_2)+(\epsilon_2-1)\psi_2^{t1}(\epsilon_1,\epsilon_2)-\epsilon_1\psi_1^{t2}(\epsilon_1,\epsilon_2)+(\epsilon_1-1)\psi_1^{t3}(\epsilon_1,\epsilon_2)-\epsilon_1\psi_1^{t4}(\epsilon_1,\epsilon_2)-(\epsilon_2-1)\psi_2^{t4}(\epsilon_1,\epsilon_2),$$

$$IOM \approx \psi_1^{t1}(\epsilon_1,\epsilon_2)+(\epsilon_2-1)\big(\psi_2^{t1}(\epsilon_1,\epsilon_2)-\psi_2^{t4}(\epsilon_1,\epsilon_2)\big)-\epsilon_1\big(\psi_1^{t2}(\epsilon_1,\epsilon_2)+\psi_1^{t4}(\epsilon_1,\epsilon_2)\big)+(\epsilon_1-1)\psi_1^{t3}(\epsilon_1,\epsilon_2),$$

$$IOM \approx \psi_1^{t1}(\epsilon_1,\epsilon_2)+\psi_2^{t4}(\epsilon_1,\epsilon_2)-\psi_2^{t1}(\epsilon_1,\epsilon_2)-\psi_1^{t3}(\epsilon_1,\epsilon_2)+\epsilon_1(\psi_1^{t3}(\epsilon_1,\epsilon_2)-\psi_1^{t2}(\epsilon_1,\epsilon_2)-\psi_1^{t4}(\epsilon_1,\epsilon_2))+\epsilon_2(\psi_2^{t1}(\epsilon_1,\epsilon_2)+\psi_1^{t6}(\epsilon_1,\epsilon_2)-\psi_2^{t4}(\epsilon_1,\epsilon_2)-\psi_1^{t5}(\epsilon_1,\epsilon_2))$$

$$IOM \approx \tilde{\psi}_1(\epsilon_1,\epsilon_2)-\tilde{\psi}_2(\epsilon_1,\epsilon_2)+\epsilon_1(\tilde{\psi}_3(\epsilon_1,\epsilon_2)-\tilde{\psi}_4(\epsilon_1,\epsilon_2))+\epsilon_2\big(\tilde{\psi}_5(\epsilon_1,\epsilon_2)-\tilde{\psi}_6(\epsilon_1,\epsilon_2)\big)$$

where ψ_j^{ti} and $\tilde{\psi}_j$ are positive and non-linear functions of ϵ_1 and ϵ_2.

In a nutshell, depending on the exact heterogeneity in elasticities, and the severity of the distortion, the IOM can be positive (Star supplier amplifies distortions) or negative (Star supplier mitigates distortions).

A5. Sectoral shock: homogeneous elasticity

Term 1

$$\text{Term 1} = \frac{(1-a)a\vartheta_1^{-\epsilon_1}\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}}{\Big(a+(1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)\big(\vartheta_1^{1-\epsilon_1}-(1-a)\big)^2}(1-\beta)\Big(\frac{\Big(a+(1-a)\phi_1^{\frac{1-\epsilon_{Q_2}}{1-\epsilon_{Q_1}}}\Big)+a\vartheta_2(1+\Pi^S)(\epsilon_2-1)}{\Big(a+(1-a)\phi_1^{\frac{1-\epsilon_{Q_2}}{1-\epsilon_{Q_1}}}\Big)}\Big),$$

becomes

$$\text{Term } 1_{\epsilon_1=\epsilon_2} = \frac{(1-a)a(1-\beta)\vartheta_1^{-\epsilon}}{\big(\vartheta_1^{1-\epsilon}-(1-a)\big)^2}\frac{\Big(a+(1-a)\phi_1+a\vartheta_2(1+\Pi^S)(\epsilon-1)\Big)}{\Big(a+(1-a)\phi_1\Big)^2},$$

$$\text{Term } 1_{\epsilon_1=\epsilon_2} = \frac{(1-a)a(1-\beta)\vartheta_1^{-\epsilon}\big(a+(1-a)\phi_1\big)}{\big(\vartheta_1^{1-\epsilon}-(1-a)\big)^2\Big(a+(1-a)\phi_1\Big)^2}+\frac{a\vartheta_2(1+\Pi^S)(\epsilon-1)}{\big(\vartheta_1^{1-\epsilon}-(1-a)\big)^2\Big(a+(1-a)\phi_1\Big)^2},$$

$$\text{Term } 1_{\epsilon_1=\epsilon_2} = \frac{(1-a)a(1-\beta)\vartheta_1^{-\epsilon}}{\left(\vartheta_1^{1-\epsilon}-(1-a)\right)^2\Big(a+(1-a)\phi_1\Big)} + \frac{a\vartheta_2(1+\Pi^S)(\epsilon-1)}{\left(\vartheta_1^{1-\epsilon}-(1-a)\right)^2\Big(a+(1-a)\phi_1\Big)^2},$$

$$\text{Term } 1_{\epsilon_1=\epsilon_2} = \psi_1^{t1}(\epsilon) + (\epsilon-1)\psi_2^{t2}(\epsilon),$$

in which $\psi_1^{t1} > 0, \psi_2^{t2} > 0$ and depending on ϵ in a non-linear way.

This term is positive and increasing in ϵ (whenever $\epsilon > 1$ and $\vartheta_2 << 1$), implying that larger flexibility generates a larger decline in rents (downstream rents), from shrinking production more given the shock to the supplier. From this term, a higher elasticity increases IOM and the star supplier amplifies shocks compared to the Island.

$$\text{Term } 2 = -a\epsilon\vartheta_1^{\epsilon-1}P_1^{\epsilon-1}\left[(1+\Pi^S)s_1^S - (1+\Pi^I)s_1^I\right]$$

$$\text{Term } 2 = -\epsilon\psi_1^{t2}(\epsilon),$$

in which $\psi_1^{t2} > 0$ and depending on ϵ in a non-linear way.

Term 2 is negative as s_1^S is larger than s_1^I, $s_1^S = \underbrace{\frac{\beta}{1-(1-a)\vartheta_1^{\epsilon}}}_{s_1^I} + \underbrace{\frac{\beta_2(P_1^S)^{1-\epsilon}(P_2^S)^{\epsilon-1}\vartheta_2^{\epsilon}(1-a)}{1-(1-a)\vartheta_1^{\epsilon}}}_{s_1^{S_2}>0}),$ while $\Pi^S \approx \Pi^I$. Through this, when sector 1 is slightly constrained ($\vartheta_1 \approx 1$), a shock to sector 1 is mitigated in the star supplier economy, more so the higher ϵ. Intuitively, if $\epsilon = 0$ this term is irrelevant because the distorted sector in both networks is optimally not changing its production plan (M, L). When $\epsilon > 0$ in the Star supplier economy, a larger fraction of the economy is better able to couple with the shock. However, when sector 1 is heavily distorted, this term shrinks when ϵ is large, and larger than 1. Thus, when the distortion is severe, the composition effect dominates the relocation effect, and the Star supplier economy displays a larger reduction in real GDP, all else equal.

Term 3 is

$$\text{Term } 3 = -a(\epsilon-1)\vartheta_1^{\epsilon}P_1^{\epsilon-2}\frac{\partial P_1}{\partial \vartheta_1}\big[(1+\Pi^S)s_1^S-(1+\Pi^I)s_1^I\big]$$

is positive when $\epsilon > 1$ (as $s_1^S > s_1^I$ and $\frac{\partial P_1}{\partial \vartheta_1} < 0$), but negative when ϵ_1. Here a higher elasticity amplifies further (if $1 < \epsilon < \bar{\epsilon}$ and distortion is not too tight). This effect is not the direct effect on P_1, as that is the same for both networks, but it is the effect on sector 1's rents. When the sectors are very flexible (so the distorted sector is very flexible), it optimally shrinks more, reducing households rents (a function of revenue). However, $\frac{\partial P_1}{\partial \vartheta_1}$ is less negative the larger ϵ. When the distortion is initially very tight, or the elasticity very large, a further increase in the elasticity reduces the value Term 3. A larger ϵ also reduces the value of $\frac{\partial P_1}{\partial \vartheta_1}$.

Term 3 can be rewritten as

$$\text{Term } 3 = (\epsilon-1)\psi_1^{t3}(\epsilon)$$

in which $\psi_1^{t3} > 0$ and depending on ϵ in a non-linear way.

Term 4 is

$$\text{Term } 4 = -a\vartheta_1^{\epsilon}P_1^{\epsilon-1}\big[(1+\Pi^S)\frac{\partial s_1^S}{\partial \vartheta_1}-(1+\Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1}\big],$$

where we use the fact that $s_1^S = \underbrace{\frac{\beta}{1-(1-a)\vartheta_1^{\epsilon}}}_{s_1^I} + \underbrace{\frac{\beta_2(P_1^S)^{1-\epsilon}(P_2^S)^{\epsilon-1}\vartheta_2^{\epsilon}(1-a)}{1-(1-a)\vartheta_1^{\epsilon}}}_{s_1^{S_2}>0}$ to obtain

$$-a\vartheta_1^{\epsilon_1}P_1^{\epsilon_1-1}\big[(1+\Pi^S)\big(\frac{\partial s_1^I}{\partial \vartheta_1}+\frac{\partial s_1^{S_2}}{\partial \vartheta_1}\big)-(1+\Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1}\big]$$
$$-a\vartheta_1^{\epsilon_1}P_1^{\epsilon_1-1}\big[(\Pi^S-\Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1}+(1+\Pi^S)\frac{\partial s_1^{S_2}}{\partial \vartheta_1}\big],$$

where $(\Pi^S - \Pi^I)\frac{\partial s_1^I}{\partial \vartheta_1} = (\Pi^S - \Pi^I)\frac{\epsilon_1(1-a)\beta_1\vartheta^{\epsilon-1}}{(1-(1-a)\vartheta^\epsilon)^2} \approx 0$. Regarding the term $\frac{\partial s_1^{S_2}}{\partial \vartheta_1}$, we have

$$s_1^{S_2} = \frac{\beta_2 (P_1^S)^{1-\epsilon}(P_2^S)^{\epsilon-1}\vartheta_2^\epsilon(1-a)}{1-(1-a)\vartheta_1^\epsilon},$$

in which

$$\begin{aligned}(P_1^S)^{1-\epsilon} =& \frac{a}{\vartheta_1^{1-\epsilon}-(1-a)} = \phi_1,\\ (P_2^S)^{1-\epsilon} =& a\vartheta_2^{\epsilon-1} + (1-a)\vartheta_2^{\epsilon-1}\Big(\frac{a}{\vartheta_1^{1-\epsilon}-(1-a)}\Big) = \vartheta_2^{\epsilon-1}\big(a+(1-a)\phi_1\big),\end{aligned}$$

implying

$$s_1^{S_2} = \frac{\beta_2\phi_1\vartheta_2(1-a)}{\big(1-(1-a)\vartheta_1^{\epsilon_1}\big)\big(a+(1-a)\phi_1\big)},$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta_1} =& (1-\epsilon)\frac{\vartheta_2(1-a)^2a^2\beta_2\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^\epsilon\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)^3\big(a+(1-a)\phi_1\big)^2}\\ &+\epsilon\Big[\frac{\vartheta_2(1-a)^2a\beta_2\vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^\epsilon\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big] - (1-\epsilon)\Big[\frac{\vartheta_2\beta_2(1-a)a\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^\epsilon\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big],\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta_1} =& \epsilon\Big[\frac{\vartheta_2(1-a)^2a\beta_2\vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^\epsilon\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big]\\ &+(1-\epsilon)\Big[\frac{\vartheta_2(1-a)^2a^2\beta_2\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^\epsilon\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)^3\big(a+(1-a)\phi_1\big)^2} - \frac{\vartheta_2\beta_2(1-a)a\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^\epsilon\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big],\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta_1} =& \epsilon\Big[\frac{\vartheta_2(1-a)^2a\beta_2\vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^\epsilon\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big]\\ &+(1-\epsilon)\frac{\vartheta_2\beta_2(1-a)a\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^\epsilon\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big[\frac{(1-a)a}{\big(1-(1-a)\vartheta_1^\epsilon\big)^{-1}\big(\vartheta_1^{1-\epsilon}-(1-a)\big)^2\big(a+(1-a)\phi_1\big)} - 1\Big],\end{aligned}$$

$$\frac{\partial s_1^S}{\partial \vartheta_1} = \epsilon\Big[\frac{\vartheta_2(1-a)^2 a\beta_2\vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big] + (1-\epsilon)\frac{\vartheta_2\beta_2(1-a)a\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big[\frac{(1-a)a\big(1-(1-a)\vartheta_1^{\epsilon}\big)}{\big(\vartheta_1^{1-\epsilon}-(1-a)\big)^2\big(a+(1-a)\frac{a}{\vartheta_1^{1-\epsilon}-(1-a)}\big)}-1\Big],$$

$$\frac{\partial s_1^S}{\partial \vartheta_1} = \epsilon\Big[\frac{\vartheta_2(1-a)^2 a\beta_2\vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big] + (1-\epsilon)\frac{\vartheta_2\beta_2(1-a)a\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big[\frac{(1-a)a\big(1-(1-a)\vartheta_1^{\epsilon}\big)}{\big(\vartheta_1^{1-\epsilon}-(1-a)\big)^2\frac{a\vartheta_1^{1-\epsilon}}{\vartheta_1^{1-\epsilon}-(1-a)}\big)}-1\Big],$$

$$\frac{\partial s_1^S}{\partial \vartheta_1} = \epsilon\Big[\frac{\vartheta_2(1-a)^2 a\beta_2\vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big] + (1-\epsilon)\frac{\vartheta_2\beta_2(1-a)a\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big[\frac{(1-a)a\big(1-(1-a)\vartheta_1^{\epsilon}\big)}{\big(\vartheta_1^{1-\epsilon}-(1-a)\big)a\vartheta_1^{1-\epsilon}\big)}-1\Big],$$

$$\frac{\partial s_1^S}{\partial \vartheta_1} = \epsilon\Big[\frac{\vartheta_2(1-a)^2 a\beta_2\vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big] + (1-\epsilon)\frac{\vartheta_2\beta_2(1-a)a\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big[\frac{(1-a)\big(1-(1-a)\vartheta_1^{\epsilon}\big)}{\big(\vartheta_1^{1-\epsilon}-(1-a)\big)\vartheta_1^{1-\epsilon}\big)}-1\Big],$$

$$\frac{\partial s_1^S}{\partial \vartheta_1} = \epsilon\underbrace{\Big[\frac{\vartheta_2(1-a)^2 a\beta_2\vartheta_1^{\epsilon-1}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}\Big]}_{>0} + (1-\epsilon)\underbrace{\frac{\vartheta_2\beta_2(1-a)a\vartheta_1^{-\epsilon}}{\big(1-(1-a)\vartheta_1^{\epsilon}\big)^2\big(a+(1-a)\phi_1\big)\big(\vartheta_1^{1-\epsilon}-(1-a)\big)}}_{>0}\underbrace{\Big[\frac{(1-a)\big(1-(1-a)\vartheta_1^{\epsilon}\big)-\big(\vartheta_1^{1-\epsilon}-(1-a)\big)\vartheta_1^{1-\epsilon}}{\big(\vartheta_1^{1-\epsilon}-(1-a)\big)\vartheta_1^{1-\epsilon}}\Big]}_{><0},$$

To figure out whether $\frac{\partial s_1^{S_2}}{\partial \vartheta_1} >< 0$ we look at the last term $\Big[\frac{(1-a)\big(1-(1-a)\vartheta_1^{\epsilon}\big)-\big(\vartheta_1^{1-\epsilon}-(1-a)\big)\vartheta_1^{1-\epsilon}}{\big(\vartheta_1^{1-\epsilon}-(1-a)\big)\vartheta_1^{1-\epsilon}}\Big]$, which can be expressed as

$$\Big[\frac{(1-a)\big(1-(1-a)\big)-\big(1-(1-a)\big)}{\big(1-(1-a)\big)}\Big],$$

$$\Big[\frac{(1-a)a-a}{\big(1-(1-a)\big)}\Big]=\Big[\frac{a(1-a-1)}{\big(1-(1-a)\big)}\Big]=\Big[\frac{-a^2}{a}\Big]=-a<0,$$

when ($\vartheta_1 \approx 1$). In this case, if $\epsilon > 1$, Term 4 is negative, implying a smaller, potentially negative, IOM. This effect is smaller the larger the elasticity (as ϑ^ϵ decreases with ϵ). Now, term ψ_3 could be positive if sector $\vartheta_1 << 1$ is very distorted. In that case, a large elasticity could imply that Term 4 is positive, in which the Star supplier network amplifies shocks. However, this effect would be mitigates by the fact that $\vartheta_1^\epsilon \approx 0$ in this case. In both situations, a larger elasticity would imply a smaller miitgation effect of the Star supplier, compared to the Island economy, or a mild amplification effect in the Star supplier.

Recall that

$$\text{Term 4} \approx -\underbrace{a\vartheta_1^\epsilon P_1^{\epsilon-1}(1+\Pi^S)}_{>0}\frac{\partial s_1^{S_2}}{\partial \vartheta_1}.$$

$$\text{Term 4} \approx -\epsilon\psi_1^{t4}-(1-\epsilon)\psi_2^{t4},$$

in which ψ_1^{t4} and a non-linear function of ϵ, while ψ_2^{t4} can be positive or negative and it is a non-linear function of ϵ.

Putting Term 1, Term 2, Term 3, and Term 4 together yields:

$$IOM \approx \psi_1^{t1}(\epsilon)+(\epsilon-1)\psi_2^{t2}(\epsilon)-\epsilon\psi_1^{t2}(\epsilon)+(\epsilon-1)\psi_1^{t3}(\epsilon)-\epsilon\psi_1^{t4}(\epsilon)-(1-\epsilon)\psi_2^{t4}(\epsilon),$$

$$IOM \approx \psi_1^{t1}(\epsilon)+(\epsilon-1)\big(\psi_2^{t2}(\epsilon)+\psi_1^{t3}(\epsilon)\big)-\epsilon\big(\psi_1^{t2}(\epsilon)+\psi_1^{t4}(\epsilon)\big)-(1-\epsilon)\psi_2^{t4}(\epsilon),$$

$$IOM \approx \psi_1^{t1}(\epsilon)-\big(\psi_2^{t3}(\epsilon)+\psi_1^{t3}(\epsilon)\big)-\psi_2^{t4}(\epsilon)+\epsilon\big(\psi_2^{t2}(\epsilon)+\psi_1^{t3}(\epsilon)+\psi_2^{t4}(\epsilon)-\psi_1^{t2}(\epsilon)-\psi_1^{t4}(\epsilon)\big)$$

$$IOM \approx \hat{\psi}_1(\epsilon)-\hat{\psi}_2(\epsilon)-\hat{\psi}_3(\epsilon)+\epsilon\big(\hat{\psi}_4(\epsilon)-\hat{\psi}_5(\epsilon)-\hat{\psi}_2(\epsilon)\big)$$

where $\psi_1^{t1},\psi_2^{t1},\psi_1^{t2},\psi_1^{t3},\psi_1^{t4}$ are positive and non-linear functions of ϵ. On the other hand, ψ_2^{t4} can be positive or negative depending on ϑ_1 and ϵ and it is a non-linear function of ϵ. Also, $\hat{\psi}_1, \hat{\psi}_2, \hat{\psi}_4, \hat{\psi}_5$ are positive and non-linear functions of ϵ. On the other hand, $\hat{\psi}_3$ is a non-linear function of ϵ and can take positive or negative values.

A6. *Aggregate shock: homogeneous elasticity*

PROOF PROPOSITION 5: AGGREGATE SHOCK:

In the homogeneous elasticity case we have

$$
\begin{aligned}
P_1^{1-\epsilon} =& \frac{a}{\vartheta^{1-\epsilon} - (1-a)}, \\
P_2^{1-\epsilon} =& \frac{a}{\vartheta^{1-\epsilon} - (1-a)}, \\
s_1 =& \frac{\beta_1}{1-(1-a)\vartheta^{\epsilon}} \\
s_2 =& \frac{\beta_2}{1-(1-a)\vartheta^{\epsilon}} \\
(1+\Pi) =& \frac{1}{\sum_{j=1}^N a_j (\vartheta_j^w)^{\epsilon_{Q_j}} Z_j^{\epsilon_{Q_j}-1} P_j^{\epsilon_{Q_j}-1} s_j} = \frac{1-(1-a)\vartheta^{\epsilon}}{\vartheta - (1-a)\vartheta^{\epsilon}} \geq 1.
\end{aligned}
$$

For the star supplier we have

$$
\begin{aligned}
P_1^{1-\epsilon} =& \frac{a}{\vartheta^{1-\epsilon} - (1-a)}, \\
P_2^{1-\epsilon} =& \frac{a}{\vartheta^{1-\epsilon} - (1-a)} \\
s_1 =& \frac{\beta_1 + \beta_2(1-a)\vartheta^{\epsilon}}{1-(1-a)\vartheta^{\epsilon}} \\
s_2 =& \beta_2, \\
(1+\Pi) =& \frac{1}{\sum_{j=1}^N a_j (\vartheta_j^w)^{\epsilon_{Q_j}} Z_j^{\epsilon_{Q_j}-1} P_j^{\epsilon_{Q_j}-1} s_j} = \frac{1-(1-a)\vartheta^{\epsilon}}{\vartheta - (1-a)\vartheta^{\epsilon}} \geq 1.
\end{aligned}
$$

Note here that $\frac{\partial(1+\Pi)}{\partial\vartheta} < 0$—a tighter distortion, lower ϑ, increases rents from distortions (think of increased mark-ups or rents from financial intermediary). This effect is stronger the smaller the elasticity, as in that case firms adjust production down less but prices increase more ($\uparrow PQ$).

To obtain the IOM we compute

$$\frac{\partial \log C^i}{\partial\vartheta} = \underbrace{-\beta 1 \frac{\partial \log P_1}{\partial\vartheta} - (1-\beta 1)\frac{\partial \log P_2}{\partial\vartheta}}_{\text{Real wage channel}} \underbrace{-(1+\Pi)a\Big[\epsilon s_1 \vartheta^{\epsilon-1} P_1^{\epsilon-1} + s_1\vartheta^{\epsilon}\frac{\partial P_1^{\epsilon-1}}{\partial\vartheta} + \vartheta^{\epsilon} P_1^{\epsilon-1}\frac{\partial s_1}{\partial\vartheta} + s_2\vartheta^{\epsilon}\frac{\partial P_2^{\epsilon-1}}{\partial\vartheta} + s_2\epsilon\vartheta^{\epsilon-1}P_2^{\epsilon-1} + \vartheta^{\epsilon}P_2^{\epsilon-1}\frac{\partial s_2}{\partial\vartheta}\Big]}_{\text{Rents channel}}$$

$$\frac{\partial \log C^s}{\partial\vartheta} = \underbrace{-\beta 1 \frac{\partial \log P_1}{\partial\vartheta} - (1-\beta 1)\frac{\partial \log P_2}{\partial\vartheta}}_{\text{Real wage channel}} \underbrace{-(1+\Pi)a\Big[\epsilon s_1 \vartheta^{\epsilon-1} P_1^{\epsilon-1} + s_1\vartheta^{\epsilon}\frac{\partial P_1^{\epsilon-1}}{\partial\vartheta} + \vartheta^{\epsilon} P_1^{\epsilon-1}\frac{\partial s_1}{\partial\vartheta} + s_2\vartheta^{\epsilon}\frac{\partial P_2^{\epsilon-1}}{\partial\vartheta} + s_2\epsilon\vartheta^{\epsilon-1}P_2^{\epsilon-1}\Big]}_{\text{Rents channel}}$$

Implying

$$IOM = (1+\Pi)a\Big[\epsilon\vartheta^{\epsilon-1}P_1^{\epsilon-1}\Delta s_1 + \vartheta^{\epsilon}\frac{\partial P_1^{\epsilon-1}}{\partial\vartheta}\Delta s_1 + \vartheta^{\epsilon}P_1^{\epsilon-1}(\frac{\partial s_1^I}{\partial\vartheta} - \frac{\partial s_1^S}{\partial\vartheta}) + \vartheta^{\epsilon}\frac{\partial P_2^{\epsilon-1}}{\partial\vartheta}\Delta s_2 + \epsilon\vartheta^{\epsilon-1}P_2^{\epsilon-1}\Delta s_2 + \vartheta^{\epsilon}P_2^{\epsilon-1}\frac{\partial s_2^I}{\partial\vartheta},\Big]$$

where $\Delta s_j = s_j^I - s_j^S$. We now use the fact that $P_1 = P_2$ in both networks to obtain

$$IOM = (1+\Pi)a\Big[\epsilon\vartheta^{\epsilon-1}P^{\epsilon-1}(\Delta s_1 + \Delta s_2) + \vartheta^{\epsilon}\frac{\partial P^{\epsilon-1}}{\partial\vartheta}(\Delta s_1 + \Delta s_2) + \vartheta^{\epsilon}P_1^{\epsilon-1}(\frac{\partial s_1^I}{\partial\vartheta} - \frac{\partial s_1^S}{\partial\vartheta}) + \vartheta^{\epsilon}P_2^{\epsilon-1}\frac{\partial s_2^I}{\partial\vartheta}\Big],$$

using the solution for sectoral sales, we can easily show that $\Delta s_1 = -\Delta s_2$, which implies that

$$IOM = (1+\Pi)a\Big[\vartheta^{\epsilon}P^{\epsilon-1}(\frac{\partial s_1^I}{\partial\vartheta} - \frac{\partial s_1^S}{\partial\vartheta} + \frac{\partial s_2^I}{\partial\vartheta})\Big],$$

in which $(\frac{\partial s_1^i}{\partial\vartheta} - \frac{\partial s_1^s}{\partial\vartheta} + \frac{\partial s_2^i}{\partial\vartheta}) = 0$, implying

$$IOM = (1+\Pi)a\Big[\vartheta^{\epsilon}P^{\epsilon-1}(\frac{\partial s_1^i}{\partial\vartheta} - \frac{\partial s_1^s}{\partial\vartheta} + \frac{\partial s_2^i}{\partial\vartheta})\Big] = 0.$$

Thus, we have shown that when $\epsilon_1 = \epsilon_2$ the Star supplier economy is isomorphic to the Island economy.

A7. *Aggregate shock: heterogeneous elasticity*

We now study the heterogeneous elasticities case. We have in the Island economy

$$
\begin{aligned}
P_1^{1-\epsilon_1} =& \frac{a}{\vartheta^{1-\epsilon_1} - (1-a)}, \\
P_2^{1-\epsilon_2} =& \frac{a}{\vartheta^{1-\epsilon_2} - (1-a)}, \\
s_1 =& \frac{\beta_1}{1-(1-a)\vartheta^{\epsilon_1}} \\
s_2 =& \frac{\beta_2}{1-(1-a)\vartheta^{\epsilon_2}},
\end{aligned}
$$

and in the Star supplier economy

$$
\begin{aligned}
P_1^{1-\epsilon_{Q_1}} =& \frac{a}{\vartheta^{1-\epsilon_{Q_1}} - (1-a)}, \\
P_2 =& \frac{1}{\vartheta}\left(a + (1-a)\left(\frac{a}{\vartheta^{1-\epsilon_{Q_1}} - (1-a)}\right)^{\frac{1-\epsilon_{Q_2}}{1-\epsilon_{Q_1}}}\right)^{\frac{1}{1-\epsilon_{Q_2}}} \\
s_1 =& \frac{\beta_1}{1-(1-a)\vartheta^{\epsilon_1}} + \frac{\beta_2 \phi^{\frac{1-\epsilon_2}{1-\epsilon_1}} \vartheta(1-a)}{\left(1-(1-a)\vartheta^{\epsilon_1}\right)\left(a + (1-a)\phi^{\frac{1-\epsilon_2}{1-\epsilon_1}}\right)}, \\
s_2 =& \beta_2
\end{aligned}
$$

To obtain the IOM we compute

$$
\begin{aligned}
\frac{\partial \log C^I}{\partial \vartheta} =& \underbrace{-\beta 1 \frac{\partial \log P_1}{\partial \vartheta} - \beta_2 \frac{\partial \log P_2^I}{\partial \vartheta}}_{\text{Real wage channel}} \\
& \underbrace{-(1+\Pi^I)a\Big[\epsilon_1 s_1^I \vartheta^{\epsilon 1 - 1} P_1^{\epsilon 1 - 1} + s_1^I \vartheta^{\epsilon 1} \frac{\partial P_1^{\epsilon 1 - 1}}{\partial \vartheta} + \vartheta^{\epsilon 1} P_1^{\epsilon 1 - 1} \frac{\partial s_1^I}{\partial \vartheta} + s_2^I \vartheta^{\epsilon 2} \frac{\partial (P_2^I)^{\epsilon 2 - 1}}{\partial \vartheta} + s_2^I \epsilon_2 \vartheta^{\epsilon 2 - 1} (P_2^I)^{\epsilon 2 - 1} + \vartheta^{\epsilon 2} (P_2^I)^{\epsilon 2 - 1} \frac{\partial s_2^I}{\partial \vartheta}\Big]}_{\text{Rents channel}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \log C^s}{\partial \vartheta} = & \underbrace{-\beta 1 \frac{\partial \log P_1}{\partial \vartheta} - \beta_2 \frac{\partial \log P_2^S}{\partial \vartheta}}_{\text{Real wage channel}} \\
& \underbrace{-(1+\Pi^S)a\Big[\epsilon_1 s_1^S \vartheta^{\epsilon 1-1} P_1^{\epsilon 1-1} + s_1^S \vartheta^{\epsilon 1} \frac{\partial P_1^{\epsilon 1-1}}{\partial \vartheta} + \vartheta^{\epsilon 1} P_1^{\epsilon 1-1} \frac{\partial s_1^S}{\partial \vartheta} + s_2^S \vartheta^{\epsilon 2} \frac{\partial (P_2^S)^{\epsilon 2-1}}{\partial \vartheta} + s_2^S \epsilon_2 \vartheta^{\epsilon 2-1} (P_2^S)^{\epsilon 2-1}\Big]}_{\text{Rents channel}}
\end{aligned}
$$

Using the fact that $\frac{\partial \log P_2}{\vartheta} = \frac{1}{P_2}\frac{\partial P_2}{\vartheta}$, that $\frac{\partial P_2^{\epsilon_2-1}}{\vartheta} = (\epsilon_2-1)P_2^{\epsilon_2-2}\frac{\partial P_2}{\vartheta}$, and that $s_2^S = 1-\beta$, we reorganize the IOM as follows

$$
\begin{aligned}
IOM = & \underbrace{-\frac{\partial \log P_2^S}{\partial \vartheta}(1-\beta)\Big(1 + a\vartheta^{\epsilon_2}(P_2^S)^{\epsilon_2-1}(1+\Pi^S)(\epsilon_2-1)\Big)}_{\text{Term 1}} \\
& \underbrace{-a\epsilon_1\vartheta^{\epsilon_1-1}P_1^{\epsilon_1-1}\big[(1+\Pi^S)s_1^S - (1+\Pi^I)s_1^I\big]}_{\text{Term 2}} \\
& \underbrace{-a\vartheta^{\epsilon_1}(\epsilon_1-1)P_1^{\epsilon_1-2}\frac{\partial P_1}{\partial \vartheta}\big[(1+\Pi^S)s_1^S - (1+\Pi^I)s_1^I\big]}_{\text{Term 3}} \\
& \underbrace{-a\vartheta^{\epsilon_1}P_1^{\epsilon_1-1}\big[(1+\Pi^S)\frac{\partial s_1^S}{\partial \vartheta} - (1+\Pi^I)\frac{\partial s_1^I}{\partial \vartheta}\big]}_{\text{Term 4}} \\
& \underbrace{-\epsilon_2\vartheta^{\epsilon_2-1}\big[(1+\Pi^S)s_2^S(P_2^S)^{\epsilon_2-1} - (1+\Pi^I)s_2^I(P_2^I)^{\epsilon_2-1}\big]}_{\text{Term 5}} \\
& \underbrace{+(1+\Pi^S)\vartheta^{\epsilon_2}(P_2^I)^{\epsilon_2-1}\frac{\partial s_2^I}{\partial \vartheta}}_{\text{Term 6}}
\end{aligned}
$$

Term 1

$$
-(1-\beta_1)\frac{\partial \log P_2}{\partial \vartheta} = -\frac{(1-\beta_1)}{P_2}\frac{\partial P_2}{\partial \vartheta} = -\frac{(1-\beta 1)}{P_2}\Big[-\frac{P_2}{\vartheta} - P_2^{\epsilon_2}\frac{(1-a)a\vartheta^{-\epsilon_1}\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}\vartheta^{-1}}{\big(\vartheta^{1-\epsilon_1}-(1-a)\big)^2}\Big],
$$

$$
-(1-\beta_1)\frac{\partial \log P_2}{\partial \vartheta} = \frac{(1-\beta 1)}{\vartheta} + (1-\beta_1)P_2^{\epsilon_2-1}\frac{(1-a)a\vartheta^{-\epsilon_1-1}\phi^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}}{\big(\vartheta^{1-\epsilon_1}-(1-a)\big)^2}.
$$

$$\text{Term 1} = \psi_1^{t1}(\epsilon_1, \epsilon_2) + (\epsilon_2 - 1)\psi_2^{t1}(\epsilon_1, \epsilon_2),$$

in which $\psi_1^{t1}(\epsilon_1, \epsilon_2)$ and $\psi_2^{t1}(\epsilon_1, \epsilon_2)$ are positive and non-linear functions of ϵ_1 and ϵ_2.

Term 2

$$\text{Term 2} = -a\epsilon\vartheta^{\epsilon_1 - 1}P_1^{\epsilon_1 - 1}\big[(1+\Pi^S)s_1^S - (1+\Pi^I)s_1^I\big]$$

$$\text{Term 2} = -\epsilon_1\psi_1^{t2}(\epsilon_1, \epsilon_2),$$

in which $\psi_1^{t2} > 0$ and a non-linear function of ϵ_1 and ϵ_2.

Term 3 is

$$\text{Term 3} = -a(\epsilon_1 - 1)\vartheta^{\epsilon_1}P_1^{\epsilon_1 - 2}\frac{\partial P_1}{\partial \vartheta}\big[(1+\Pi^S)s_1^S - (1+\Pi^I)s_1^I\big]$$

$$\text{Term 3} = (\epsilon_1 - 1)\psi_1^{t3}(\epsilon_1, \epsilon_2).$$

where $\psi_1^{t3}(\epsilon_1, \epsilon_2)$ is positive and non-linear function of ϵ_1 and ϵ_2.

Let us compute Term 4

$$-a\vartheta^{\epsilon_1}P_1^{\epsilon_1 - 1}\big[(1+\Pi^S)\big(\frac{\partial s_1^I}{\partial \vartheta} + \frac{\partial s_1^{S_2}}{\partial \vartheta}\big) - (1+\Pi^I)\frac{\partial s_1^I}{\partial \vartheta}\big]$$

$$-a\vartheta^{\epsilon_1}P_1^{\epsilon_1 - 1}\big[(\Pi^S - \Pi^I)\frac{\partial s_1^I}{\partial \vartheta} + (1+\Pi^S)\frac{\partial s_1^{S_2}}{\partial \vartheta}\big],$$

where $(\Pi^S - \Pi^I)\frac{\partial s_1^I}{\partial \vartheta} = (\Pi^S - \Pi^I)\frac{\epsilon_1(1-a)\beta_1\vartheta^{\epsilon_1 - 1}}{(1-(1-a)\vartheta^{\epsilon_1})^2} \approx 0$. Regarding the term $\frac{\partial s_1^{S_2}}{\partial \vartheta}$, we have

$$s_1^{S_2} = \frac{\beta_2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\vartheta(1-a)}{\big(1-(1-a)\vartheta^{\epsilon_1}\big)\big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\big)},$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta} = & \frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a+(1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})} + \epsilon_1 \frac{(1-a)^2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})} \\ & - (1-\epsilon_2)\Big[\frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon}-(1-a))^{\frac{\epsilon_1+\epsilon_2-2}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})}\Big] + (1-\epsilon_2)\Big[\frac{\vartheta^{1-\epsilon_1}(1-a)^2 a^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon}-(1-a))^{\frac{\epsilon_2-1}{1-\epsilon_1}}\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})\Big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)^2}\Big],\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta} = & \frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a+(1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})} + \epsilon_1 \frac{(1-a)^2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})} \\ & + (1-\epsilon_2)\Big\{\frac{\vartheta^{1-\epsilon_1}(1-a)^2 a^{\frac{\epsilon_1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon}-(1-a))^{\frac{\epsilon_2-1}{1-\epsilon_1}}\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})\Big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)^2} - \frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon}-(1-a))^{\frac{\epsilon_1+\epsilon_2-2}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})}\Big\},\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta_1} = & \frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a+(1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})} + \epsilon_1 \frac{(1-a)^2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})} \\ & + (1-\epsilon_2)\frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta_1^{1-\epsilon}-(1-a))^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{(1-(1-a)\vartheta_1^{\epsilon})\Big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)}\Big\{\frac{(1-a)a^{-1}\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})} - (\vartheta_1^{1-\epsilon}-(1-a))^{-1}\Big\},\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta} = & \frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a+(1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})} + \epsilon_1 \frac{(1-a)^2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})} \\ & + (1-\epsilon_2)\frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon}-(1-a))^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})\Big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)}\Big\{\frac{(1-a)a^{-1}\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}}}{(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})} - \frac{\phi_1}{a}\Big\},\end{aligned}$$

$$\begin{aligned}\frac{\partial s_1^S}{\partial \vartheta} = & \frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a+(1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})} + \epsilon_1 \frac{(1-a)^2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})} \\ & + (1-\epsilon_2)\frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon}-(1-a))^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})\Big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)}\Big\{\frac{(1-a)\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}} - \phi(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})}{(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})a}\Big\},\end{aligned}$$

$$\frac{\partial s_1^S}{\partial \vartheta} = \frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a+(1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})} + \epsilon_1 \frac{(1-a)^2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})}$$
$$+ (1-\epsilon_2)\frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon}-(1-a))^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})\Big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)}\left\{\frac{(1-a)\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}} - \phi a - (1-a)\phi_1^{\frac{2-(\epsilon_1+\epsilon_2)}{1-\epsilon_1}})}{(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})a}\right\},$$

$$\frac{\partial s_1^S}{\partial \vartheta} = \frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a+(1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})} + \epsilon_1 \frac{(1-a)^2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})}$$
$$+ (1-\epsilon_2)\frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon}-(1-a))^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})\Big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)}\left\{\frac{-\phi a}{(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})a}\right\},$$

$$\frac{\partial s_1^S}{\partial \vartheta} = \underbrace{\frac{(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2}{(1-(1-a)\vartheta^{\epsilon_1})(a+(1-a)\phi_1^{\frac{1-\epsilon_1}{1-\epsilon_2}})}}_{>0} + \epsilon_1 \underbrace{\frac{(1-a)^2\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2\vartheta^{\epsilon_1}}{(1-(1-a)\vartheta^{\epsilon_1})^2(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})}}_{>0}$$
$$(\epsilon_2-1)\underbrace{\frac{\vartheta^{1-\epsilon_1}(1-a)a^{\frac{1-\epsilon_2}{1-\epsilon_1}}\beta_2(\vartheta^{1-\epsilon}-(1-a))^{\frac{\epsilon_2-1}{1-\epsilon_1}}}{(1-(1-a)\vartheta^{\epsilon})\Big(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}}\Big)}\frac{\phi a}{(a+(1-a)\phi_1^{\frac{1-\epsilon_2}{1-\epsilon_1}})a}}_{>0}$$

Recall that

$$\text{Term 4} \approx -a\vartheta^{\epsilon_1}P_1^{\epsilon_1-1}(1+\Pi^S)\frac{\partial s_1^S}{\partial \vartheta},$$

implying

$$\text{Term 4} \approx -\epsilon_1\psi_1^{t4}(\epsilon_1,\epsilon_2) - (\epsilon_2-1)\psi_2^{t4}(\epsilon_1,\epsilon_2),$$

where $\psi_1^{t4}(\epsilon_1,\epsilon_2)$ and $\psi_2^{t4}(\epsilon_1,\epsilon_2)$ are positive and non-linear functions of ϵ_1 and ϵ_2

We have Term 5

$$\text{Term 5} = -\epsilon_2\vartheta^{\epsilon_2-1}\big[(1+\Pi^S)s_2^S(P_2^S)^{\epsilon_2-1} - (1+\Pi^I)s_2^I(P_2^I)^{\epsilon_2-1}\big]$$

$$\text{Term } 5 = -\epsilon_2 \psi_1^{t5}(\epsilon_1, \epsilon_2),$$

in which $\psi_1^{t5}(\epsilon_1, \epsilon_2)$ is a non-linear function of ϵ_1 and ϵ_2 and it could take positive or negative values.

Term 6

$$\text{Term } 6 = (1+\Pi^S)\vartheta^{\epsilon_2}(P_2^I)^{\epsilon_2 - 1}\frac{\partial s_2^I}{\partial \vartheta}$$

$$\text{Term } 6 = (1+\Pi^S)\vartheta^{\epsilon_2}(P_2^I)^{\epsilon_2 - 1}\frac{(1-a)\vartheta^{\epsilon_2 - 1}\beta_2\epsilon_2}{\left(1-(1-a)\vartheta^{\epsilon_2}\right)^2}$$

$$\text{Term } 6 = \epsilon_2 \psi_1^{t6}(\epsilon_2),$$

where $\psi_1^{t6}(\epsilon_2)$ is positive and a non-linear function of ϵ_2.

Putting Term 1, Term 2, Term 3, Term 4, Term 5, and Term 6 together yields:

$$IOM \approx \psi_1^{t1}(\epsilon_1,\epsilon_2)+(\epsilon_2-1)\psi_2^{t1}(\epsilon_1,\epsilon_2)-\epsilon_1\psi_1^{t2}(\epsilon_1,\epsilon_2)+(\epsilon_1-1)\psi_1^{t3}(\epsilon_1,\epsilon_2)-\epsilon_1\psi_1^{t4}(\epsilon_1,\epsilon_2)-(\epsilon_2-1)\psi_2^{t4}(\epsilon_1,\epsilon_2)-\epsilon_2\psi_1^{t5}(\epsilon_1,\epsilon_2)+\epsilon_2\psi_1^{t6}(\epsilon_2),$$

$$IOM \approx \psi_1^{t1}(\epsilon_1,\epsilon_2)+(\epsilon_2-1)\left(\psi_2^{t1}(\epsilon_1,\epsilon_2)-\psi_2^{t4}(\epsilon_1,\epsilon_2)\right)-\epsilon_1\left(\psi_1^{t2}(\epsilon_1,\epsilon_2)+\psi_1^{t4}(\epsilon_1,\epsilon_2)\right)+(\epsilon_1-1)\psi_1^{t3}(\epsilon_1,\epsilon_2)+\epsilon_2\left(\psi_1^{t6}(\epsilon_2)-\psi_1^{t5}(\epsilon_1,\epsilon_2)\right),$$

$$IOM \approx \bar\psi_1(\epsilon_1,\epsilon_2) - \bar\psi_2(\epsilon_1,\epsilon_2) + \epsilon_1\left(\bar\psi_3(\epsilon_1,\epsilon_2) - \bar\psi_4(\epsilon_1,\epsilon_2)\right) + \epsilon_2\left(\bar\psi_5(\epsilon_1,\epsilon_2) - \bar\psi_6(\epsilon_1,\epsilon_2) - \bar\psi_7(\epsilon_1,\epsilon_2)\right)$$

where $\psi_1^{t1}, \psi_2^{t1}, \psi_1^{t2}, \psi_1^{t3}, \psi_1^{t4}, \psi_2^{t4}, \psi_1^{t6}$ are positive and non-linear functions of ϵ_1 and ϵ_2. On the other hand, $\psi_1^{t5}(\epsilon_1, \epsilon_2)$ is a non-linear function of ϵ_1 and ϵ_2 and it could take positive or negative values. Also, $\bar\psi_1, \bar\psi_2, \bar\psi_3, \bar\psi_4, \bar\psi_5, \bar\psi_6$ are positive and non-linear functions of ϵ_1 and ϵ_2. On the other hand, $\bar\psi_7$ is a non-linear function of ϵ_1 and ϵ_2 and it could take positive or negative values.

Additional Empirical Results

B1. Spreads and Flexibility using statistically significant elasticity at 95% confidence

Table B1 shows that the same negative relationship between flexibility and spreads holds when we define statistically significant point estimates based on the 95% confidence rather than 90% confidence.

Table B1—Spreads and Flexibility (95% confidence)

VARIABLES	(1) Δ Spread	(2) Δ Spread	(3) Δ Spread	(4) Δ Spread
$\epsilon_Q^{IV} \cdot GR$	-0.342***			
	(0.117)			
$\epsilon_Q^{IV} \cdot EBP$		-0.151**		
		(0.068)		
High $\epsilon_Q^{IV} \cdot GR$			-1.486***	
			(0.459)	
High $\epsilon_Q^{IV} \cdot EBP$				-0.744***
				(0.243)
Observations	2,917	2,917	2,917	2,917
Number of sector	53	53	53	53
Adjusted R-squared	0.434	0.435	0.436	0.440
Time FE	Yes	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes	Yes

Note: ϵ_Q^{IV} is the IV estimate of sectoral elasticity. High ϵ_Q^{IV} is a dummy that takes the value of 1 for sectors with an elasticity above median and the value of 0 otherwise. Standard errors presented in parentheses are clustered at the sector level. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

B2. Spreads and Flexibility OLS Elasticities

Table B2 shows that similar results to Table 3 hold when we use our biased OLS estimate grouping sectors by high and low flexibility. We see that high-flexibility sectors experienced an increase in spreads that was 1.09 percentage points than in low-flexibility sectors.

B3. Complementary Evidence Using Firm-Level Data on Short Term Liquidity

In this Appendix, we use firm-level data to estimate the relationship between production flexibility and short-term liquidity. We obtain firms' working capital (current assets - current liabilities) to sales ratio. We have a balanced panel 2002q1-2015q4. We drop outliers (1% and 99% percentiles) in terms of sales

Table B2—Spreads and Flexibility OLS

VARIABLES	(1) Δ Spread	(2) Δ Spread
High $\epsilon_Q^{FE} \cdot GR$	-1.098*	
	(0.624)	
High $\epsilon_Q^{FE} \cdot EBP$		-0.608*
		(0.316)
Observations	2,917	2,917
Adjusted R-squared	0.433	0.437
Time FE	Yes	Yes
Sector FE	Yes	Yes

Note: This table presents an OLS regression using the 4-quarters change in sectoral credit spread as the dependent variable. The independent variables are sectoral sales, the value of property and plants, inventories, leverage (total debt divided by assets), the excess bond premium (EBP), time fixed-effects, sector fixed-effect, the elasticity, the interaction between the elasticity and a Great Recession dummy, and the interaction between the elasticity and the EBP. High ϵ_Q^{FE} is a dummy that takes the value of 1 for sectors with an elasticity above median, and that takes the value of 0 otherwise. Standard errors presented in parentheses are clustered at the sector level. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

growth, working capital to sales growth, and leverage growth during the Great Recession. The results in Table B4 show that high flexibility firms experienced growth in their working capital to sales ratio that is 59 percentage points larger than low flexibility firms. During the Great Recession, the average working capital to sales ratio growth in the sample is -3.94%, the 1st percentile is -%228, the 99th percentile is 862%, and the standard deviation is 533%.

Table B3—Average Spreads and Flexibility

VARIABLES	(1) Δ Spread	(2) Δ Spread	(3) Δ Spread	(4) Δ Spread
$\epsilon_Q^{IV} \cdot GR$	-0.343***			
	(0.115)			
$\epsilon_Q^{IV} \cdot EBP$		-0.153**		
		(0.067)		
High $\epsilon_Q^{IV} \cdot GR$			-1.147**	
			(0.486)	
High $\epsilon_Q^{IV} \cdot EBP$				-0.603**
				(0.238)
Observations	2,917	2,917	2,917	2,917
Number of sector	53	53	53	53
Adjusted R-squared	0.525	0.526	0.524	0.527
Time FE	Yes	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes	Yes

Note: This table presents an OLS regression using the four-quarter change in average sectoral credit spreads as the dependent variable. The independent variables are sectoral sales, the value of property and plants, inventories, leverage (total debt divided by assets), the excess bond premium (EBP), time fixed-effects, sector fixed-effects, the estimates sectoral elasticity of substitution, the interaction between the elasticity and a Great Recession dummy, and the interaction between the elasticity and the EBP. ϵ_Q^{IV} are the IV estimates of sectoral elasticity in Table 2. High ϵ_Q^{IV} is a dummy that takes the value of 1 for sectors with an elasticity above median and the value of 0 otherwise. Standard errors presented in parentheses are clustered at the sector level. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

VARIABLES	(1) % Δ WCS	(2) % Δ WCS	(3) % Δ WCS	(4) % Δ WCS
$\epsilon_Q^{IV} \cdot GR$	0.155**			
	(0.063)			
$\epsilon_Q^{IV} \cdot EBP$		0.052**		
		(0.026)		
High $\epsilon_Q^{IV} \cdot GR$			0.594**	
			(0.245)	
High $\epsilon_Q^{IV} \cdot EBP$				0.185*
				(0.100)
Observations	82,998	82,998	82,998	82,998
Adjusted R-squared	0.002	0.002	0.002	0.002
Time FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes

Table B4—Working Capital to Sales (WCS) Growth and Flexibility

Note: This table presents an OLS regression using firm-level working capital to sales ratio as the dependent variable. The independent variables are sectoral sales, the value of property and plants, inventories, leverage (total debt divided by assets), the excess bond premium (EBP), time fixed-effects, firm fixed-effects, the high elasticity dummy, the interaction between the high elasticity dummy and a Great Recession dummy, and the interaction between the high elasticity dummy the EBP. Standard errors presented in parentheses are clustered at the firm level. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.