A Note on Optimal Sectoral Policies in Production Networks

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Abstract

I study optimal sectoral policies in a model with input-output linkages and sectoral distortions. I characterize network pecuniary externalities and then derive the conditions on the set of input subsidies – as rank conditions to the system of linear equations – that decentralize the first best allocation. In general network structures, labor subsidies are not sufficient to implement the first best allocation. Subsidies to the input that connects firms – intermediate inputs – can fully correct for the externalities. The framework also allows to back out multiple combinations of labor and intermediate inputs subsidies that decentralize the first best, which is desirable when subsidizing some sectors (inputs) is unpopular. The exact combinations of subsidies that decentralize the first best depend on the underlying network structure of the economy.

Keywords: Pecuniary externality, input-output linkages, constrained efficient, sectoral subsidies

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1 Introduction

The network origins of aggregate fluctuations are a new, but widely accepted, phenomenon in macroeconomics. It has been shown, theoretically and empirically, that disruptions in the production of a firm or industry – be sectoral productivity or financial shocks – can have significant aggregate effects by means of sectoral input-output connections (Horvath (1998), Foerster et al. (2011), Acemoglu et al. (2012), Baqaee and Farhi (2017a), Baqaee and Farhi (2017b), Bigio and La'O (2016), Carvalho et al. (2016), Atalay (2017), and Miranda-Pinto (2018)).

However, there are no policy lessons to be drawn from this literature. Bigio and La'O (2016), Luo (2015), and Miranda-Pinto and Young (2018) show that during the Great Recession, sectoral financing constraints distorted firms' optimal input choice. The effect of these sectoral *wedges* is amplified by sectoral linkages. However, linkages might also offer a way out. In particular, can the authority – conditional on tighter credit conditions and existent input-output connections – mitigate a recession by relocating sectoral production via sectoral input subsidies? Finding an answer to this question is the goal of this paper.

I study the normative aspects of multisector economies with input-output linkages and sectoral distortions as in Bigio and La'O (2016) and Baqaee and Farhi (2017b). Firms in any of the N sectors of the economy behave competitively and produce final output using labor and intermediate inputs from other firms. I assume constant elasticity of substitution (CES) sectoral production functions. While the specific nature of sectoral distortions is not crucial, I take a stand and assume that firms face working capital constraints that distort the optimal choice of inputs in production.[1] This constraint can arise due to firms' limited commitment to repay their working capital loans.

To study the scope for policy intervention I follow Bianchi (2011) and solve the constrained efficient planner problem. The social planner faces the same working capital constraints as private firms but internalizes the price effects of firms' production-borrowing decisions. When sectoral constraints bind, there exist network pecuniary externalities at work that open the scope for policy intervention. Firms in downstream (upstream) sectors do not internalize how their decisions affect the severity of the constraint of upstream (downstream) firms.

[1] Bigio and La'O (2016) show that an economy with sectoral distortions and input-output linkages can well account for the decline in aggregate output and labor during the Great Recession in the U.S. On the other hand, Miranda-Pinto and Young (2018) find that an economy with sectoral working capital constraint and heterogeneous sectoral elasticities of substitution between inputs is able to account for the observed relationship between sectoral elasticities and sectoral corporate corporate bond spreads during financial crisis.

I show that the constrained planner solution is first best. Therefore, I proceed to study the set of optimal input subsidies that decentralize the first best allocation. I provide a framework that makes use of the model's system of linear equations on prices and subsidies to implement the first best allocation. Using a rank condition on the system of linear equations, I show that, when all firms use intermediates in production and when all firms are credit constrained, a set of intermediate input subsidies on each sector can decentralize the first best. On the other hand, when all sectors use intermediates, labor subsidies alone do not have enough degrees of freedom to relocate inputs and undo the constraints. There are also multiple combinations of labor and intermediate subsidies that can implement the first best allocation.

This paper makes several contributions to the literature. The first one is to provide a mathematical characterization of the feasible set of policy instruments in the form of a rank condition in the system of linear equations that characterize the economy. The rank condition depends on the set of instruments considered – be subsidies to labor and/or intermediate input to a given sector – and the structure of input-output connections. For example, while a set of labor subsidies is not optimal when all sectors use intermediates, a set labor subsidies is optimal when one sector produces without intermediates.

Another contribution of this paper is that it provides multiple sets of input subsidies that achieve the same goal: fully undo sectoral constraints by internalizing network pecuniary externalities. The sets of optimal policies depend on the policy instruments available and the details of sectoral connections. In this regard, this paper emphasizes the importance of the microeconomic structure of economy in the design of optimal policy. Finally, the existence of multiple policy tools that effectively relocate sectoral activity is crucial from a political economy perspective, when bailing out some industries or banks is unpopular or when injecting liquidity to the financial sector is an inefficient way of increasing credit supply.

Related Literature: Contemporaneous to this study, Liu (2017) studies sectoral interventions that have the highest social value. The author proposes development policies targeting the most upstream industries of the economy. The framework provided in this paper instead provides a variety of optimal input subsidies that implement the first best allocation. Indeed, the existence of input-output linkages, heterogeneous frictions, and different policy tools, provide a variety of corrective policies that not only target large upstream industries. In addition, unlike Liu (2017) who focuses on long-term development policies, I study business cycle stabilization policies.

2 The Model Economy

There are N sectors in the economy. Firms in sector j produce output Q_j using labor L_j and intermediate inputs M_{ij} from other sectors. The production technologies are general CES of the form:

$$Q_j = Z_j \Big[a_j^{1/\epsilon_Q} L_j^{\frac{\epsilon_Q - 1}{\epsilon_Q}} + (1-a_j)^{1/\epsilon_Q} M_j^{\frac{\epsilon_Q - 1}{\epsilon_Q}} \Big]^{\frac{\xi \epsilon_Q}{\epsilon_Q - 1}}, \tag{1}$$

where Z_j is sectoral total factor productivity, a_j is the importance of labor in production, $1-a_j$ is the importance of intermediates in production, and ϵ_Q is the elasticity of substitution between labor and the bundle of intermediates. When $\epsilon_Q = 1$, a_j is exactly the expenditure share of labor in gross output, while $1-a_j$ is the expenditure share of intermediates in gross output. The parameter ξ controls the returns to scale. When $\xi < 1$, there are decreasing returns to scale and when $\xi = 1$ the technology displays constant returns to scale (CRS). The benchmark case assumes CRS in the limit. The intermediate input bundle is:

$$M_j = \left(\sum_{i=1}^{N} \omega_{ij}^{1/\epsilon_M} M_{ij}^{\frac{\epsilon_M - 1}{\epsilon_M}} \right)^{\frac{\epsilon_M}{\epsilon_M - 1}}, \tag{2}$$

where ω_{ij} is the expenditure share of intermediate inputs from sector i in the total intermediate input expenditure of sector j. The elasticity of substitution between intermediates is ϵ_M.

Firms in each sector face the following working capital constraint:

$$wL_j + \sum_{i=1}^{N} P_i M_{ij} \leq \eta_j P_j Q_j. \tag{3}$$

As in Bigio and La'O (2016) firms need to pay input before production. The external funds that a firm can obtain are limited by a fraction η_j of total sales. This assumption is the result of an enforcement problem. Firms could run away with revenues without paying back the intra-period loan to the financial intermediary. In an equilibrium without default on debt, where the intermediary can seize a fraction η_j of expected revenue, the participation constraint implies the constraint in (3).

In this environment, firms are exogenously and permanently (un) constrained if the value of the collateral constraint parameter η_j is smaller (larger) than the degrees of scale (ξ).

The representative household utility is:

$$U(C, L) = \log C - L, \tag{4}$$

where

$$C = \prod_{j=1}^{N} C_j^{\phi_j}. \tag{5}$$

The consumption share of sector j in total consumption expenditure is ϕ_j. Labor is elastically supplied and freely mobile across sectors. The household budget constraint is $wL = \sum_{j=1}^{N} P_j C_j$, where w is the wage rate. I abstract from firms' profits, which is the same as interpreting the results in the limit CRS case, as in Bigio and La'O (2016). To simplify the analysis, I also assume that the excess revenues generated by sectoral distortions are thrown into the ocean.

In the following sections I define the equilibrium of the economy and the benchmark allocations for the study of optimal policy. I start by defining the decentralized competitive equilibrium (constrained and unconstrained) to then study the planner problem.

2.1 Competitive Equilibrium

I assume that markets are perfectly competitive. All agents in this economy make static decisions which is why I suppress the time subscripts from the model. I assume that the wage rate is the numeraire of the economy, implying $w = 1$.

Definition 1 *(Competitive equilibrium) The competitive equilibrium is defined by prices $\{P_j\}_j$ and allocations $(\{C_j, Q_j, L_j, M_j\}_j)$, such that, given prices,*

- *firms maximize profits subject to technology (1)-(2) and frictions (3),*
- *households maximize utility (4) subject to the budget constraint,*
- *and markets clear for all j*

$$Q_j = C_j + \sum_{i=1}^{N} M_{ji},$$

$$L = \sum_{i=1}^{N} L_j.$$

The optimality conditions for firms and households, in the CRS limit case, are:

$$P_j \psi_j Z_j^{\rho_Q} \left(\frac{a_j Q_j}{L_j}\right)^{1-\rho_Q} = w = 1 \tag{6}$$

$$P_j \psi_j Z^{\rho_Q} \left(\frac{(1-a_j) Q_j}{M_j}\right)^{1-\rho_Q} = P_j^M, \tag{7}$$

$$\frac{1}{C P_c} = w, \tag{8}$$

where $\rho_Q = \frac{\epsilon_Q - 1}{\epsilon_Q}$ and ψ_j is a measure of sectoral distortions. The value of ψ_j is 1 when the constraint is not binding. Sectors are constrained whenever the pledgeability parameter η_j is smaller than the degree of scales (ξ). Therefore, when the constraint binds we have $\psi_j = \frac{\eta_j}{\xi} \leq 1$.[2]

The price of the intermediate input bundle P_j^M and the price of the consumption bundle P_c are obtained by cost minimization and are equal to:

$$P_j^M = \Big(\sum_{i=1}^{N} \omega_{ij} P_i^{1-\epsilon_M}\Big)^{\frac{1}{1-\epsilon_M}},$$

$$P_c = \prod_{i=1}^{N} \Big(\frac{P_j}{\phi_j}\Big)^{\phi_j}.$$

Proposition 1 *In the constant returns to scale (CRS) limit and assuming $\epsilon_Q = \epsilon_M$, the constrained competitive equilibrium vector of prices P and allocations (Q, L, M) are:*

$$\log P = \frac{1}{1-\epsilon_Q} \log\Big([I - Z^{\epsilon_Q - 1} \circ \psi^{\epsilon_Q - 1} \circ (((1-a) \circ \Omega')]^{-1} (Z^{\epsilon_Q - 1} \circ \psi^{\epsilon_Q - 1} \circ a) \Big),$$

$$\log(P \circ Q) = \log\Big([I - (\varrho^{1-\epsilon_Q} \circ Z^{\epsilon_Q - 1} \circ (1-a) \circ \psi) \circ \Omega]^{-1} \phi \Big).$$

$$\log Q = \log P \circ Q - \log P.$$

$$\log L = \frac{1}{1-\rho_Q}\Big((1-\rho_Q)\log a + \rho_Q \log Z + \log \psi + \log P + (1-\rho_Q)\log Q \Big),$$

and

$$\log M = \frac{1}{1-\rho_Q}\Big((1-\rho_Q)\log(1-a) + \rho_Q \log Z + \log \psi + \log P + (1-\rho_Q)\log Q - \log P^M \Big),$$

where $\circ$ represent the Hadamard element-wise product. The vector ϱ is a function of the parameters of the model.

See proof in the Appendix.

Unconstrained Equilibrium

By first welfare theorem, when all sectors are unconstrained ($\psi_j = 1 \ \forall j$) the competitive equilibrium is first best. This is, the competitive equilibrium allocations in the absence of borrowing constraints coincide with the solution to the planner problem. We will see later

[2] The distortion ψ_j can also be represented as function of the Lagrange multiplier of the collateral constraint and the parameter η_j. In particular, $\psi_j = \frac{1+\eta_j \mu_j}{1+\mu_j}$.

that the unconstrained allocation can be attained when the government has enough policy instruments.

Definition 2 *The unconstrained competitive equilibrium vector of prices* P^* *and first best allocations* (Q^*, L^*, M^*) *correspond to those in Proposition 1 when* $\psi_j = 1$ *for all* j.

3 A Network Pecuniary Externality

The scope for policy intervention arises from the fact that individual firms do not internalize how their decisions affect other firms in the production network. To formally characterize these network externalities I follow Bianchi (2011) and Benigno et al. (2013) and define the constrained efficient planner problem.

Definition 3 *(Constrained efficient planner) The planner chooses allocations* $(\{C_j, Q_j, L_j, M_j\}_j)$, *by maximizing the households utility (4) subject to sectoral technology (1)-(2), the working capital constraints (3), and subject to the competitive equilibrium optimality conditions for firms (6)-(8) that pin down the vector of prices* $\{P_j\}_j$.

The planner chooses allocations facing the same working capital constraints while letting goods and input markets to clear competitively. Therefore, the planner internalizes how production decisions affect sectoral prices and then the value of sectors' collateral, which is determined endogenously. It is instructive to study the externality in a simple two sector model.

A Two Sector Example

Suppose there are only two sectors in the economy. Sector one produces using only labor and sector 2 produces using labor and intermediates from sector 1. For simplicity, assume that $\epsilon_Q = \epsilon_M = 1$. The planner's optimality conditions for the use of intermediates and the price of intermediates are:[3]

$$M_2 : -\lambda_1 + \lambda_2(1-a_2)\frac{C}{M_2} - \mu_2 P_1 + \underbrace{\mu_1\eta_1 P_1}_{\textit{downstream externality}} - \gamma_1 P_1 + \gamma_3 \frac{P_1}{\psi_2(1-a_2)} = 0$$

$$P_1 : \underbrace{-\mu_2 M_2}_{\textit{upstream externality}} + \mu_1\eta_1 M_2 - \gamma_1 M_2 + \gamma_3 \frac{M_2}{\psi_2(1-a_2)} = 0, \quad (9)$$

[3]The variables λ_j and γ_j represent the Lagrange multiplier for technologies and firms' first order conditions, respectively.

In equation (9), the collateral constraint multiplier of the intermediate good sector, μ_1, appears in the choice of intermediates for the final good sector. The term $\mu_1\eta_1 P_1$ is absent in the private optimality decision for M_2 in equation (7), which is why I call this term *downstream externality*. The intuition behind this result is that, when firms in the intermediate good sector are credit constrained, downstream firms do not internalize that there is a social gain of using more intermediates M_2. The increased demand for intermediates would rise the relative price of intermediate good firms, which in turns relaxes their borrowing constraint.

On the other hand, if firms in the final good sector were constrained, $\mu_2 > 0$, the planner internalizes the existence of an extra social benefit of increasing the supply of intermediate goods ($Q_1 = M_2$), and therefore reducing the price of intermediates. The social benefit is represented in equation (9) by the term $\mu_2 M_2$ in the optimal choice of P_1, which I call *upstream externality*.[4]

The next proposition links the constrained efficient solution with the decentralized unconstrained equilibrium. The planner can completely undo the constraints by relocating sectoral activity.

Proposition 2 *The allocations (C^p, Q^p, L^p, M^p) in the constrained efficient planner problem are equal to the first best allocations (C^*, Q^*, L^*, M^*). See proof in the Appendix.*

Having described the first best allocation and the pecuniary externality at work, I proceed to study optimal policy.

4 Primal Ramsey Problem

In this section, I seek to decentralize the planner's (first best) allocation for general network structures. I assume the existence of labor subsidies (s_j^l) and intermediate input bundle subsidies (s_j^m). I assume that these subsidies are financed via lump-sum taxes to households (T). The government follows a balanced budget rule ($T = \sum_{j=1}^N s_j^l L_j + \sum_{j=1}^N s_j^m P_j^m M_j$). I do not choose a subsidy on sales as a tool because that simply gets rid of the constraint by giving more collateral.

As emphasized in the introduction, the goal of this paper is to study what combinations of sectoral subsidies are able to implement the first best allocation. Thus, the primal Ramsey problem studies the firms individual optimality conditions, evaluated at the first best allocations, with the set of sectoral subsidies in place. While the approach presented in this paper – Lemma 1 – applies for general CES technologies, to simplify exposition and

[4] Benigno et al. (2013) study a different but related *pecuniary externality* problem. When households are borrowing constrained, they do not internalize how their consumption and saving decisions affect the value of their collateral.

computation I proceed by assuming Cobb-Douglas technologies ($\epsilon_M = \epsilon_Q = 1$) as in Bigio and La'O (2016).[5]

Lemma 1 *Let Q^*, L^*, and M^* be the vector of first best allocations of output, labor, and intermediate input bundle, respectively. Suppose that a vector S of K sectoral subsidies are considered. The following system of equations characterizes the set of prices and subsidies that, eventually, decentralize the first best allocation:*

$$AX = B,$$

where X is a vector of dimension $N+K$ that contains the N sectoral prices and the K policy instruments to be considered. B is a function of technology and frictions (ψ, Ω) and the first best allocations (Q^*, L^*, M^*), and A is a matrix of dimension $2N$ by $N+K$ that depends on the input-output structure of the economy Ω and the vector of sectoral subsidies S. By Rouché-Capelli theorem, when the matrices A and $A|B$ have the same rank there is at least one solution of sectoral subsidies that implements the first best. If their rank is also equal to the number of endogenous variables $2N$, there is a unique solution, otherwise there are infinitely many solutions. When $rank(A) \neq rank(A|B)$, the set of subsidies S considered cannot implement the first best allocation.

Lemma 1 establishes that a sufficient condition to decentralize a first best allocation is that the rank of A and $A|B$ are the same. One can immediately see that when there are more subsidies available than number of sectors ($K > N$), the system is under determined and displays infinitely many solutions. I study the minimum number of subsidies (to labor and/or intermediates) that are able to decentralize the first best.

The next propositions establish how many subsidies and what type of subsidies are needed to decentralize the first best allocation.

Proposition 3 *Assume that all sectors use intermediates in production ($\sum_{i=1}^{N} \omega_{ij} = 1 \ \forall j$). Then, a set of N intermediate input subsidies $\{s_j^{m,}\}_{j=1}^{N}$ suffices to decentralize the first best allocation.*
See proof in the Appendix.

Intuitively, Proposition 3 establishes that network externalities can be corrected with subsidies that affect the input that connects firms in different industries, intermediate inputs. There must be at least N subsidies. With less than N instruments, the rank condition is not met, and it is not possible to attain the first best. When more than N instruments are available, say labor and intermediate subsidies, the system is under determined and there are multiple ways of decentralizing the first best. With exactly N intermediate input subsidies there is a unique solution that backs out the set of optimal subsidies.

[5] Strictly speaking, one needs to linearize the general CES model as in Atalay (2017).

Proposition 4 *Assume that all sectors use intermediates in production* ($\sum_{i=1}^{N} \omega_{ij} = 1\ \forall j$)*. Then, a set of* $N-1$ *intermediate input subsidies* $\{s_j^{m,}\}_{j \neq k}^{N}$ *and one labor subsidy* s_k^l *suffices to decentralize the first best allocation if only if* $\omega_{kk} < 1$.
See proof in the Appendix.

A combination of labor and intermediate input subsidies is able to decentralize the first best as long as the sector receiving the labor subsidy is also demanding intermediates from other sectors ($\omega_{kk} < 1$). When this is the case, a labor subsidy to sector k changes the demand from k to other sectors' output in a way that mimics the effect of an intermediate input subsidy. However, if sector k is not using others output in production, the labor subsidy to sector k does not have enough degrees of freedom to optimally relocate sectoral production.

Proposition 5 *Assume that all sectors use intermediates in production* ($\sum_{i=1}^{N} \omega_{ij} = 1\ \forall j$)*. Then, a set of* N *labor input subsidies* $\{s_j^l\}_{j=1}^{N}$ *cannot decentralize the first best allocation. Nevertheless, if at least one sector (k) does not use intermediates in production* ($\sum_{i=1}^{N} \omega_{ik} = 0$)*. Then, a set of* N *labor input subsidies* $\{s^{l,j}\}_{j=1}^{N}$ *suffices to decentralize the first best allocation.*
See proof in the Appendix.

Proposition 5 establishes that when all sectors are using inputs from other sectors, labor subsidies are not able to correct for network pecuniary externalities. As in Proposition 4, without additional conditions on the network, labor subsidies do not have enough degrees of freedom to attack externalities among connected firms. For labor subsidies alone to be optimal, there must be at least one sector not using intermediates. Intuitively, suppose that there exist $N-1$ labor subsidies that directly relax sectoral constraints of $N-1$ sectors. An additional labor subsidy to sector N will then distort sectoral allocation by reducing the demand for intermediates from sector N to other sectors. This does not occur when firms in sector N do not use intermediates from other industries.

4.1 Discussion

This paper studies optimal sectoral policies that correct for network pecuniary externalities. A key assumption in this paper is the absence of informational frictions. For the design and implementation of sectoral policies one must identify and measure sectoral frictions. Bigio and La'O (2016) and Miranda-Pinto and Young (2018) provide different strategies to identify sectoral frictions using sectoral bond spreads as a proxy for financial distortions.

With a fully calibrated model and a full set of input subsidies available, the framework provided in this paper can be generalized to generate an algorithm that selects all the possible set of sectoral intermediate input and/or labor subsidies that meet the rank condition in Lemma 1. With the set of optimal sectoral policies at hand, policy makers can pick the most feasible of implementing during macroeconomic downturns.

5 Conclusions

This paper studies the normative implications of models with input-output linkages and sectoral frictions as in Bigio and La'O (2016) and Baqaee and Farhi (2017b). The model displays network pecuniary externalities that can be corrected by subsidizing inputs of production. While previous work highlights how network connections amplify the effect of distortions, this paper emphasizes how linkages offer multiple sets of intermediates and/or labor subsidies that decentralize the first best allocation. In a general production network model, the use of labor subsidies is not enough to undo the constraints. A set of intermediate input subsidies to all sectors – or the right mix between labor and intermediate input subsidies – is able to implement the first best allocation. The existence of multiple alternatives of corrective policies is desired from a political economy point of view, especially when subsidizing some industries is unpopular.

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Online Appendix (not for publication)

Proof Proposition 1:.

Let's first define $\rho_Q = \frac{\epsilon_Q - 1}{\epsilon_Q}$. Using equations (6) and (7) and sectoral production functions to obtain

$$Z_j^{-\rho_Q} = a_j^{1/\epsilon_Q}\left(\frac{L_j}{Q_j}\right)^{\rho_Q} + (1-a_j)^{1/\epsilon_Q}\left(\frac{M_j}{Q_j}\right)^{\rho_Q},$$

which combined with the FONC gives and assuming w is the numeraire gives

$$P_j^{1-\epsilon_Q} = Z_j^{\epsilon_Q - 1} a_j \psi_j^{\epsilon_Q - 1} + Z_j^{\epsilon_Q - 1}(1-a_j)\psi_j^{\epsilon_Q - 1}\left(\sum_{i=1}^{N}\omega_{ij}P_i^{1-\epsilon_M}\right)^{\frac{1-\epsilon_Q}{1-\epsilon_M}},$$

assuming $\epsilon_Q = \epsilon_M$ we have, in matrices, the solution for prices

$$P^{1-\epsilon_Q} = [I - Z^{\epsilon_Q - 1} \circ \psi^{\epsilon_Q - 1} \circ (((1-a)\circ\Omega')]^{-1}(Z^{\epsilon_Q - 1}\circ\psi^{\epsilon_Q - 1}\circ a)$$

$$\log P = \frac{1}{1-\epsilon_Q}\log\left([I - Z^{\epsilon_Q - 1} \circ \psi^{\epsilon_Q - 1} \circ (((1-a)\circ\Omega')]^{-1}(Z^{\epsilon_Q - 1}\circ\psi^{\epsilon_Q - 1}\circ a)\right)$$

To obtain the vector of sectoral output I use the market clearing conditions (times P_j)

$$P_jQ_j = P_jC_j + \sum_{i=1}^{N} P_jM_{ji}.$$

We also need the household optimality conditions. Assuming $U(C,L) = \log C - L$ and using the wage rate as the numerarie, the households optimality conditions imply

$$\frac{1}{C} = P_c,$$

where P_c is the consumption bundle price index. In addition, the household cost minimization condition implies

$$P_jC_j = \phi_j P_c C = \phi_j.$$

On the other hand, the profit maximizing condition for firms with respect to M_{ij} is

$$P_jM_{ji} = \Big(\frac{P_j}{P_i}\Big)^{1-\epsilon_Q} Z_i^{\epsilon_Q-1}(1-a_j)\omega_{ji}\psi_i P_i Q_i.$$

As we already solved for prices, let's define $\varrho_i = \frac{P_j}{P_i}$.

Replacing these equations into the goods market clearing condition and defining sectoral sales as S_j, we obtain the following expression

$$P_jQ_j = \phi_j + \sum_{i}^{N} \varrho_i^{1-\epsilon_Q} Z_i^{\epsilon_Q-1}(1-a_i)\psi_i\omega_{ji}P_iQ_i,$$

that in matrices becomes

$$P \circ Q = [I - (\varrho^{1-\epsilon_Q} \circ Z^{\epsilon_Q-1} \circ (1-a) \circ \psi) \circ \Omega]^{-1}\phi.$$

Therefore, using the fact that $\log P \circ Q = \log P + \log Q$ and that

$$\log P = \frac{1}{1-\epsilon_Q} \log\Big([I - Z^{\epsilon_Q-1} \circ \psi^{\epsilon_Q-1} \circ (((1-a) \circ \Omega')]^{-1}(Z^{\epsilon_Q-1} \circ \psi^{\epsilon_Q-1} \circ a)\Big),$$

the vector of sectoral output is

$$\log Q = \log P \circ Q - \log P.$$

The vector of sectoral labor is

$$\log L = \frac{1}{1-\rho_Q}\Big((1-\rho_Q)\log a + \rho_Q \log Z + \log\psi + \log P + (1-\rho_Q)\log Q\Big),$$

and the vector of sectoral intermediate input bundle is

$$\log M = \frac{1}{1-\rho_Q}\Big((1-\rho_Q)\log(1-a) + \rho_Q \log Z + \log\psi + \log P + (1-\rho_Q)\log Q - \log P^M\Big).$$

■

Proof Proposition 2:. The planner solves the following problem

$$\max_{\{C_j, L_j, M_j, Q_j\}_j} \quad \log C - L,$$

subject to (1), (3), (5), (6), (7) and (8). Assume all sectors are constrained, therefore $\psi_j = \frac{\eta_j}{\xi} < 1$. Then, combine the competitive equilibrium first order conditions for inputs (6) and (7) with the collateral constraint (3) to obtain

$$wL_j + P_j^M M_j = a_j \psi_j P_j Q_j + (1-a_j)\psi_j P_j Q_j \leq \eta_j P_j Q_j,$$

which by definition of ψ_j (and CRS, $\xi = 1$) simply becomes

$$1 \leq 1.$$

Therefore, the constrained efficient planner is simply the solution to the standard planner problem where household welfare is maximized subject to sectoral technology. By first welfare theorem, the unconstrained competitive equilibrium allocation coincides with the constrained efficient planner.

■

Proof Lemma 1.

Assume $\epsilon_Q = \epsilon_M = 1$. The system equations defined by (6) and (7), for all j, with sectoral subsidies ($\{s_j^l, s_j^m\}_j$) is described by the following set of $2N$ equations

$$\begin{aligned} -\log P_j + \log(1-s_j^l) = &\quad \log a_j\psi_j + \log Q_j^* - \log L_j^* \\ \sum_{i=1}^N \omega_{ij}\log P_i - \log P_j + \log(1-s_j^m) = &\quad \log(1-a)_j\psi_j + \log Q_j^* - \textstyle\sum_{i=1}^N \omega_{ij}\log M_{ij}^*. \end{aligned}$$

In matrices, we have

$$A \cdot X = B,$$

where A is a matrix of dimension $2N$ by $N+K$, where K is the number of endogenous variables, which is at least equal to $N+1$, N sectoral prices and one policy instrument, and at most equal to $3N$, when $2N$ policy instruments are available. The vector X is

$$X = \Big[\log P_1 \quad \dots \quad \log P_N \quad \log(1-s^1) \quad \dots \quad \log(1-s^N)\Big],$$

where P_i are sectoral prices and s_i policy instruments. The subsidies s^1 to s^N are generic and do not correspond to a particular sector or input. The matrix A is

$$A = \begin{bmatrix} -I_n & A_L \\ \Omega - I_n & A_M \end{bmatrix} \quad A|B = \left[\begin{array}{cc|c} -I_n & A_L & B_1 \\ \Omega - I_n & A_M & B_2 \end{array}\right],$$

where I_n is the identity matrix of dimension N. The matrix Ω represents the input-output matrix of the economy (N by N matrix), and the matrices A_L and A_M – dimension N by K – are function of the set of subsidies considered. The vector B has a dimension of $2N$ and is a function of the parameters of the model (technologies a_j and frictions ψ_j) and the first best allocations (Q^*, L^*, M^*). When A and $A|B$ have both full rank, there is a unique solution of prices and policy instruments that decentralizes the first best. If their rank is smaller than N, there are multiple solutions. If $rank(A) \neq rank(A|B)$, there is no solution.

■

Proof Proposition 3.

I start by assuming $K \geq N$. This is, there are at least N subsidies available. In particular, as $N > K$implies an under determined system, I focus on $K = N$. I come back to the case $K < N$ at the end. In this case, with only intermediate input subsidies we have

$$A_L = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & & & & \vdots \\ 0 & \dots & & & 0 \end{bmatrix} \quad \text{and} \quad A_M = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & & & & \vdots \\ 0 & \dots & & & 1 \end{bmatrix},$$

implying

$$A = \begin{bmatrix} -1 & 0 & \dots & 0 & 0 & \dots & \dots & 0 \\ 0 & -1 & 0 & \vdots & 0 & 0 & 0 & \vdots \\ \vdots & & & \vdots & & & & \vdots \\ 0 & \dots & \dots & -1 & \vdots & & & 0 \\ \omega_{11}-1 & \omega_{12} & \dots & \omega_{1n} & 1 & \dots & & 0 \\ \omega_{21} & \omega_{22}-1 & \dots & \omega_{2n} & 0 & 1 & 0 & \vdots \\ \vdots & & & \vdots & & & & \vdots \\ \omega_{n1} & \dots & \dots & \omega_{nn}-1 & 0 & \dots & & 1 \end{bmatrix}.$$

As A is a square matrix ($2N$ by $2N$), if $det|A| \neq 0$ then A is full rank. The determinant of order n is expressed as

$$\det|A| = \sum (\pm) a_{1,r1} a_{2,r2} \cdots a_{2n,r2n},$$

where $a_{i,ri}$ is an element of A and ri represents all the possible permutations of element i such that each term of the product has exactly one term from each row and column of A. As column $2n$ has all terms zero except the term $a_{2n,2n}$, the only way of having a non-zero product of terms is using the term a_{2n2n} from column $2n$. The same applies for terms $a_{2n-1,2n-1}$ in column $2n-1$, $a_{2n-2,2n-2}$ in column $2n-2$, and so on. Thus, the determinant of matrix A will be non zero as long as we find a combination of non zero elements in the upper left matrix $-I_n$. It is easy to see that the only way that we can pick a non-zero combination of terms from each column and row (in the upper-left part of A) is by picking up the diagonal of $-I_n$. Indeed, using the formula above we have that

$$\det|A| = \sum(\pm)a_{1,r1}a_{2,r2}\cdots a_{2n,r2n} = \pm 1,$$

where the sign depends on n. The fact that $det(A) \neq 0$ proves that when only intermediate input subsidies are used, A has full rank. It then suffices to show that $A|B$ has also a rank of $2N$. As A is full rank and $A|B$ can have at most a rank of $2N$ (the lesser between the number of rows $2N$ and columns $2N+1$), it follows that A and $A|B$ have the same rank. This proves that a set of intermediate input subsidies decentralizes the first best allocation.

When $K < N$, A can have at most a rank of $N+K < 2N$. On the other hand, the matrix $A|B$ can have a rank of at most $N+K+1$. As long as the elements of vector B are not a linear combination of the columns of A, $rank(A|B) = N+K+1 > N+K \geq rank(A)$. An interior solution implies that B is not a vector of zeros. Therefore, the system does not admit a solution. However, it can be the case that sectoral distortions ψ_j are such that B is a linear combination of the columns of A, implying $rank(A) = rank(A|B)$ and that there are multiple solutions. This can occur when several sectors are unconstrained ($\psi_j = 1$). I rule out this case by assuming $\psi_j < 1\ \forall_j$.

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Proof Proposition 4.

I focus on $K = N$. Without loss of generality, assume that sector k receiving the labor subsidy is sector $k = N$. Therefore, we have

$$A_L = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & & & & \vdots \\ 0 & \dots & & & 1 \end{bmatrix} \quad \text{and} \quad A_M = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & & & & \vdots \\ 0 & \dots & & & 0 \end{bmatrix},$$

implying

$$A = \begin{bmatrix} -1 & 0 & \dots & 0 & 0 & \dots & \dots & 0 \\ 0 & -1 & 0 & \vdots & 0 & 0 & 0 & \vdots \\ \vdots & & & \vdots & & & & \vdots \\ 0 & \dots & \dots & -1 & \vdots & & & 1 \\ \omega_{11}-1 & \omega_{12} & \dots & \omega_{1n} & 1 & \dots & & 0 \\ \omega_{21} & \omega_{22}-1 & \dots & \omega_{2n} & 0 & 1 & 0 & \vdots \\ \vdots & & & \vdots & & & & \vdots \\ \omega_{n1} & \dots & \dots & \omega_{nn}-1 & 0 & \dots & & 0 \end{bmatrix}.$$

Using the same approach, column $2n$ has all terms zero except the term $a_{n,2n}$, the only way of having a non-zero product of terms is using the term a_{n2n} from column $2n$. The same applies for terms $a_{2n-1,2n-1}$ in column $2n-1$, $a_{2n-2,2n-2}$ in column $2n-2$, and so on. Thus, the determinant of matrix A will be non zero as long as we find a combination of non zero elements in the matrix formed by concatenating the first $N-1$ rows and columns of $-I_{n-1}$ with the last row and column of matrix $\Omega - I_n$. It is easy to see that the only way that we can pick a non-zero combination of terms from each column and row (in the upper-left part of A) is by picking up the diagonal of such a matrix and assuming $w_{nn} \neq 1$. When $w_{nn} = 1$, $det(A) = 0$ and the set of subsidies cannot implement a first best.

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Proof Proposition 5. I focus on $K = N$. When only labor subsidies are used, we have that

$$A_L = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & & & & \vdots \\ 0 & \dots & & & 1 \end{bmatrix} \quad \text{and} \quad A_M = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 0 & & 0 & \dots & 0 \\ \vdots & & & & \vdots \\ 0 & \dots & & & 0 \end{bmatrix},$$

implying

$$A = \begin{bmatrix} -1 & 0 & \dots & 0 & 1 & \dots & \dots & 0 \\ 0 & -1 & 0 & \vdots & 0 & 1 & 0 & \vdots \\ \vdots & & & \vdots & & & & \vdots \\ 0 & \dots & \dots & -1 & \vdots & & & 1 \\ \omega_{11}-1 & \omega_{12} & \dots & \omega_{1n} & 0 & \dots & & 0 \\ \omega_{21} & \omega_{22}-1 & \dots & \omega_{2n} & 0 & \dots & & 0 \\ \vdots & & & \vdots & & & & \vdots \\ \omega_{n1} & \dots & \dots & \omega_{nn}-1 & 0 & \dots & & 0 \end{bmatrix},$$

Matrix A is a square matrix ($2N$ by $2N$). Therefore, we can use the same approach. As column $2n$ has all terms zero except the term $a_{n,2n}$, the only way of having a non-zero product of terms is using the term $a_{n,2n}$ from column $2n$. Similarly, we must use the term $a_{n-1,2n-1}$ from column $2n-1$, the $a_{n-2,2n-2}$ from column $2n-2$, and so on. Therefore, the rest of the terms fully depend on the input-output shares ω_{ij}, in the bottom-left part of A. If $det|\Omega - I_n| = 0$, there are not non-zero products among the elements of $\Omega - I_n$, which also implies that $det|A| = 0$. Note that Ω is a Markov matrix with all columns adding up to one, which implies that all columns of $\Omega - I_n$ add up to 0. This is enough to show that $\Omega - I_n$ is singular and has rank equal to 0. Adding up the first $N-1$ rows of matrix $\Omega - I_n$ yields exactly $-\omega_{nj}\ \forall j$, meaning that the last row is a linear combination of the previous $N-1$ rows. Now, we need to show that $A|B$ has different (larger) rank than A. This holds as long as B is not a column of zeros and as long as B is not a linear combination of the columns in A. An interior solutions ensures that B is not a zero vector. I rule out the case where B is a linear combination of the columns in A by assuming that $\psi_j < 1\ \forall_j$.

which holds by definition of input shares. Therefore, as long

To prove the second part of Proposition 5, assume that industry k does not use labor at all, meaning that $\omega_{ik} = 0\ \forall\ k$. Then, we have that column k adds up to -1, while all the other columns add up to 0. Therefore, it is not the anymore the case that one row of $\Omega - I_n$ is a linear combination of all the other rows, implying that $det(\Omega - I_n) \neq 0$, so $det(A) \neq 0$, implying that A has full rank. Now, it suffices to show that $A|B$ has the same rank of A to prove that a set of labor subsidies can decentralize the first best allocation. In this particular case, matrix A has full rank ($2N$). On the other hand, matrix $A|B$ can have at most a rank of $2N$ (the lesser between the number of rows $2N$ and columns $2N+1$). As A has rank of $2N$, it then follows that $A|B$ has also a rank of $2N$. This proves that a set of labor input subsidies decentralizes the first best allocation when at least one sector does not use intermediates. $\blacksquare$