Production Network Structure, Service Share, and Aggregate Volatility

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Abstract

This paper shows that GDP growth volatility declines with production network diversification. To account for this evidence, I build a multisector model with CES technologies and a cost of complexity in the bundle of intermediates. Production network diversification decreases volatility when intermediate inputs and labor are substitute inputs. U.S. sectoral data suggest that labor and intermediates are substitutes in service sectors. Therefore, a calibrated model that then also matches each country's production network can quantitatively generate the empirical patterns. The model also explains why service-oriented countries are less volatile: service sectors have a more diversified set of suppliers.

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1 Introduction

The production process, such as that of cell phones in Korea, entails complex connections between firms in different sectors of the economy. Samsung, for example, uses not only chips, plastic, and financial services, but also equipment, water, and gas. Every economy produces different goods and services, which then translate into a particular structure of input-output connections. What features of the production network structure amplify or mitigate sectoral shocks? The current literature highlights the role of the intensity of input-output connections—e.g., the existence of star intermediate input suppliers—while ascribing no role to production network diversification—e.g., the number of non-zero input-output connections. In this paper, I revisit these results, empirically and theoretically.

From an empirical standpoint, this paper shows that production network diversification—as measured by the fraction of non-zero input-output connections or by the diversification of the sectoral intermediate input bundle—is a key driver of countries' sales shares concentration (as measured by the Herfindahl-Hirschman Index (HHI)) and GDP growth volatility. Specifically, in a panel of 48 OECD and non-OECD countries for the period 1970-2014, I document two main facts: in countries with a more-diversified production network, i) sales shares are less concentrated, and ii) GDP growth volatility is lower.

From a more theoretical standpoint, I generalize the canonical economy of Acemoglu et al. (2012) to help explain the cross-country patterns I document. I allow for non-unitary elasticities of substitution in production—in particular, non-unitary elasticity between the bundle of intermediate inputs and the labor input. I then emphasize the role played by the intermediate input bundle diversification embedded in nested CES production technologies with input-output weights. In this environment, not only sectoral supplier importance, but also the diversification of the network, matters for the propagation and amplification of sectoral shocks. Using U.S data, I calibrate the model to match each country's input-output structure and estimate elasticities of substitution in production. To the extent that U.S sectoral technologies also describe sectoral technologies across countries, I use the model to perform several quantitative exercises.

The main result in this paper is as follows. The model economy suggests that a higher diversification of the production network reduces the economy's HHI of sales shares and, therefore, volatility, as long as production technologies display high substitutability between intermediate inputs and labor. Standard CES production technologies with input-output weights embed a *cost of complexity* in the bundle of intermediates. In particular, firms that source intermediate inputs from a more-diversified set of suppliers display a complexity cost. Therefore, all else equal, when labor and intermediates are substitute inputs, an increase in production diversification leads to an increase in labor demand to overcome the complexity cost. This generates a decline in firms' intermediate input shares which then reduces sectoral interdependence and mitigates the effect of sectoral shocks along the production chain.

Using U.S. sectoral data, I show that the elasticity of substitution between labor and intermediates is larger than one in service sectors. I then use countries' sectoral data on intermediate input shares and intermediate input bundle diversification to show that, as predicted by the model, sectors with more-diversified intermediate input bundles display lower intermediate input shares. This effect is stronger in service sectors, consistent with their higher production flexibility. I also show that service sectors have more-diversified intermediate input bundles than do non-service sectors. This, together with the fact that services have higher production flexibility, can help explain, through the lens of the model, why service sectors display lower intermediate input shares and, therefore, why service-oriented economies are less volatile.[1]

The model is able to qualitatively and quantitatively replicate the fact that production network diversification is associated with lower macroeconomic volatility. The calibrated model, which matches each country's input-output structure, is quantitatively successful in replicating the observed empirical patterns. The model's implied relationship among production network diversification, service share, and volatility is of an order of magnitude similar to that observed in the data.

Contribution to the literature: This paper contributes to the literature on production networks and aggregate fluctuations in Horvath (1998), Foerster et al. (2011), Acemoglu et al. (2012), Carvalho et al. (2016), Atalay (2017), and Baqaee and Farhi (2019). With respect to these studies, this paper illustrates the importance of a new production network feature, production network diversification.[2] In doing so, I emphasize the relevance of deviating from homogeneous and Cobb-Douglas sectoral technologies and of taking into account the production complexity of using a more-diversified intermediate input bundle.[3] This paper also provides international evidence of the importance of the input-output network structure in shaping the HHI of sectoral sales and macroeconomic volatility.[4]

[1] Moro (2015) show that the lower intermediate input share in service sectors can help explain why service-oriented economies are less volatile.

[2] In an environment with endogenous production networks, Acemoglu and Azar (2020) study the relationship between production network diversification—as measured by network density—and development. Moreover, in a Cobb-Douglas economy, Herskovic (2018) studies the role of production diversification or sparsity in shaping asset prices.

[3] The relationship between production complexity—through higher production diversification—and the macroeconomy provided in this paper is related to those proposed by Costinot et al. (2013) and Jaimovich et al. (2017). The former study postulates that lengthier and more complex production processes require more inputs to produce a given level of output, while the latter documents that higher-quality products require a higher labor share.

[4] A series of recent paper have studied the relationship between the structure of inter-sectoral linkages

This paper also contributes to the literature on sectoral composition and macroeconomic volatility in Moro (2012), Moro (2015), Koren and Tenreyro (2007), Carvalho and Gabaix (2013), di Giovanni and Levchenko (2012), and Gabaix (2011). These papers highlight the importance of sectoral composition and the HHI of sales shares in driving macroeconomic volatility. This paper complements the aforementioned papers by providing empirical and theoretical support for the idea that the HHI of sales shares is shaped by the production network structure, with an emphasis on production network diversification. This paper also complements previous studies by showing that the lower intermediate input share in service sectors can be explained by the high production diversification and high production flexibility observed in services.

Finally, this paper contributes to the literature on production diversification and volatility in Koren and Tenreyro (2013) and Krishna and Levchenko (2013). Different from these papers—in which production diversification shapes volatility via the law of large numbers (LLN), regardless of the details of the production function and in a framework without input-output linkages—this article proposes a different mechanism. A more-diversified network of producers reduces volatility only when the substitutability between intermediates and labor is high, and the mechanism operates through lowering intermediate input shares, which then mitigate the effect of sectoral shocks along the production chain.[5]

2 Empirical motivation

In this section, I document two empirical observations between countries' production network structure and the HHI of sales shares and macroeconomic volatility. The main motivation behind the empirical analysis in this section is based on Gabaix (2011) and Acemoglu et al. (2012). Gabaix (2011) shows that the HHI of sectoral sales share is a sufficient statistic for macroeconomic volatility, while Acemoglu et al. (2012) show that the structure of intersectoral linkages—in particular, sectoral outdegree concentration—shapes the HHI of sectoral sales.

The results in this section show: i) that the HHI of sectoral sales shares is decreasing in countries' production network diversification—as measured by the number of non-zero input-output connections or by the average sectoral intermediate input bundle diversifi-

and macroeconomic outcomes, such as a country's aggregate productivity and income level, as in Jones (2011), Bartelme and Gorodnichenko (2015), and Fadinger et al. (2015). The key difference between those studies and this paper is that the focus here is on business cycles instead of income level.

[5]The LLN does not drive the results in this paper as the number of sectors N is fixed. Even when industries differ in terms of how connected they are to other sectors, it is not clear that a less (more) diversified network amplifies (mitigates) shocks. If sectors use intermediates from only one other sector, there still exist indirect connections embedded in the input-output network. These connections render each sector vulnerable to other sectors' idiosyncratic shocks.

cation; and ii) that GDP growth volatility is decreasing in countries' production network diversification. These empirical observations are not implied by the existing literature on multisector models with input-output linkages. Indeed, Dupor (1999) and Acemoglu et al. (2012) show that, with Cobb-Douglas production technologies, production network diversification plays no role in amplifying or mitigating the effect of sectoral productivity shocks.[6]

2.1 Data

The variables of interest are i) the HHI of sectoral sales shares and ii) the volatility of GDP growth. I measure the HHI using OECD data on sectoral gross output and value added for each country for the period 1995-2011. To measure macroeconomic volatility, I use countries' real GDP at constant national prices in millions of 2011 U.S. dollars from the Penn World Tables 9.0 (rgdpna) for the period 1970-2014. To characterize the production structure, I use input-output matrix data from the OECD input-output database.[7] For each country, I have information for 33 sectors of the economy for the period 1995-2011.

The sample retains the countries in the OECD database for which I have real GDP data since 1970, resulting in a final sample of 48 countries—25 with developed economies and 23 with emerging economies. I also use countries' GDP per-capita data from the IMF databases.

2.2 Heterogeneity in input-output structure

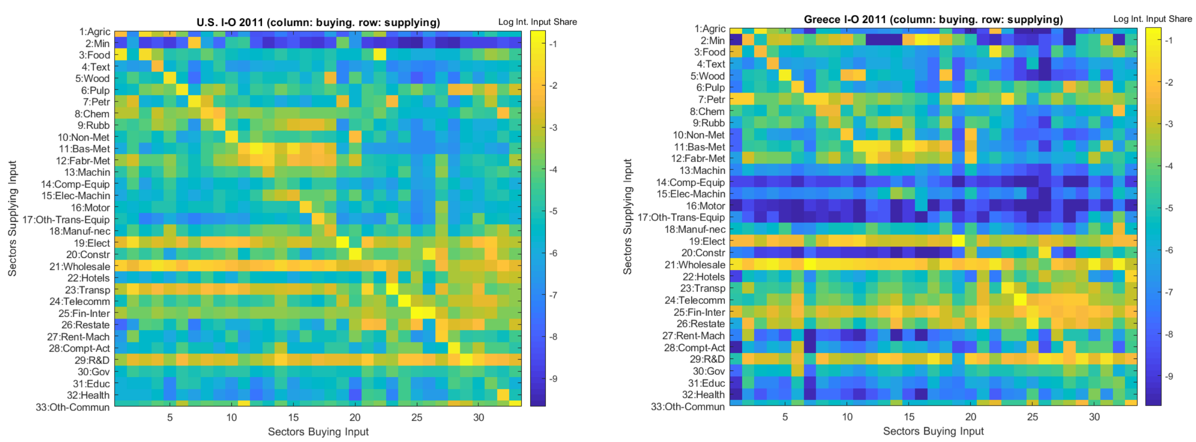
Let $\tilde{\Omega}$ be an N by N matrix that describes the observed input-output structure of the economy. An element $\tilde{\omega}_{ij}$ is an observed input-output share that represents the share of intermediate inputs shipped from sector i to sector's j total expenditure on intermediates. These input-output shares are not symmetric; it might be the case, for example, that sector i provides inputs to sector j, but sector j does not supply intermediates to sector i. The column i sum of $\tilde{\Omega}$ equals 1, as total intermediate expenses must be allocated to some (all) sector(s) of the economy. The row j sum represents how important sector j is as an intermediate input supplier to the economy as a share of other sectors' intermediate expenditures.

[6] Atalay (2017), Carvalho et al. (2016), and Baqaee and Farhi (2019) generalize the model in Acemoglu et al. (2012) by using CES production technologies, as in the present paper. However, the aforementioned papers do not focus on studying the relationship between production network diversification and aggregate volatility.

[7] In particular, I collect the Input-Output Tables ISIC Rev. 3 available at `http://www.oecd.org/sti/ind/input-outputtables.htm`. While a large fraction of countries in the sample have official data for the whole period (1995-2015), in some years, some countries have imputed information. I use only official information. For example, when I claim to use data for 2011, if a country has imputed information for 2011 but official data only for 2010, I effectively use 2010 data.

Figure 1 illustrates the input-output structures of the U.S and Greece. Each row represents an industry supplying intermediates of production, while each column represents industries using intermediates to produce final output. I report the logarithm of industries' intermediate input shares as a fraction of total intermediate input expense. Bright (yellow) colors represent high shares, while dark (blue) colors are small shares. The production network of the U.S. appears more diversified than that of Greece. Sectors in the U.S are more interconnected (panel a), while in Greece (panel b), sectors are more isolated. In particular, the U.S. has 264 more non-zero input-output shares than Greece, which is equivalent to setting seven entire columns—out of 33—of the U.S. input-output matrix to be almost zero.[8] Regarding the intensity of connections, the U.S. and Greece have few sectors that supply intermediate inputs to all other sectors. This is expressed by few very bright rows of the input-output matrix (outdegrees) in Figure 1. Wholesale trade, Transportation, Financial Intermediation, and R&D sectors are central suppliers in both countries.

Figure 1
Production network of the U.S. and Greece



Note: This figure plots the U.S's and Greece's input-output structure in 2011. Log intermediate input shares are $\log(\tilde{\omega}_{ij} + 1e^7)$. Bright-yellow colors indicate large intermediate input shares, while dark-blue colors indicate small intermediate input shares.

In the next section, I describe the production network measures I use to characterize each country's production network structure. I consider two production network features in my analysis: i) the concentration of sectoral outdegrees, and ii) the diversification of the production network.

[8]Throughout the paper, and to avoid counting spurious connections due to data harmonization, I define non-zero input-output connections as shares that are larger than a small threshold. The benchmark threshold for $\underline{\omega}_{ij}$ is 0.001.

2.2.1 Outdegrees concentration

The outdegrees concentration describes the extent to which a country has few star intermediate input suppliers. This production network measure is theoretically founded (see Horvath (1998), Acemoglu et al. (2012), and section 3.1 in this paper) and describes how sectoral productivity shocks amplify and propagate from upstream sectors (intermediate input suppliers) to downstream sectors (intermediate input users), via affecting the cost of intermediates in production. The concentration of outdegrees is defined as follows:

$$Outdegree = \sqrt{\sum_{j=1}^{N} \left((I-\tilde{\Gamma})^{-1}\tilde{\beta}\right)_j^2}, \tag{1}$$

where $\tilde{\Gamma}$ is the observed N by N matrix of input-output shares. An element $\tilde{\gamma}_{ij}$ of $\tilde{\Gamma}$ represents the cost share of intermediates from sector i in total gross output of sector j, and $\tilde{\beta}$ is the observed N dimensional vector of sectoral final consumption shares. The $Outdegree$ measure in Equation (1) captures the concentration of sectors' direct and indirect supplier importance, contained in the vector of Leontief inverse elements $(I-\tilde{\Gamma})^{-1}\beta$. Intuitively, economies with higher outdegrees will be more volatile as sectoral shocks, especially shocks to star supplier sectors, will be amplified and propagated more strongly through the network.

2.2.2 Production network diversification

Production network diversification describes the extent to which sectors of the economy rely on few versus many intermediates to produce. Intuitively, in a more-diversified network, sectoral shocks will be spread out among more sectors and will have smaller aggregate effects. However, as Dupor (1999) and Acemoglu et al. (2012) show, this is not the case in production network models with Cobb-Douglas technologies.[9]

I consider two different measures to proxy for production network diversification. First, I consider the production network density:

$$Density = \frac{\sum_{i=1}^{N}\sum_{j}^{N} 1[\tilde{\omega}_{ij} > \underline{\omega}]}{N(N-1)}, \tag{2}$$

where $1[\tilde{\omega}_{ij} > 0]$ is an indicator function that counts input-output connections that are greater than a small threshold $\underline{\omega} \in [0.001, 0.01]$. When the $Density$ is 0, the economy displays 0 non-diagonal links above $\underline{\omega}$ out of $N(N-1)$ potential links, whereas when

[9] Koren and Tenreyro (2013) and Krishna and Levchenko (2013) have also studied the role of production diversification, though in a theoretical context absent of input-output linkages. The authors show that production diversification reduces volatility. The mechanism operates through the LLN via increasing the number of sectors.

the *Density* is 1, all the potential non-diagonal connections ($N(N-1)$) are larger than the threshold. This network measure is widely used in network science and captures the notion of sparsity studied in Dupor (1999) and Acemoglu et al. (2012).

Second, motivated by the model's implications in section 3, I also consider countries' diversification of sectoral intermediate input shares:

$$Divers = \frac{1}{N}\sum_{j=1}^{N}\Big(\sum_{i=1}^{N}\tilde{\omega}_{ij}^{\epsilon_M}\Big)^{\frac{1}{1-\epsilon_M}}, \tag{3}$$

where ϵ_M is the elasticity of substitution between intermediates. Note that for any non-unitary value of ϵ_M, *Divers* is always increasing in the diversification of the vector of sectoral intermediate input shares $\{\omega_{ij}\}_{i=1}^{N}$.[10] Levchenko (2007) uses a similar diversification measure, though in a very different context and from an atheoretical perspective. The author takes the average sectoral Herfindahl index of intermediate input shares and multiplies it by -1 (to describe diversification instead of concentration). Herskovic (2018) also derives a similar diversification measure (for $\epsilon_M = 1$) to study asset prices' determination.

Figure 2 depicts the cross-country distribution of production network structure. In particular, I obtain the average of the production network measures in years 1995, 2005, and 2011. Figure 2 panel a shows the cross-country distribution of *Outdegree* in Equation (1). An economy with homogeneous sectoral outdegrees, where every sector has the same Leontief inverse element, would display an *Outdegree* of about 0.33. On the other hand, an economy in which one sector supplies (directly and indirectly) 18% of total intermediates, while other sectors equally split the supply of the remaining 82%, would display an *Outdegree* of about 0.45. Figure 2, panel a shows that 65% of the countries in the sample display an *Outdegree* larger than 0.45, implying the existence of star suppliers in most countries in the sample.[11]

Figure 2, panel b depicts the histogram of *Density* in Equation (2) for the 48 countries in the sample. There is substantial heterogeneity in the extent to which sectors interact in different countries. The average *Density* is 0.72, indicating that, for the average country in the sample, 72% of non-diagonal input-output shares are non-zero. There is substantial cross-country heterogeneity in *Density*, which is expressed by an interquartile range

[10] With unitary elasticity between intermediates ($\epsilon_M = 1$), the diversification measure becomes

$$Divers = \frac{1}{N}\sum_{j=1}^{N}\Big(\prod_{i=1}\tilde{\omega}_{ij}^{-\tilde{\omega}_{ij}}\Big),$$

which is also increasing in the diversification of the intermediate input bundle. In this case, it is also the case that the observed intermediate input share $\tilde{\omega}_{ij}$ equals the Cobb-Douglas parameter ω_{ij}.

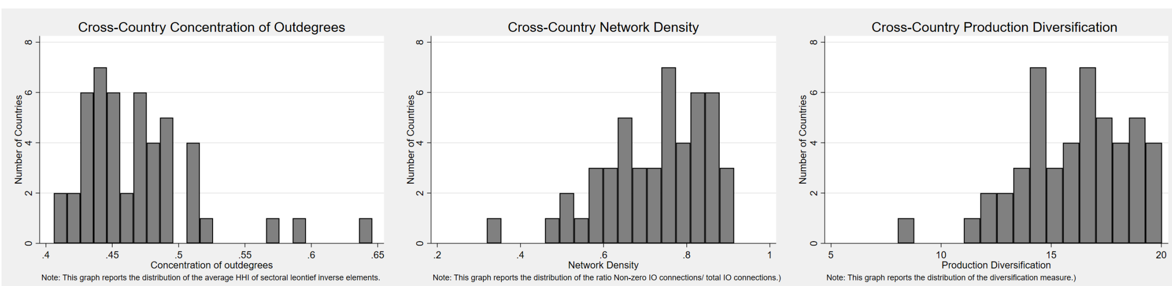
[11] Dungey and Volkov (2018) use the same OECD input-output data and document that the wholesale and R&D sectors are dominant in most countries in the sample.

of 0.2.

Panel c of Figure 2 displays the average of intermediate input diversification across countries in Equation (3), assuming that $\epsilon_M = 0.5$.[12] An economy in which the average sector has a fully diversified intermediate input bundle ($\tilde{\omega}_{ij} = 1/N$ for all j) would display a *Divers* index of $N = 33$. On the other hand, if the average sector of the economy uses only one intermediate input ($\tilde{\omega}_{ij} = 1$ and $\tilde{\omega}_{lk} = 0$ for $(k,l) \neq (i,j)$) the *Divers* index is 1. We can see substantial heterogeneity in production network diversification across countries. The average *Divers* index is 16.03 with an interquartile range of 3.74.

We observe in Figure 2 a high similarity in the cross-country distribution of *Density* and *Divers*. It is direct from Equations (2) and (3) that a production network with low (high) *Density* also displays low (high) *Divers*.

Figure 2
Cross-country distribution of production structure
panel a panel b panel c



Note: This figure plots the cross-country distribution of outdegree concentration (panel a), production network density (panel b), and production network diversification (panel c). For each country, the average input output structure of years 1995, 2005, and 2011 is considered.

While the goal of the paper is not to determine the drivers of production network structure, but to understand how the production structure affects aggregate volatility, it is interesting to examine the relationship between production network structure and development. One could imagine that more-developed countries display more-diversified production structures.[13] In Table 1 we can see that, while there is not a clear relationship between GDP per-capita and production network structure, there is a relationship between the share of services in GDP, typically used as a proxy for development, and production network structure. Countries with a higher share of services in GDP display less concentrated outdegrees, denser production networks, and more diversified intermediate input bundles. I come back to this in Section 4, where I use cross-country sectoral data

[12] In section 4, I estimate $\epsilon_M \approx 0$ for U.S. non-service sectors and $\epsilon_M \approx 1$ for U.S. service sectors. Thus, $\epsilon_M = 0.5$ is a simple average. The same results hold for any value of ϵ_M.

[13] Krishna and Levchenko (2013) show a positive relationship between production diversification, as measured by the number of intermediates used in production, and GDP per-capita. However, the authors use only information for manufacturing industries.

to investigate how diversified service sectors are in production.[14] I also observe that production network density and production diversification are highly correlated (correlation of 0.955), which expresses the fact that both are good measures of countries' production network diversification.

Table 1
Production Network Structure and Development

VARIABLES	GDPpc	Service share	Outdegree	Density	Diversification
GDPpc	1				
Service share	0.2576*	1			
	(0.0771)				
Outdegree	-0.1422	-0.3295**	1		
	(0.3350)	(0.0222)			
Density	-0.0581	0.3192**	-0.1145	1	
	(0.6946)	(0.0270)	(0.4382)		
Diversification	-0.0486	0.3260**	-0.1035	0.9816***	1
	(0.7431)	(0.0503)	(0.4837)	(0.0000)	

Note: This table presents the pairwise correlations between production network structure measures and development, as measured by GDP per-capita and by the share of services in GDP. The p-values are reported in parentheses. *** Significant at the 1-percent level; ** Significant at the 5-percent level; * Significant at the 10-percent level.

Empirical Observations

Here, I present the main empirical observations that motivate the theoretical model. These observations relate countries' production network diversification to countries' HHI sectoral sales shares and aggregate volatility. I divide the sample into three sub-periods: 1970-1995, 1996-2005, and 2006-2014.[15]

Empirical Observation 1: The HHI of sectoral sales shares declines with production network diversification.

The HHI of sectoral sales shares is defined as follows:

[14]The definition of service sectors includes OECD sectors 19, 21-33 in Table 8.

[15]Given that the OECD input-output data cover the period 1995-2011, when official data are available, I use the end of period HHI and production network measures. As explained in footnote 7, when official input-output data are not available at the end of the sub-period, I use the official data for the year that is closest to the end of period.

$$HHI = \sqrt{\sum_{j=1}^{N} \left(\frac{S_j}{GDP}\right)^2},$$

where S_j are sector j's sales.

The relationship between the HHI of sectoral sales shares and production diversification is illustrated in Table 2. There is a strong negative relationship between production network diversification ($Density$ and $Divers$) and the HHI of sectoral sales shares. Moreover, as Acemoglu et al. (2012) predicts, there is a strong positive relationship between the concentration of sectoral outdegrees ($Outdegree$) and the HHI of sectoral sales shares. Industries that are more central suppliers of intermediates are also larger in equilibrium. Therefore, a more concentrated $Outdegree$ distribution generates a more concentrated HHI of sectoral sales shares. In Appendix A (Tables 10-13), I show that the same results hold when controlling for GDP per-capita and when using total production network measures that include imported intermediates.

Table 2
Herfindahl and Production Network Structure

VARIABLES	(1) $\log HHI_T^k$	(2) $\log HHI_T^k$	(3) $\log HHI_T^k$	(4) $\log HHI_T^k$
log Service	-0.233*			
	(0.120)			
log Density		-0.552***		
		(0.118)		
log Divers			-0.603***	
			(0.125)	
log Outdegrees				1.265***
				(0.293)
Observations	126	126	126	126
Adjusted R-squared	0.041	0.229	0.205	0.273

Note: This table presents an OLS regression using the log Herfindahl index of sales shares as the dependent variable. The independent variables describe each country's production network structure and the service share. The sample is divided into three sub-periods: 1970-1995, 1996-2005, and 2006-2014. Robust standard errors in parentheses. *** Significant at the 1-percent level; ** Significant at the 5-percent level; * Significant at the 10-percent level.

Empirical Observation 2: GDP growth volatility declines with production network diversification.

Volatility is measured using the standard deviation of GDP growth for each sub-period. Table 3 illustrates the relationship among volatility, production network diversification, and the service share. There is a strong negative relationship between production network diversification and GDP growth volatility. Table 3 also shows that—consistent with the previous literature—volatility increases with the HHI of sectoral sales shares (Gabaix (2011) and Carvalho and Gabaix (2013)) and the concentration of *Outdegrees* (Acemoglu et al. (2012)) but decreases with the service share of the economy (Carvalho and Gabaix (2013) and Moro (2015)). The same results hold when controlling for GDP per-capita and when using total production network measures that include imported goods (see Tables 11, 14, and 15 in Appendix A).

Table 3
Volatility and Production Network Structure

VARIABLES	(1) $\log Vol_T^k$	(2) $\log Vol_T^k$	(3) $\log Vol_T^k$	(4) $\log Vol_T^k$	(5) $\log Vol_T^k$
log HHI	0.524***				
	(0.141)				
log Service		-0.522**			
		(0.241)			
log Density			-0.621**		
			(0.245)		
log Divers				-0.646**	
				(0.274)	
log Outdegrees					0.775*
					(0.406)
Observations	126	126	126	126	126
Adjusted R-squared	0.042	0.036	0.046	0.036	0.011

Note: This table presents an OLS regression using real GDP growth volatility as the dependent variable. The independent variables describe each country's domestic production network structure, the HHI of sectoral sales shares, the service share, and the interaction between production network diversification and the service share. The sample is divided into three sub-periods: 1970-1995, 1996-2005, and 2006-2014. Robust standard errors in parentheses. *** Significant at the 1-percent level; ** Significant at the 5-percent level; * Significant at the 10-percent level.

Overall, the facts presented in this section contribute to understanding the drivers of countries' HHI of sales shares and macroeconomic volatility. While these facts support the predictions in Gabaix (2011), Acemoglu et al. (2012), Carvalho and Gabaix (2013), and Moro (2015), they also pose a challenge to existing multisector models with intersectoral linkages. The existing literature predicts no relationship between production network diversification and aggregate volatility. Therefore, the next section aims to give economic interpretation to the cross-country facts presented in this section.

3 The model economy

The model economy in this paper extends the model in Acemoglu et al. (2012) to allow for general CES production technologies, as in Atalay (2017), Carvalho et al. (2016), and Baqaee and Farhi (2019). Unlike the aforementioned papers, this paper highlights the theoretical link between production network diversification and volatility, via the existence of a cost of complexity in the bundle of intermediates, which operates when the elasticity of substitution between labor and intermediates is non-unitary.

Firms

There are N sectors, each of which has a continuum of homogeneous firms that behave competitively. The CES technology of firms in sector j is:[16]

$$Q_j = Z_j\left(a_j L_j^{\frac{\epsilon_Q - 1}{\epsilon_Q}} + (1 - a_j) M_j^{\frac{\epsilon_Q - 1}{\epsilon_Q}}\right)^{\frac{\epsilon_Q}{\epsilon_Q - 1}}, \tag{4}$$

where the intermediate input bundle is

$$M_j = \left(\sum_{i=1}^{N} \omega_{ij} M_{ij}^{\frac{\epsilon_M - 1}{\epsilon_M}}\right)^{\frac{\epsilon_M}{\epsilon_M - 1}}. \tag{5}$$

The output of the representative firm in sector j is denoted by Q_j. Z_j is total factor productivity; L_j is labor; M_j is the intermediate input bundle of sector j; and M_{ij} is the amount of intermediates that sector j purchases from sector i. The parameter a_j represents how important labor is in the total value of production. The element ω_{ij} reflects the importance of sector i as an input supplier to sector j. Therefore, the square matrix Ω—of dimension N and typical element ω_{ij}—represents the input-output structure of the economy. Note that when $\epsilon_M = 1$, ω_{ij} is also the observed cost share of intermediates from sector i in sector's j total intermediate expenditure.

The elasticity of substitution between labor and intermediates is denoted by ϵ_Q. The elasticity of substitution among material varieties is ϵ_M.

Households

The representative household maximizes utility

[16] This is the original CES production technology derived by Arrow et al. (1961).

$$U(C_1,...,C_N) = \Big(\sum_{j=1}^{N} \beta_j C_j^{\frac{\epsilon_D - 1}{\epsilon_D}}\Big)^{\frac{\epsilon_D}{\epsilon_D - 1}}, \tag{6}$$

subject to the budget constraint

$$w\bar{L} + \sum_{j=1}^{N} \pi_j = \sum_{j=1}^{N} P_j C_j, \tag{7}$$

where C_j is the consumption of sector j's output. The consumption shares β_j satisfy $\sum_{j=1}^{N} \beta_j = 1$. The elasticity of substitution between goods from different sectors is ϵ_D. In the household budget constraint, $\bar{L}$ is labor supply (inelastically supplied); π_j is profit from firms in sector j; w is the wage rate; and P_j is the price of sector j's good.

Definition 1 *(Competitive Equilibrium) A decentralized competitive equilibrium is a set of prices* $\{w, (P_j)_j^N\}$ *and allocations* $\{(C_j, Q_j, M_j, L_j)_j^N\}$, $\{(M_{ij})_{ij}^N\}$ *such that, for a given vector of sectoral productivity shocks* $\{Z_j\}_{j=1}^N$ *and prices:*

- *the representative consumer maximizes utility* (6) *subject to the budget constraint* (7);
- *firms maximize profits; and*
- *the goods and labor markets clear:*

$$Q_j = C_j + \sum_{i=1}^{N} M_{ji},$$

$$\bar{L} = \sum_{j=1}^{N} L_j.$$

Notation

Let β, Z, and a be the $N \times 1$ vector of consumption shares, the $N \times 1$ vector of sectoral productivities, and the $N \times 1$ vector containing the importance of labor in each sector's technology, respectively. An expression $e \circ f$, where e and f are vectors of the same dimension, should be interpreted as an element-by-element multiplication (Hadamard product). An expression e^f should be interpreted as an element-wise exponent.

3.1 The network amplification

I study the general model's implications for the relationship between the network structure and aggregate GDP. The next proposition establishes the relationship between the network structure and sectoral centrality. Sector j's centrality is defined by the GDP effect of a productivity shock to sector j (Z_j). Also, define $\tilde{\Gamma}$ as the N by N matrix of model-implied intermediate input-output shares, where an element $\{\tilde{\Gamma}\}_{ij} = \tilde{\gamma}_{ij}$ represents the cost share of intermediate input expenses from sector i in sector j's gross output.

Proposition 1 *Assume that $\epsilon_Q \neq 1$, $\epsilon_D = 1$, $Z_j = 1$ for all j, and a labor endowment $\bar{L} = 1$.*

(i) When $\epsilon_M = \epsilon_Q$, the vector of sectoral centrality, as well as the vector of sectoral sales shares, is

$$s = [I - \tilde{\Gamma}]^{-1}\beta = [I - P^{1-\epsilon_Q} \circ \left((P)^{\epsilon_Q - 1} 1'\right)' \circ \left(((1-a)1')' \circ \Omega\right)^{\epsilon_Q}]^{-1}\beta, \tag{8}$$

while the vector of sectoral prices is

$$P^{1-\epsilon_Q} = [I - \left((1-a)^{\epsilon_Q} 1' \circ \Omega'^{\epsilon_Q}\right)]^{-1}(a^{\epsilon_Q}),$$

(ii) When $\epsilon_M = 1$, the vector of sectoral network centrality, also the vector of sectoral sales shares, is

$$s = [I - \tilde{\Gamma}]^{-1}\beta = [I - (\overline{P} \circ (1-a)^{\epsilon_Q} 1')' \circ \Omega]^{-1}\beta, \tag{9}$$

where $\overline{P} = \left(\frac{P}{P^M}\right)^{\epsilon_Q - 1}$ and the jth element of P^M is $P_j^M = \prod_{i=1}^N \left(\frac{P_i}{\omega_{ij}}\right)^{\omega_{ij}}$. The vector of sectoral prices are

$$(1-\epsilon_Q) \circ \log P = \log\left(a^{\epsilon_Q} \circ + (1-a)^{\epsilon_Q} \circ exp[(1-\epsilon_Q) \circ (\Omega' \log P - diag(\Omega' \log \Omega))]\right).$$

Proof: See Appendix B.

Proposition 1 extends the results in Acemoglu et al. (2012). Sectoral centrality (or influence) is determined by how interconnected a sector is with the rest of the economy, which is summarized by the vector of extended Leontief inverse elements in Eq. (8). The extended Leontief inverse depends on the model-implied intermediate input shares $\tilde{\gamma}_{ij}$, which, in turn, depend on the input-output structure Ω, the elasticity ϵ_Q, and the vector of distribution parameters a. Note that, as implied by Hulten (1978), up to a first order, the vector of sectoral centrality is also equal to the vector of sectoral sales shares.

To directly link the predictions of the model to the empirical evidence presented in Section 2, I now formally study the relationship among the production network structure, the HHI of sectoral sales shares, and GDP volatility, emphasizing the role of production network diversification. Assume that sectoral productivity follows a random walk:

$$\log Z_{jt} = \log \bar{Z} + \log Z_{jt-1} + \kappa_{jt}, \tag{10}$$

where the productivity shock κ is normally distributed with mean zero and standard deviation σ_j. The term $\bar{Z}$ is the steady-state level of technology, assumed to be common across sectors.

Proposition 2 *Define σ_{GDP} the volatility of the log change of real GDP. Assume that $\epsilon_Q \neq 1$; $\epsilon_D = 1$; steady-state sectoral productivity is $\bar{Z} = 1$ for all j; and a labor endowment $\bar{L} = 1$. Assume that sectoral productivity shocks κ_{jt} are independent and have a volatility of σ_j. Then, up to a first order,*

(i) when $\epsilon_Q = \epsilon_M \neq 1$, the volatility of the log change of GDP is:

$$\sigma_{GDP} = \sqrt{\sum_{j=1}^{N} \Big([I - P^{1-\epsilon_Q} \circ \Big((P^{\epsilon_Q - 1} 1')' \circ \Big(((1-a)1')' \circ \Omega \Big)^{\epsilon_Q}]^{-1} \beta \Big)_j \sigma_j^2}, \tag{11}$$

(ii) when $\epsilon_Q \neq 1$ and $\epsilon_M = 1$, the volatility of the log change of GDP is:

$$\sigma_{GDP} = \sqrt{\sum_{j=1}^{N} \Big([I - (\overline{P} \circ (1-a)^{\epsilon_Q} 1')' \circ \Omega]^{-1} \beta \Big)_j \sigma_j^2}, \tag{12}$$

where $\overline{P}$ is defined in Proposition 1.

Proof: See Appendix B.

Proposition 2 shows that GDP growth volatility is determined by the HHI index of extended Leontief inverse elements in Proposition 1. By the Hulten theorem, aggregate volatility is also equal to the HHI index of sectoral sales shares. In the next section, I study the role of production network diversification embedded in Proposition 1 and Proposition 2.

The role of production diversification

To better examine the role of production diversification, I analyze two symmetric networks. In symmetric networks, the matrix Ω, which represents the network structure of the economy, displays homogeneous row sums (first-order outdegree). One useful feature of these networks is that—within network and given common sectoral a, ϵ_Q, and ϵ_M—sectoral prices are the same across sectors. In this case, the vector of sectoral centrality in Proposition 1, part i) becomes

$$s = [I - \Big(((1-a)1')' \circ \Omega\Big)^{\epsilon_Q}]^{-1}\beta,$$

while the vector of sectoral centrality in Proposition 1, part ii) becomes

$$s = [I - (Divers^{1-\epsilon_Q} \circ (1-a)^{\epsilon_Q} 1')' \circ \Omega]^{-1}\beta,$$

in which an element i of the vector $Divers$ is $Divers_i = \prod_{j=1}^{N} \omega_{ji}^{-\omega_{ji}}$. This sectoral diversification measure corresponds to the one used in Section 2, Eq. (3), for the case when $\epsilon_M = 1$. We can see that when $\epsilon_Q > 1$, higher production diversification reduces sectoral centrality.

I define the following symmetric networks:

$$\Omega^{sparse} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad \text{and} \qquad \Omega^{denser} \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}.$$

Assume that $\epsilon_M = 1$, $a = [0.6, 0.6, 0.6]$ and $\beta = [1/3, 1/3, 1/3]$ are common across networks. According to Proposition 1, part ii), when $\epsilon_Q = 1.2$, the vector of sectoral centralities is

$$s^{sparse} = [0.50, 0.50, 0.50] \qquad s^{denser} = [0.47, 0.48, 0.49],$$

while when $\epsilon_Q = 0.8$, we have

$$s^{sparse} = [0.64, 0.64, 0.64] \qquad s^{denser} = [0.73, 0.71, 0.68].$$

We can see that the denser network displays lower sectoral centralities when intermediates and labor are substitute inputs. However, when intermediates and labor are complement inputs the denser network displays larger sectoral centralities instead.[17] The reason, which I explain in more detail in the next section, is the existence of a cost of complexity in the bundle of intermediates. When $\epsilon_Q > 1$, diversified firms can use more labor to compensate for the complexity of the bundle, which then reduces their intermediate input share and, therefore, their sales share. On the other hand, when $\epsilon_Q < 1$, diversified firms cannot fully compensate for the complexity of the bundle, which then increases their intermediate input share and sales share.

It is interesting to examine the network effects. In the denser network, sector 1 is the only sector providing intermediates to two sectors and using intermediates from two sectors. Therefore, besides its own complexity effect, the fact that other sectors demand

[17] The same result holds with $\epsilon_Q = \epsilon_M \neq 1$ (Proposition 1, part i)). I use Proposition 1, part ii) to highlight that the key elasticity is ϵ_Q, as opposed to ϵ_M.

less (more) when $\epsilon_Q > 1$ ($\epsilon_Q < 1$) further reduces (increases) sector 1's size. For example, when $\epsilon_Q > 1$, the other downstream complex sector (sector 2) uses fewer intermediates from sector 1, implying that sector 1 is the smallest sector in equilibrium. This effect is not present for sector 2, as sector 2 supplies only to the non-diversified sector 3. The opposite happens when $\epsilon_Q < 1$.

These differences in network centralities also imply differences in volatilities. Based on Proposition 2, when labor is easily substitutable with intermediates, the denser network is less volatile. However, when inputs are complements, a denser network displays larger sectoral centralities and, therefore, higher volatility.

Note that, as Dupor (1999) and Acemoglu et al. (2012) predict, when $\epsilon_Q = 1$, both networks behave the same. In fact, when $\epsilon_Q = 1$, we have

$$s^{sparse} = [0.56, 0.56, 0.56] \qquad s^{denser} = [0.56, 0.56, 0.56].$$

Two symmetric networks that differ only in terms of their production diversification display the same sectoral centralities and aggregate volatility.

3.2 Cost of Complexity

To understand the mechanism by which production network diversification affects shocks propagation, it is instructive to examine the cost of complexity embedded in Equation (5). For the sake of exposition, and to compare the results of this paper with the implied results from other papers, I generalize Equation (5) as follows:

$$M_j = \left(\sum_{i=1}^{N} \omega_{ij}^{\varrho_M} M_{ij}^{\frac{\epsilon_M - 1}{\epsilon_M}} \right)^{\frac{\epsilon_M}{\epsilon_M - 1}}, \tag{13}$$

where the new parameter ϱ_M controls the so-called cost of complexity. Recall that $\varrho_M = 1$ in standard CES production functions (as in Eq. 5). To clearly observe the trade-off between a concentrated intermediate input bundle versus a diversified one, one can compare a firm in sector j using only one intermediate input in production versus a firm in sector j using all of the N intermediate inputs available in production (diversified firm). In the first case, $\omega_{ij} = 1$ for the main input and $\omega_{kj} = 0$ for all $k \neq i$. The diversified firm, instead, has $\omega_{ij} = 1/N$ for all i.

Suppose, also, that for the non-diversified firm, we have $M_{ij} = M_1 > 0$ and $M_{kj} = 0$ for all $k \neq i$, while for the diversified firm, we have $M_{ij} = M_2 = \frac{1}{N} M_1$ for all i. Equation (13) indicates that firm 1's intermediate input bundle is equal to $M_j^1 = M_1$. On the other hand, the intermediate input bundle of the diversified firm is

$$M_j^2 = (\omega_{1j}^{\rho_M} \cdot M_2^{\rho_M} + ... + \omega_{Nj}^{\rho_M} \cdot M_2^{\rho_M})^{1/\rho_M},$$

where $\rho_M = \frac{\epsilon_M - 1}{\epsilon_M}$. Using the assumption of fully diversified input-output weights ($\omega_{ij} = \frac{1}{N}$ for all i) and that $M_2 = \frac{1}{N} M_1$, we obtain

$$M_j^2 = \Big((\frac{1}{N})^{\rho_M} M_2^{\rho_M} N \Big)^{1/\rho_M} = M_2 N^{\frac{1-\rho_M}{\rho_M}} = M_1 N^{\frac{1-\rho_M - \rho_M}{\rho_M}}.$$

We can see that when $\frac{1-\rho_M-\rho_M}{\rho_M} = 0$, the concentrated bundle firm and the completely diversified firm produce exactly the same (M_1). This is the case in the implicit normalization case $\rho_M = 1 - \rho_M = \frac{1}{\epsilon_M}$. There exists a cost of complexity in the bundle of intermediates whenever $\frac{1-\rho_M-\rho_M}{\rho_M} < 0$. In this case, the firm with the diversified bundle produces $M_j^2 = M_1 N^{\frac{1-\rho_M-\rho_M}{\rho_M}}$ smaller than M_1 (when $N > 1$). More generally, a cost of complexity exists ($\frac{1-\rho_M-\rho_M}{\rho_M} < 0$) when

$$\rho_M > \frac{1}{\epsilon_M} \quad when \quad \epsilon_M > 1,$$

and

$$\rho_M < \frac{1}{\epsilon_M} \quad when \quad \epsilon_M < 1.$$

In any case, regardless of the value of ϵ_M, the standard CES production function ($\rho_M = 1$) meets both conditions above and implies a cost of complexity in the intermediate input bundle. Note that exactly the same complexity cost exists when $\epsilon_M = 1$ (see Appendix C).

To link production diversification with production complexity, it is useful to think of an example. Consider the production of educational services. In some countries, schools and universities offer educational services using few intermediate input types (e.g., paper, blackboard, and chalk) in large quantities (concentrated bundle), while in other countries, educational services are more sophisticated and require more intermediate input types (e.g., paper, blackboard, chalk, projectors, microphones, iPads, Apple Pens, and learning soft-wares.) in relatively smaller amounts (diversified bundle). Therefore, a more-diversified production network is also a network that produces more-complex goods.

The role of ϵ_Q

The cost of complexity manifests in the relative price of the intermediate input bundle P_j^M / P_j, which can have important effects when ϵ_Q is non-unitary. In symmetric networks, in which sectoral prices P_j are the same across sectors, the intermediate input price index of the simple bundle firm ($\omega_{ij} = 1$ and $\omega_{lk} = 0$ for $(l,k) \neq (i,j)$) is

$$\frac{P_j^{M,1}}{P_j} = \frac{1}{P_j}\Big(\sum_{i=1}^{N}\omega_{ij}^{\epsilon_M}P_i^{1-\epsilon_M}\Big)^{\frac{1}{1-\epsilon_M}} = \frac{P_i}{P_j}\Big(\sum_{i=1}^{N}\omega_{ij}^{\epsilon_M}\Big)^{\frac{1}{1-\epsilon_M}} = Divers_j^1 = 1,$$

while for the diversified firm,

$$\frac{P_j^{M,2}}{P_j} = \frac{1}{P_j}\Big(\sum_{i=1}^{N}\omega_{ij}^{\epsilon_M}P_i^{1-\epsilon_M}\Big)^{\frac{1}{1-\epsilon_M}} = \frac{P_i}{P_j}\Big(\sum_{i=1}^{N}\omega_{ij}^{\epsilon_M}\Big)^{\frac{1}{1-\epsilon_M}} = Divers_j^2 = N.$$

Higher production diversification of firm 2 implies a higher relative cost of the intermediate input bundle. We can see the implications of this complexity cost in the optimality conditions for M_j:

$$\frac{P_j^M M_j}{P_j Q_j} = (1-a_j)^{\epsilon_Q}\Big(\frac{P_j^M}{Z_j P_j}\Big)^{1-\epsilon_Q}. \tag{14}$$

Given a_j, Z_j, when firms use the bundle of intermediates and labor as gross substitutes ($\epsilon_Q > 1$), firms in a denser network are able to use more labor to compensate for the more complex production process. Hence, a denser network displays smaller sectors with a lower intermediate input share, which then mitigate the effect of sectoral shocks along the production chain, as implied by Proposition 1. When firms have low production flexibility ($\epsilon_Q < 1$), in equilibrium, a denser network displays larger intermediate shares and larger sectors, which then amplify the effect of sectoral shocks and increase volatility.

Relationship to the literature

Atalay (2017) and Carvalho et al. (2016) also build a multisector model with CES technologies. While the authors focus on a different question, these two studies assume the knife-edge case of $\varrho_M = 1/\epsilon_M$. This assumption is an implicit normalization that eases the calibration of the distribution parameters. However, as seen above it eliminates the role of production network diversification by linking the model implied intermediate input share with the distribution parameter $1-a$, entirely (as in the Cobb-Douglas case).

Baqaee and Farhi (2019) also build a production network model with general CES technologies. The authors focus on understanding what features of the network shape the change in sales shares, as opposed to the sales share level. Baqaee and Farhi (2019) use normalized-CES technologies, which also ease calibration by pinning down the intermediate input share entirely by $1-a$ (at the normalization point), leaving no role for production diversification.

Therefore, the normalized CES technologies in Atalay (2017), Carvalho et al. (2016), and Baqaee and Farhi (2019) are observationally equivalent to a production function with a cost of complexity, if we only consider factor shares. The empirical difference arises in the prediction that there is a specific relationship between equilibrium observed intermediate shares and production diversification which comes out of the cost of complexity. Such a relationship could also be rationalized by a normalized CES, in reduced-form through the distribution parameter, but without explanation.

Finally, Acemoglu and Azar (2020) develop a model of an endogenous production network to study the role of network density in economic growth. The CES technology used in Acemoglu and Azar (2020) follows Equation (5) (with $\rho_M = 1$) and defines $M_{ij} = A_{ij}\tilde{M}_{ij}$, where A_{ij} is the input-specific productivity and $\tilde{M}_{ij}$ the quantity of the input used. Therefore, the authors have a cost of complexity in the bundle of intermediates, in efficiency unit terms ($M_{ij} = A_{ij}\tilde{M}_{ij}$). In their set-up, when firms adopt more varieties (more-diversified intermediate input bundle), they consider the input productivity A_{ij} and also that the overall gain in the new diversified bundle defies the cost of complexity.

4 Testing the model implications with sectoral data

This section has three goals. First, it aims to estimate the elasticities of substitution between inputs (ϵ_M and ϵ_Q). Second, it assesses the relationship between sectoral intermediate input diversification and the intermediate input share. Third, it studies production diversification in service sectors.

4.1 Estimation of elasticities

To estimate the elasticities, I follow Atalay (2017) and use the model's implied cost minimization condition for inputs. While Atalay (2017) uses the implicitly normalized CES, instead of the original Arrow et al. (1961) CES, both models have the following implied equation for log changes in intermediate input shares (see details in Appendix E)

$$\Delta \log\left(\frac{P_{it}M_{ijt}}{P_{jt}Q_{jt}}\right) = (\epsilon_M - 1)\Delta \log\left(\frac{P_{jt}^M}{P_{it}}\right) + (\epsilon_Q - 1)\Delta \log\left(\frac{P_{jt}}{P_{jt}^M}\right) + (\epsilon_Q - 1)\Delta \log Z_{jt}, \qquad (15)$$

where the elasticity between intermediates (ϵ_M) is pinned down by changes in the relative price between the intermediate input bundle price and the price of a given intermediate input (P_j^M/P_i). The elasticity between labor and intermediates (ϵ_Q) is pinned down by changes in the relative price of sectoral output and the price of the bundle (P_j/P_j^M).

The empirical counterpart of Eq. (15) is:

$$\Delta \log\left(\frac{P_{it}M_{ijt}}{P_{jt}Q_{jt}}\right) = \gamma_{ij} + \alpha \Delta \log\left(\frac{P_{jt}^M}{P_{it}}\right) + \beta \Delta \log\left(\frac{P_{jt}}{P_{jt}^M}\right) + \nu_{ijt}, \qquad (16)$$

where γ_{ij} is a buyer-seller fixed effect that aims to control for unobserved time-invariant intermediate-input trade partner relationships. The elasticity between intermediates is $\epsilon_M = 1 + \alpha$, while the elasticity between value-added and intermediates is $\epsilon_Q = 1 + \beta$. The error term is denoted by ν_{ijt}. Estimating Equation (16) via OLS would yield biased coefficients due to the endogeneity of sectoral prices. Sectoral productivities (Z_j) are part of the error term ν_{ijt} and are correlated with sectoral prices (P_j, P_i, P_j^M).

Therefore, I use military spending and the input-output structure to create three instruments for sectoral prices (see Atalay (2017) and Miranda-Pinto and Young (2019) for more details). Since the instruments are relevant to the United States, I assume that sectoral technologies in the U.S are similar to sectoral technologies elsewhere. I use BEA annual input-output data on sectoral prices and intermediate shares. The data contain 66 non-government sectors of the economy and cover the period 1997-2018.[18]

Guided by Miranda-Pinto and Young (2019), I allow for the elasticities to differ across sectors service and non-service sectors. Service sectors are sectors 6 and 27-66 in Table 9. This classification is consistent with the service-sector classification in Section 2 (sectors 19, 21-33 in the OECD data in Table 8). Non-service sectors represent the rest of the economy (sectors 1-5 and 7-26 in Table 9 and sectors 1-18 and 20 in the OECD data in Table 8).

Table 4 reports the estimated IV elasticities for all sectors, for non-service sectors, and for service sectors. The instruments are strong, indicated by a large Cragg-Donald F statistic of weak identification in each column. The first stage coefficients are also consistent with military spending working as a demand shifter. For example, the instrument for P_j is positively correlated with $\Delta \log\left(\frac{P_{jt}}{P_{jt}^M}\right)$, indicating that increased demand from the military to sector j tends to increase the price of sector j. The same applies for the relationship between the instruments for P_j^M and P_i.

The second-stage results in column 1 of Table 4 are similar to findings by Atalay (2017). Note that the null hypothesis is that production elasticities are unitary (e.g., $H_0 : \epsilon_Q - 1 = 0$). Under the assumption of homogeneous production elasticity across sectors, the elasticity of substitution between different intermediates is close to zero, while the elasticity of substitution between labor and intermediates is statistically not different from one.[19] However, as in Miranda-Pinto and Young (2019), important differences arise when we relax the assumption of homogeneous elasticities across sectors. Non-service sectors display very low substitutability between intermediates, $\epsilon_M \approx 0$, and unitary elasticity between intermediates and labor, $\epsilon_Q \approx 1$. On the other hand, service sectors are

[18]For each sector, I keep the top 25 intermediate goods' supplier sectors. The results are similar when using the top 20 or 30 suppliers.

[19]While the point estimate of ϵ_M is negative (-0.1), the upper-end value of the 95 percent confidence interval of ϵ_M is 0.35

significantly more flexible in production. In particular, service sectors display $\epsilon_M \approx 1$ and a point estimate of ϵ_Q of 3.38.[20]

Table 4
Estimated Elasticities

Second stage	(1) All	(2) Non-services	(3) Services
$\epsilon_M - 1$	-1.10***	-1.23***	-0.45
	(0.232)	(0.228)	(0.591)
$\epsilon_Q - 1$	0.34	-0.38	2.38**
	(0.539)	(0.490)	(0.927)
First stage $\Delta \log\left(\frac{P^M_{jt}}{P_{it}}\right)$			
P^{IV}_j	-0.47	-5.19***	0.52
	(0.399)	(0.918)	(0.403)
$P^{M,IV}_j$	8.45***	17.82***	3.95***
	(0.57)	(1.33)	(0.572)
P^{IV}_i	-5.58***	-9.91***	-3.30***
	(0.389)	(0.879)	(0.386)
F statistic	125.41	94.16	30.31
First stage $\Delta \log\left(\frac{P_{jt}}{P^M_{jt}}\right)$			
P^{IV}_j	2.33***	6.57***	0.40**
	(0.220)	(0.536)	(0.201)
$P^{M,IV}_j$	-3.93***	-8.82***	-2.61***
	(0.317)	(0.778)	(0.284)
P^{IV}_i	0.06	0.13	0.22**
	(0.214)	(0.513)	(0.192)
F statistic	76.54	59.52	49.92
Observations	32,980	12,500	20,480
Cragg-Donald F Statistic	52.63^r	36.24^r	24.06^r

Note: This table reports the estimated coefficients of Eq. (16), using IV and military spending instruments. In the row labeled Cragg-Donald Statistic, the superscript "r" indicates that the test for a weak instrument is rejected at the 10-percent threshold. Standard errors are reported in parentheses. *** Significant at the 1-percent level; ** Significant at the 5-percent level; * Significant at the 10-percent level.

4.2 Sectoral diversification and intermediate input shares

In this section, I use cross-country sectoral data to study whether production diversification is associated with the intermediate input share as predicted by the model. The

[20] The 95 percent confidence interval of service sectors ϵ_Q is $[1.56, 5.2]$.

logarithm of model-implied intermediate input share in Equation (14), around $Z_j = 1$ for all j, is:

$$\log(\frac{P_j M_j^M}{P_j Q_j}) = \epsilon_Q \log(1-a_j) + (1-\epsilon_Q)\log(P_j^M/P_j), \tag{17}$$

where the second term in Eq. (17) can become zero for two reasons: i) because $\epsilon_Q = 1$; or ii) because $\rho_M = 1/\epsilon_M$, in which case, $P_j^M = P_j$ for all j. In these cases, the model-implied intermediate input share is determined entirely by the distribution parameter $(1-a_j)$. As discussed in Section 3, production diversification affects the relative price of the intermediate input bundle (P_j^M/P_j). To better illustrate this relationship, and for the sake of exposition, assume that $\epsilon_M = 1$ (which is, indeed, accurate for service sectors in Table 4). In this case, the second term in Eq. 17 can be decomposed into

$$\log \frac{P_j^M}{P_j} = -\log P_j + \sum_{i=1}^{N} \omega_{ij} \log P_i - \sum_{i=1}^{N} \omega_{ij} \log \omega_{ij},$$

implying

$$\log(\frac{P_j M_j^M}{P_j Q_j}) = \epsilon_Q \log(1-a_j) + (1-\epsilon_Q)\Big(-\log P_j + \sum_{i=1}^{N} \omega_{ij} \log P_i\Big) + (1-\epsilon_Q)(-\sum_{i=1}^{N} \omega_{ij} \log \omega_{ij}). \tag{18}$$

In symmetric networks, the term $\Big(-\log P_j + \sum_{i=1}^{N} \omega_{ij} \log P_i\Big)$ is zero, and production diversification, captured by $\log Divers_j = -\sum_{i=1}^{N} \omega_{ij} \log \omega_{ij}$, accounts for differences in the intermediate input share, given a_j and ϵ_Q. In non-symmetric networks, the model-implied prices depend on the input-output structure Ω (as seen in Proposition 1).

Thus, investigating the causal link between sectoral production diversification and intermediate input shares is complex due to unobserved distribution parameter (a_j) and sectoral prices. Therefore, the goal of this section is simply to provide suggestive evidence of the mechanism by looking at the correlation between production diversification and intermediate input share. I use cross-country sectoral data to estimate the following empirical specification of Eq. (18):

$$\log\Big(\frac{P_{jkt} M_{jkt}^M}{P_{jkt} Q_{jkt}}\Big) = \alpha_j + \beta_k + \eta \log Divers_{jkt} + \epsilon_{jkt}, \tag{19}$$

where α_j and β_k are sector and country fixed-effects that control for unobserved sectoral characteristics—in this case, the distribution parameter $(1-a_{jk})$ and the term $-\log P_j + \sum_{i=1}^{N} \omega_{ij} \log P_i$ (if time-invariant). The key coefficient to be estimated is η, which underlines the relationship between sectoral production diversification and the intermediate input share. For sectors with $\epsilon_Q > 1$, η is expected to be negative.

The estimation of Eq. (19) without sectoral fixed-effects uses cross-sectoral heterogeneity in $Divers_{jk}$ to identify the relationship between diversification and intermediate input shares.[21] On the other hand, the estimation of Eq. (19) using sectoral fixed-effects uses time variation in $Divers_{jkt}$ to capture the relationship between $Divers_{jkt}$ and intermediate input shares. In the regression, I proxy for production diversification using

$$\log Divers_j = \log(\sum_{i=1}^{N} \omega_{ij}^{\epsilon_M})^{\frac{1}{1-\epsilon_M}}, \tag{20}$$

with $\epsilon_M = 0.5$.[22]

Table 5 reports the results of estimating Eq. (19) for service and non-service sectors. Column 1 reports a negative cross-sectional relationship between production diversification and the intermediate input share (significant at the 95% confidence level). Column 2 shows that this relationship is non-existent in non-service sectors. Columns 3 and 4 control for sectoral fixed-effects and show the same results. The negative relationship between production diversification and intermediate input shares is observed only within service sectors. Overall, the results in Tables 4 and 5 support the mechanism of the model: in high ϵ_Q sectors, production diversification is associated with lower intermediate input shares.

Table 5
Intermediate Input Shares and Production Diversification

VARIABLES	(1) Services	(2) Non-services	(3) Services	(4) Non-services
log Divers	-0.129**	-0.003	-0.186***	0.044
	(0.052)	(0.040)	(0.047)	(0.050)
Observations	1,764	2,394	1,764	2,394
Adjusted R-squared	0.206	0.378	0.554	0.539
Country FE	Yes	Yes	Yes	Yes
Sector FE	No	No	Yes	Yes

Note: This table presents the regression results of estimating Eq. (19). The dependent variable is the log of sectoral total intermediate input shares. Divers is measured using Eq. (20), with $\epsilon_M = 0.5$. Standard errors are in parentheses. *** Significant at the 1-percent level; ** Significant at the 5-percent level; * Significant at the 10-percent level.

[21] Note that omitted variable bias causes the OLS estimate to be inconsistent

[22] The same results hold when using the diversification measure implied by $\epsilon_M = 1$, $\log Divers_j = -\sum_{i=1}^{N} \omega_{ij} \log \omega_{ij}$. The same is also true when using $\epsilon_M \approx 0$ for non-service sectors and $\epsilon_M = 1$ for service sectors.

4.3 Production Diversification of Service Sectors

This final section shows that the well-documented fact that service sectors display lower intermediate input shares (Moro (2015)) can be explained, in part, by the fact that service sectors display high production diversification, as measured by *Divers*. Column 1 of Table 6 confirms that service sectors have a smaller intermediate input share. Columns 2 and 3 show that, consistent with the results in Table 1, service sectors have more-diversified intermediate input bundles. These results indicate that, through the lens of the model, high elasticity and high diversification can help explain why service sectors display a lower intermediate input share and, therefore, why service-oriented countries are less volatile.

Table 6
Production Diversification and Service Share

VARIABLES	(1) $\log P_j^M M_j / P_j Q_j$	(2) $\log Divers_{(\epsilon_M=0.5)}$	(3) $\log Divers_{(\epsilon_M=1)}$
Services	-0.325***	0.016*	0.038***
	(0.012)	(0.009)	(0.014)
Observations	4,158	4,158	4,158
Adjusted R-squared	0.343	0.315	0.069
Country FE	Yes	Yes	Yes
Sector FE	No	No	No

Note: Column 1 presents the estimated coefficient of a service-sector dummy as the independent variable and log intermediate input shares as dependent variable, controlling for country fixed-effects. Columns 2 and 3 present the estimated coefficient of a service-sector dummy as the independent variable and log *Divers* as the dependent variable for different values of ϵ_M and controlling for country fixed-effects.

5 Quantitative assessment

In this section, I calibrate the model using each country's input-output matrix. The goal is to study the quantitative predictions of the model regarding the empirical cross-country correlations among production diversification, service share, and volatility documented in Section 2.

Calibration

The previous section showed that the value of ϵ_M is not relevant to understanding the role of production diversification. Therefore, to simplify the computational burden and to focus on the role played by production diversification when ϵ_Q is non-unitary and different than ϵ_M, I set $\epsilon_M = 1$. I then calibrate ϵ_Q using the U.S. estimates in Table 4 for the service and non-service sectors. In particular, the benchmark calibration assumes that $\epsilon_Q = 1$ for non-service sectors and $\epsilon_Q = 1.56$ for service sectors, which is the lower-end value of the 95-percent confidence interval for ϵ_Q in column 3. Alternatively, I reestimate sectoral ϵ_Q under the assumption that $\epsilon_M = 1$ (for all sectors). The results of estimating Equation (16) under the imposition of $\epsilon_M = 1$ are reported in Table 16 of Appendix A. The 95 percent confidence intervals for ϵ_Q are: $\epsilon_Q \in [1.55, 3.49]$ for all sectors; $\epsilon_Q \in [0.98, 2.71]$ for non-service sectors ; and $\epsilon_Q \in [2.06, 5.3]$ for service sectors.

The countries' intermediate input shares ω_{ij} are matched to the observed shares in 2005 using the OECD input-output tables at a level of disaggregation of 33 sectors. The elasticity of substitution between consumption goods ϵ_D is assumed to be unitary. Therefore, the consumption shares β_j are calibrated to be the observed consumption shares in the 2005 input-output tables for each country.

The calibration of the distribution parameter a_j, which captures the importance of labor in sectoral production, deserves more discussion. With Cobb-Douglas production functions, the distribution parameter $(1-a_j)$ equals the observed intermediate input share in sector j. However, when ϵ_Q is non-unitary, and there is a role for production diversification in determining intermediate input shares, the distribution parameter differs from the observed shares. Therefore, given ϵ_Q and given Ω, I calibrate a_j using an iterative procedure that matches the model-implied sectoral intermediate input shares with the observed ones.

The process for sectoral productivity follows a random walk as in Equation (10), in which shocks κ_{jt} are independent and normally distributed with zero mean and variance σ_j^2.[23] I calibrate the variance for sectoral productivity following Horvath (2000), who uses the Jorgenson dataset to estimate sectoral productivities for the U.S. sectors at a level of disaggregation of 36 sectors. The estimates apply to annual productivity; thus, they represent a good benchmark for my analysis, as the empirical results in Section 2 are at an annual frequency and at a level of disaggregation of 33 sectors. The persistence of sectoral productivities estimated in Horvath (2000) is high and does not differ much across sectors. However, the author finds important differences in the volatility of sectoral shocks σ_j across sectors, especially between manufacturing and services. To focus on the role of the service share coming from their lower intermediate input share, the benchmark calibration assumes $\sigma = 0.02$ for service sectors and non-service sectors. To account for the heterogeneity in sectoral shocks' volatility documented in Horvath (2000), I also consider

[23] The steady-state value of productivity $Z_j = \bar{Z}$ (for all j) is irrelevant given that I study the volatility of GDP growth.

the case in which non-service sectors are more volatile and have $\sigma = 0.04$.[24]

I simulate series of sectoral productivities and aggregate GDP for every country in the sample. I then reestimate the cross-sectional relationship among the model's implied GDP volatility, the input-output structure, and the service share. In particular, I simulate series of size T for each economy S times.[25] With the implied path for real GDP, I calculate each country's times series of real GDP growth. I then calculate the standard deviation of growth for each economy and simulation. The countries' average volatility of GDP growth over the S simulations is the dependent variable. The independent variables are the model implied service share, network density, production diversification, and outdegree concentration.

Production Network Structure and Volatility

Table 7 reports the data and the model-implied relationship between production network structure and volatility. Column 1 reports the empirical relationships from Table 3 in Section 2. Columns 2 and 3 show the model-implied correlations under the assumption that service and non-service sector shocks have the same volatility. In column 2, I report the model-implied regression coefficients when ϵ_Q of service sectors (ϵ_Q^s) is 1.56 and ϵ_Q of non-service sectors (ϵ_Q^{ns}) is one. The model is quantitatively successful in matching the observed relationship between production diversification and volatility. While in the data a 10% increase in network density (diversification) is associated with a 6.21% (6.46%) decline in volatility, the model in column 2 predicts that the same 10% increase in network density (diversification) is associated with a 5.24% (5.49%) reduction in volatility. Moreover, the model is able to match the positive relationship between volatility and the concentration of sectoral outdegrees. The data suggest that a 10% increase in the concentration of outdegrees is associated with a 7.75% increase in volatility, while the model predicts a 10.09% increase in volatility.

Column 3 of Table 7 reports the model-implied relationship between production diversification and volatility when all elasticities are one. As predicted by the model, when the elasticity of substitution between labor and intermediates is one, heterogeneity in production network diversification across countries generates no cross-country differences in macroeconomic volatility. The only production network structure that shapes sales shares and volatility is the concentration of outdegrees (see the last row of column 3).

[24] Koren and Tenreyro (2007) find similar results for the volatility of sectoral shocks. Service sectors tend to be less volatile than non-service sectors.

[25] The baseline results use $T = 50$ years and $S = 50$ simulations.

Service Share and Volatility

The model-implied relationship between service shares and volatility deserves additional discussion. To guide the discussion, I decompose the expression for aggregate volatility in Equation (12), Proposition 2, following the approach in Carvalho and Gabaix (2013):

$$\sigma = \sqrt{\sum_{j=1}^{N} \left(\frac{S_j}{GDP_j} \frac{GDP_j}{GDP} \right)^2 \sigma_j^2}, \tag{21}$$

where S_j represent sector j's sales. Service sectors are different from non-service sectors in all three aspects highlighted in Equation (21). Services have lower sales to value added shares ($\frac{S_j}{GDP_j}$). Service sectors represent a larger fraction of GDP ($\frac{GDP_j}{GDP}$). Finally, services receive less-volatile shocks σ_j. Columns 2 and 3 of Table 7 shut down the effect from heterogeneous volatility of shocks highlighted in Carvalho and Gabaix (2013). As in Moro (2015), the negative relationship between the service shares and volatility is driven by the fact that services have smaller intermediate input shares and, therefore, smaller sales to value added shares. The key difference between column 2 and column 3 is the role of production diversification. When $\epsilon_Q^s > 1$, service sectors display smaller intermediate input shares and smaller sales to value added shares because of their higher production diversification (see Tables 1 and 6). Thus, the negative relationship between the service shares and volatility in column 2 is explained by the mechanism highlighted in Section 3. On the other hand, when $\epsilon_Q = 1$ for all sectors (column 3), there is no role for production diversification, and the negative relationship between service shares and volatility is a result of the implied heterogeneity in the calibrated distribution parameter a_j, necessary to match the observed intermediate input share of services.

Finally, in column 4 of Table 7, I calibrate the model, incorporating the fact that services receive less-volatile shocks. Following Horvath (2000), I assume that $\sigma_j = 0.02$ for service sectors and $\sigma_j = 0.04$ for non-service sectors. In this case, the model is more successful in quantitatively matching the relationship between services share and volatility across countries. While columns 2 and 3 predict that a 10% increase in service shares reduces volatility by 1.9%-2.7%, column 4 predicts a 6.55% reduction in volatility, which is closer to the 5.22% decline observed in the cross-country facts.

6 Conclusion

This paper provides cross-country evidence on the importance of input-output structure in accounting for the heterogeneity in cross-country HHI of sales shares and macroeconomic volatility. I show that higher production network diversification is associated with

Table 7
Production Network Structure, Service Shares, and Volatility

	(1)	(2)	(3)	(4)
VARIABLES	Data	$\sigma_s = \sigma_{ns}$	$\sigma_s = \sigma_{ns}$	$\sigma_s < \sigma_{ns}$
		$\epsilon_Q^s = 1.56$	$\epsilon_Q^s = 1$	$\epsilon_Q^s = 1.56$
log Service	-0.522**	-0.199**	-0.272***	-0.655***
	(0.241)	(0.0793)	(0.055)	(0.128)
log Density	-0.621**	-0.524**	-0.0945	-0.788***
	(0.245)	(0.200)	(0.0635)	(0.248)
log Divers	-0.646**	-0.549***	-0.112	-0.900***
	(0.274)	(0.178)	(0.0723)	(0.295)
log Outdegree	0.775*	1.089***	1.049***	1.646***
	(0.406)	(0.192)	(0.0551)	(0.349)

Note: This table presents the data and model implied OLS regression results for the relationship between production structure and volatility. All regressions include a constant. The dependent variable is GDP growth volatility, be it observed (column 1) or simulated from the model (columns 2-4). The model calibration assumes that $\epsilon_M = 1$ for all sectors and $\epsilon_Q = 1$ for non-service sectors (ϵ_Q^{ns}). The independent variables describe each country's domestic production network structure and the service shares. Robust standard errors in parentheses. *** Significant at the 1-percent level; ** Significant at the 5-percent level; * Significant at the 10-percent level.

lower HHI of sales shares and lower GDP volatility. These results stand in contrast to the prediction of existing multisector models with production linkages.

To explain these empirical observations I build a multisector model with general CES technologies that can account for these facts. I find that the production network diversification has a differential effect on volatility, depending on the flexibility in production—in particular, the elasticity of substitution between intermediates and labor. The existence of a *cost of complexity* in producing with more-diversified set of suppliers implies that a more-diversified production network mitigates shocks and reduces volatility when labor and intermediates are substitute inputs.

Using U.S data, I show that service sectors have an elasticity between labor and intermediates that is larger than one and larger than in non-service sectors. I then use cross-country sectoral data to show that, as implied by the model's mechanism, the diversification of the intermediate input bundle of flexible service sectors has a negative effect on the observed intermediate input share.

I calibrate the model to match each country's production network structure to show that, similar to the empirical results, the regressions using data simulated from the model yield a negative relationship between diversification and volatility. The model can also generate the observed negative relationship between volatility and service shares, without relying on smaller shocks affecting services. This result is explained by the higher production diversification in service sectors, which, together with the higher flexibility in services, leads to lower intermediate input shares and aggregate volatility.

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Appendix A

Table 8
OECD sectors ISIC Revision 3

Sector number	Sector name
1	Agriculture, hunting, forestry and fishing
2	Mining and quarrying
3	Food products, beverages and tobacco
4	Textiles, textile products, leather and footwear
5	Wood and products of wood and cork
6	Pulp, paper, paper products, printing and publishing
7	Coke, refined petroleum products and nuclear fuel
8	Chemicals and chemical products
9	Rubber and plastics products
10	Other non-metallic mineral products
11	Basic metals
12	Fabricated metal products
13	Machinery and equipment, nec
14	Computer, Electronic and optical equipment
15	Electrical machinery and apparatus, nec
16	Motor vehicles, trailers and semi-trailers
17	Other transport equipment
18	Manufacturing nec; recycling
19	Electricity, gas and water supply
20	Construction
21	Wholesale and retail trade; repairs
22	Hotels and restaurants
23	Transport and storage
24	Post and telecommunications
25	Financial intermediation
26	Real estate activities
27	Renting of machinery and equipment
28	Computer and related activities
29	R&D and other business activities
30	Public administration and defence; compulsory social security
31	Education
32	Health and social work
33	Other community, social and personal services

Table 9
Sectors BEA- OECD

BEA sector	Iocode	OECD sector	BEA Sector Name
1	111CA	1	Farms
2	113FF	1	Forestry, fishing, and related activities
3	211	2	Oil and gas extraction
4	212	2	Mining, except oil and gas
5	213	2	Support activities for mining
6	22	19	Utilities
7	23	20	Construction
8	321	5	Wood products
9	327	10	Nonmetallic mineral products
10	331	11	Primary metals
11	332	12	Fabricated metal products
12	333	13	Machinery
13	334	14	Computer and electronic products
14	335	15	Electrical equipment, appliances, and components
15	3361MV	16	Motor vehicles, bodies and trailers, and parts
16	3364OT	17	Other transportation equipment
17	337	18	Furniture and related products
18	339	18	Miscellaneous manufacturing
19	311FT	3	Food and beverage and tobacco products
20	313TT	4	Textile mills and textile product mills
21	315AL	4	Apparel and leather and allied products
22	322	6	Paper products
23	323	6	Printing and related support activities
24	324	7	Petroleum and coal products
25	325	8	Chemical products
26	326	9	Plastics and rubber products
27	42	21	Wholesale trade
28	441	21	Motor vehicle and parts dealers
29	445	21	Food and beverage stores
30	452	21	General merchandise stores
31	4A0	21	Other retail
32	481	23	Air transportation
33	482	23	Rail transportation
34	483	23	Water transportation
35	484	23	Truck transportation
36	485	23	Transit and ground passenger transportation
37	486	23	Pipeline transportation
38	487OS	23	Other transportation and support activities
39	493	23	Warehousing and storage
40	511	6	Publishing industries, except internet
41	512	33	Motion picture and sound recording industries
42	513	24	Broadcasting and telecommunications
43	514	28	Data processing, internet pub., and other inf. services
44	521CI	25	Federal Reserve banks, credit interm., and rel. act.
45	523	25	Securities, commodity contracts, and investments
46	524	25	Insurance carriers and related activities
47	525	25	Funds, trusts, and other financial vehicles
48	HS	26	Housing Services
49	ORE	26	Other Real Estate
50	532RL	27	Rental and leasing services and lessors of int. assets
51	5411	29	Legal services
52	5415	28	Computer systems design and related services
53	5412OP	29	Miscellaneous professional, scientific, and tech. Serv.
54	55	29	Management of companies and enterprises
55	561	29	Administrative and support services
56	562	18	Waste management and remediation services
57	61	31	Educational services
58	621	32	Ambulatory health care services
59	622	32	Hospitals
60	623	32	Nursing and residential care facilities
61	624	32	Social assistance
62	711AS	33	Performing arts, spectator sports, museums
63	713	33	Amusements, gambling, and recreation industries
64	721	22	Accommodation
65	722	22	Food services and drinking places
66	81	33	Other services, except government
67	GFGD	30	Federal general government (defense)
68	GFGN	30	Federal general government (nondefense)
69	GFE	30	Federal government enterprises
70	GSLG	30	State and local general government
71	GSLE	30	State and local government enterprises

Domestic production network structure and HHI/volatility controlling for GDP per-capita

Table 10
Herfindahl and Production Structure

VARIABLES	(1) $\log HHI_T^k$	(2) $\log HHI_T^k$	(3) $\log HHI_T^k$	(4) $\log HHI_T^k$
log GDPpc	0.088***	0.050	0.051	0.105***
	(0.031)	(0.031)	(0.031)	(0.032)
log Service	-0.377***			
	(0.091)			
log Density		-0.564***		
		(0.118)		
log Divers			-0.620***	
			(0.127)	
log Outdegrees				1.554***
				(0.282)
Observations	126	126	126	126
Adjusted R-squared	0.105	0.252	0.230	0.383

Note: This table presents an OLS regression using the log Herfindahl index of sales shares as the dependent variable. The independent variables describe each country domestic production structure and GDP per-capita. The sample is divided into three sub-periods: 1970-1995, 1996-2005, and 2006-2014. Robust standard errors in parentheses.

Table 11
Volatility and Production Structure

VARIABLES	(1) $\log Vol_T^k$	(2) $\log Vol_T^k$	(3) $\log Vol_T^k$	(4) $\log Vol_T^k$	(5) $\log Vol_T^k$
log GDPpc	-0.249***	-0.198***	-0.213***	-0.213***	-0.215***
	(0.059)	(0.059)	(0.059)	(0.059)	(0.064)
log HHI	0.642***				
	(0.141)				
log Service		-0.199			
		(0.245)			
log Density			-0.568***		
			(0.212)		
log Divers				-0.574**	
				(0.242)	
log Outdegrees					0.184
					(0.500)
Observations	126	126	126	126	126
Adjusted R-squared	0.164	0.094	0.135	0.124	0.090

Note: This table presents an OLS regression using real GDP growth as the dependent variable. The independent variables describe each country domestic production structure and GDP per-capita. The sample is divided into three sub-periods: 1970-1995, 1996-2005, and 2006-2014. Robust standard errors in parentheses.

Total production network structure and HHI/Volatility

Table 12
Herfindahl and Production Structure (total)

VARIABLES	(1) $\log HHI_T^k$	(2) $\log HHI_T^k$	(3) $\log HHI_T^k$	(4) $\log HHI_T^k$
log Service	-0.233*			
	(0.120)			
log Density		-0.372***		
		(0.133)		
log Divers			-0.610***	
			(0.139)	
log Outdegrees				1.169***
				(0.159)
Observations	126	126	126	126
Adjusted R-squared	0.041	0.044	0.112	0.340

Note: This table presents an OLS regression using the log Herfindahl index of sales shares as the dependent variable. The independent variables describe each country total production structure, that include imported inputs and goods. The sample is divided into three sub-periods: 1970-1995, 1996-2005, and 2006-2014. Robust standard errors in parentheses.

Table 13
Herfindahl and Production Structure (total)

VARIABLES	(1) $\log HHI_T^k$	(2) $\log HHI_T^k$	(3) $\log HHI_T^k$	(4) $\log HHI_T^k$
log GDPpc	0.088***	0.060	0.057	0.067**
	(0.031)	(0.037)	(0.035)	(0.032)
log Service	-0.377***			
	(0.091)			
log Density		-0.453***		
		(0.119)		
log Divers			-0.661***	
			(0.143)	
log Outdegrees				1.235***
				(0.148)
Observations	126	126	126	126
Adjusted R-squared	0.105	0.076	0.142	0.386

Note: This table presents an OLS regression using the log Herfindahl index of sales shares as the dependent variable. The independent variables describe each country total production structure, that include imported inputs and goods, and GDP per-capita. The sample is divided into three sub-periods: 1970-1995, 1996-2005, and 2006-2014. Robust standard errors in parentheses.

Table 14
Volatility and Production Structure (total)

VARIABLES	(1) $\log Vol_T^k$	(2) $\log Vol_T^k$	(3) $\log Vol_T^k$	(4) $\log Vol_T^k$	(5) $\log Vol_T^k$
log HHI	0.524***				
	(0.141)				
log Service		-0.522**			
		(0.241)			
log Density			-0.829**		
			(0.349)		
log Divers				-0.899**	
				(0.352)	
log Outdegrees					1.045***
					(0.335)
Observations	126	126	126	126	126
Adjusted R-squared	0.042	0.036	0.038	0.039	0.042

Note: This table presents an OLS regression using real GDP growth volatility as the dependent variable. The independent variables describe each country total production structure that includes imported inputs and goods. The sample is divided into three sub-periods: 1970-1995, 1996-2005, and 2006-2014. Robust standard errors in parentheses.

Table 15
Volatility and Production Structure (total)

VARIABLES	(1) $\log Vol_T^k$	(2) $\log Vol_T^k$	(3) $\log Vol_T^k$	(4) $\log Vol_T^k$	(5) $\log Vol_T^k$
log GDPpc	-0.249***	-0.198***	-0.199***	-0.205***	-0.205***
	(0.059)	(0.059)	(0.058)	(0.058)	(0.058)
log HHI	0.642***				
	(0.141)				
log Service		-0.199			
		(0.245)			
log Density			-0.561*		
			(0.289)		
log Divers				-0.716**	
				(0.325)	
log Outdegrees					0.846**
					(0.354)
Observations	126	126	126	126	126
Adjusted R-squared	0.164	0.094	0.110	0.119	0.122

Note: This table presents an OLS regression using real GDP growth volatility as the dependent variable. The independent variables describe each country total production structure, that includes imported inputs and goods, and GDP per-capita. The sample is divided into three sub-periods: 1970-1995, 1996-2005, and 2006-2014. Robust standard errors in parentheses.

Table 16
Estimated Elasticities Assuming $\epsilon_M = 1$

Second stage	(1) All	(2) Non-services	(3) Services
$\epsilon_Q - 1$	1.52***	0.84*	2.68**
	(0.493)	(0.442)	(0.828)
First stage $\Delta \log\left(\frac{P_{jt}}{P_{jt}^M}\right)$			
P_j^{IV}	2.33***	6.55***	0.39**
	(0.220)	(0.527)	(0.200)
$P_j^{M,IV}$	-3.93***	-8.72***	-2.40***
	(0.317)	(0.655)	(0.227)
Cragg-Donald F Statistic	114.81^r	89.26^r	74.18^r
Observations	32,980	12,500	20,480

Note: This table reports the estimated coefficients of Eq. (16) under the assumption of $\epsilon_M = 1$ for all sectors. Military spending is used to construct instruments for P_j and P_j^M. In the row labeled Cragg-Donald Statistic, the superscript "r" indicates that the test for a weak instrument is rejected at the 10 percent threshold. Standard errors are reported in parentheses. *** Significant at the 1 percent level; ** Significant at the 5 percent level; * Significant at the 10 percent level.

Appendix B

Proposition 1

Hulten 1978

For completeness, I re-derive Hulten (1978)'s theorem. This is a frictionless economy and the first welfare theorem holds. Therefore, I solve the planner's problem. The planner maximizes household utility subject to the output and labor resource constraints and sectoral technologies.

$$\mathcal{L} = C(C1, ..., C_N) + \sum_{j=1}^{N} \mu_j (Q_j - C_j - \sum_{i=1}^{N} M_{ji}) + \phi(\bar{L} - \sum_j L_j).$$

Where

$$Q_j = Z_j \Big(a_j L_j^{\frac{\epsilon_Q - 1}{\epsilon_Q}} + (1 - a_j) + M_j^{\frac{\epsilon_Q - 1}{\epsilon_Q}} \Big)^{\frac{\epsilon_Q}{\epsilon_Q - 1}},$$

and

$$M_j = \Big(\sum_{i=1}^{N} M_{ij}^{\frac{\epsilon_M - 1}{\epsilon_M}} \Big)^{\frac{\epsilon_M}{\epsilon_M - 1}}.$$

By envelope theorem, at the optimum $C(C_j^*, ..., C^*)$, we have that

$$\frac{\partial C^*}{\partial Z_j} = \mu_j^* \frac{\partial Q_j^*}{\partial Z_j} = \mu_j^* \frac{Q_j^*}{Z_j}$$

which evaluated at the steady state $Z_j = 1$ (for all j) yields

$$\frac{\partial GDP^*}{\partial Z_j} = \mu_j^* Q_j^*,$$

or

$$\frac{\partial \log GDP^*}{\partial Z_j} = \frac{\mu_j^* Q_j^*}{GDP},$$

The sectoral Lagrange multiplier μ_j equals sectoral price P_j in competitive equilibrium. This is trivial to show by comparing the planner first order condition for M_{ij} with the competitive equilibrium first order condition for M_{ij}. Therefore, up to a first order, the aggregate effect of a sectoral shock Z_j is determined by sectoral sales $P_j Q_j$.

Proposition 1 i) ($\epsilon_Q = \epsilon_M \neq 1$)

Let us define $\rho_Q = \frac{\epsilon_Q - 1}{\epsilon_Q}$ and $\rho_M = \frac{\epsilon_M - 1}{\epsilon_M}$. The firms' optimality conditions for inputs are

$$L_j : P_j Z_j^{\rho_Q} \Big(\frac{Q_j}{L_j}\Big)^{1-\rho_Q} a_j = w \tag{22}$$

$$M_j : P_j Z_j^{\rho_Q} \Big(\frac{Q_j}{M_j}\Big)^{1-\rho_Q} (1-a_j) = P_j^M \tag{23}$$

$$M_{ji} : P_i Z_i^{\rho_Q} Q_i^{1-\rho_Q} M_i^{\rho_Q - \rho_M} M_{ji}^{\rho_M - 1} (1-a_i)\omega_{ji} = P_j. \tag{24}$$

Multiplying sectoral market clearing condition for sector j by sectoral price P_j we obtain

$$S_j = P_j C_j + \sum_{i=1}^{N} P_j M_{ji},$$

where S_j is sectoral sales. Let's use the household optimal consumption share for each good (with $\epsilon_D = 1$ we have $P_j C_j = \beta_j P_c C$) and rearrange the firm optimality condition for M_{ij} in (24) (assuming $\epsilon_Q = \epsilon_M$)

$$P_j M_{ji}^{1-\rho_Q} = Z_i^{\rho_Q}(1-a_i)\omega_{ji} P_i Q_i^{1-\rho_Q},$$

$$P_j M_{ji} = \Big(\frac{P_i}{P_j}\Big)^{\epsilon_Q - 1} Z_i^{\epsilon_Q - 1} ((1-a_i)\omega_{ji})^{\epsilon_Q} P_i Q_i,$$

to get

$$S_j = \beta_j P_c C + P_j^{1-\epsilon_Q} \sum_{i=1}^{N} P_i^{\epsilon_Q - 1} Z_i^{\epsilon_Q - 1} ((1-a_i)\omega_{ji})^{\epsilon_Q} S_i.$$

$$\frac{S_j}{P_c C} = \beta_j + P_j^{1-\epsilon_Q} \sum_{i=1}^{N} P_i^{\epsilon_Q - 1} Z_i^{\epsilon_Q - 1} ((1-a_i)\omega_{ji})^{\epsilon_Q} \frac{S_i}{P_c C},$$

which proves Proposition 1 part i)

$$s = [I - P^{1-\epsilon_Q} \circ \Big((Z \circ P)^{\epsilon_Q - 1} 1'\Big)' \circ \Big(((1-a)1')' \circ \Omega\Big)^{\epsilon_Q}]^{-1} \beta. \tag{25}$$

To solve for sectoral prices, I express the production function of firms in sector j as:

$$Z_j^{-\rho_Q} = a_j \Big(\frac{L_j}{Q_j}\Big)^{\rho_Q} + (1-a_j)\Big(\frac{M_j}{Q_j}\Big)^{\rho_Q}, \tag{26}$$

which combined with the FONC for labor in Eq. (22)

$$P_j Z_j^{\rho_Q} \Big(\frac{Q_j}{L_j}\Big)^{1-\rho_Q} a_j = w$$

$$\Big(\frac{L_j}{Q_j}\Big)^{\rho_Q} = (a_j P_j)^{\epsilon_Q - 1} Z_j^{\frac{\rho_Q^2}{1-\rho_Q}}$$

$$\Big(\frac{L_j}{Q_j}\Big)^{\rho_Q} = (\frac{a_j P_j}{w})^{\epsilon_Q - 1} Z_j^{\frac{(\epsilon_Q - 1)^2}{\epsilon_Q}}$$

$$\Big(\frac{L_j}{Q_j}\Big)^{\rho_Q} = \Big(\frac{a_j P_j}{w}\Big)^{\epsilon_Q - 1} Z_j^{\frac{(\epsilon_Q - 1)^2}{\epsilon_Q}}$$

and the FONC for intermediates M_j in Eq. (23)

$$P_j Z_j^{\rho_Q} \Big(\frac{Q_j}{M_j}\Big)^{1-\rho_Q} (1 - a_j) = P_j^M$$

$$\Big(\frac{M_j}{Q_j}\Big)^{1-\rho_Q} = \Big(\frac{P_j}{P_j^M}\Big)(1 - a_j) Z_j^{\rho_Q}$$

$$\Big(\frac{M_j}{Q_j}\Big)^{(1-\rho_Q)\frac{\rho_Q}{1-\rho_Q}} = \Big(\Big(\frac{P_j}{P_j^M}\Big)(1 - a_j) Z_j^{\rho_Q}\Big)^{\frac{\rho_Q}{1-\rho_Q}}$$

$$\Big(\frac{M_j}{Q_j}\Big)^{\rho_Q} = \Big(\Big(\frac{P_j}{P_j^M}\Big)(1 - a_j)\Big)^{\epsilon_Q - 1} Z_j^{\frac{(\epsilon_Q - 1)^2}{\epsilon_Q}},$$

yields (replacing back into Eq. (26))

$$Z_j^{-\rho_Q} = a_j \Big(\Big(\frac{a_j P_j}{w}\Big)^{\epsilon_Q - 1} Z_j^{\frac{(\epsilon_Q - 1)^2}{\epsilon_Q}}\Big) + (1 - a_j)\Big(\Big(\Big(\frac{P_j}{P_j^M}\Big)(1 - a_j)\Big)^{\epsilon_Q - 1} Z_j^{\frac{(\epsilon_Q - 1)^2}{\epsilon_Q}}\Big),$$

$$P_j^{1-\epsilon_Q} = Z_j^{\epsilon_Q - 1} a_j^{\epsilon_Q} w^{1-\epsilon_Q} + Z_j^{\epsilon_Q - 1}(1 - a_j)^{\epsilon_Q} (P_j^M)^{1-\epsilon_Q}. \tag{27}$$

To solve for the intermediate input bundle price P_j^M, we solve the firms' cost minimizing problem. Firms minimize intermediates expenditure, subject to technology.

$$\mathcal{L} = \sum_{i=1}^{N} P_i M_{ij} + \lambda\Big((M_j - \Big(\sum_{i=1}^{N} \omega_{ij} M_{ij}^{\rho_M}\Big)^{1/\rho_M}\Big).$$

In competitive markets, the marginal cost of the bundle (λ) equals the price of the bundle (P_j^M). Taking first order conditions with respect to M_{ij}, we obtain:

$$\frac{\partial \mathcal{L}}{\partial M_{ij}} = P_i - P_j^M \Big(\sum_{i=1}^{N} \omega_{ij} M_{ij}^{\rho_M}\Big)^{\frac{1-\rho_M}{\rho_M}} \omega_{ij} M_{ij}^{\rho_M - 1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial M_{ij}} = P_i - P_j^M M_j^{1-\rho_M} \omega_{ij} M_{ij}^{\rho_M - 1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial M_{ij}} = P_i = P_j^M \Big(\frac{M_j}{M_{ij}}\Big)^{1-\rho_M} \omega_{ij},$$

implying

$$M_{ij}^{\rho_M} = \Big(\frac{P_j^M}{P_i}\Big)^{\epsilon_M - 1} \omega_{ij}^{\epsilon_M - 1} M_j^{\rho_M}.$$

Replacing the previous equation into the intermediate bundle technology $M_j = \big(\sum_{i=1}^N \omega_{ij} M_{ij}^{\rho_M}\big)^{1/\rho_M}$ yields

$$P_j^M = \Big(\sum_{i=1}^N \omega_{ij}^{\epsilon_M} P_i^{1-\epsilon_M}\Big)^{\frac{1}{1-\epsilon_M}}. \tag{28}$$

Plugging Equation (28) into the price Equation (27) yields

$$P_j^{1-\epsilon_Q} = Z_j^{\epsilon_Q - 1} a_j^{\epsilon_Q} w^{1-\epsilon_Q} + Z_j^{\epsilon_Q - 1} (1-a_j)^{\epsilon_Q} \Big(\sum_{i=1}^N \omega_{ij}^{\epsilon_M} P_i^{1-\epsilon_M}\Big)^{\frac{1-\epsilon_Q}{1-\epsilon_M}}.$$

Assuming $\epsilon_Q = \epsilon_M$ and that the numeraire of the economy is the wage rate w, we have

$$P_j^{1-\epsilon_Q} = Z_j^{\epsilon_Q - 1} a_j^{\epsilon_Q} + Z_j^{\epsilon_Q - 1} (1-a_j)^{\epsilon_Q} \Big(\sum_{i=1}^N \omega_{ij}^{\epsilon_Q} P_i^{1-\epsilon_Q}\Big),$$

which in matrix form becomes

$$P^{1-\epsilon_Q} = Z^{\epsilon_Q - 1} \circ a^{\epsilon_Q} + Z^{\epsilon_Q - 1} \circ \Big(\big((1-a)^{\epsilon_Q} 1' \circ \Omega'^{\epsilon_Q}\big) P^{1-\epsilon_Q}\Big),$$

and assuming $Z_j = 1$ for all j, proves Proposition 1 i)

$$P^{1-\epsilon_Q} = [I - \big((1-a)^{\epsilon_Q} 1' \circ \Omega'^{\epsilon_Q}\big)]^{-1} (a^{\epsilon_Q}). \tag{29}$$

Proposition 1 ii) $\epsilon_Q \neq 1$ **and** $\epsilon_M = 1$

The firms' FONC are

$$L_j : P_j Z_j^{\rho_Q} \Big(\frac{Q_j}{L_j}\Big)^{1-\rho_Q} a_j = w \tag{30}$$

$$M_j : P_j Z_j^{\rho_Q} \Big(\frac{Q_j}{M_j}\Big)^{1-\rho_Q} (1-a_j) = P_j^M \tag{31}$$

$$M_{ji} : P_i Z_i^{\rho_Q} \Big(\frac{Q_i}{M_i}\Big)^{1-\rho_Q} (1-a_i) \omega_{ji} \frac{M_i}{M_{ji}} = P_j. \tag{32}$$

Rearranging and plugging Eq. (31) into Eq. (32) yields

$$P_jM_{ji} = P_iQ_i(1-a_i)^{\epsilon_Q}\omega_{ji}\Big(\frac{Z_iP_i}{P_i^M}\Big)^{\epsilon_Q-1}, \tag{33}$$

where $P_i^M = \prod_{j=1}^N \left(\frac{P_j}{\omega_{ji}}\right)^{\omega_{ji}}$ (proved at the end of this section).

To obtain sales shares, we multiply sectoral market clearing condition for sector j by sectoral price P_j to obtain

$$S_j = P_jC_j + \sum_{i=1}^N P_jM_{ji}.$$

Use the modified firm optimality condition for M_{ji} in Eq. (33) and the household optimality condition $P_jC_j = \beta_jP_cC$ (for $\epsilon_D = 1$) to obtain

$$S_j = \beta_jP_cC + \sum_{i=1}^N (1-a_i)^{\epsilon_Q}\omega_{ji}\Big(\frac{Z_iP_i}{P_i^M}\Big)^{\epsilon_Q-1}S_i,$$

$$s_j = \beta_j + \sum_{i=1}^N (1-a_i)^{\epsilon_Q}\omega_{ji}\Big(\frac{Z_iP_i}{P_i^M}\Big)^{\epsilon_Q-1}s_i,$$

$$s = \beta + \Big((\overline{P}\circ(1-a)^{\epsilon_Q}1')'\circ\Omega\Big)s$$

$$s = [I - (\overline{P}\circ(1-a)^{\epsilon_Q}1')'\circ\Omega]^{-1}\beta, \tag{34}$$

where s is the vector of sales to GDP shares and $\overline{P} = \left(\frac{Z\circ P}{P^M}\right)^{\epsilon_Q-1}$.

To obtain sectoral prices, I express the production function of firms in sector j as:

$$Z_j^{-\rho_Q} = a_j\Big(\frac{L_j}{Q_j}\Big)^{\rho_Q} + (1-a_j)\Big(\frac{M_j}{Q_j}\Big)^{\rho_Q}, \tag{35}$$

which combined with the FONC for labor in Eq. (30) yields

$$P_jZ_j^{\rho_Q}\Big(\frac{Q_j}{L_j}\Big)^{1-\rho_Q}a_j = w,$$

$$\Big(\frac{L_j}{Q_j}\Big)^{\rho_Q} = (a_jP_j)^{\epsilon_Q-1}Z_j^{\frac{\rho_Q^2}{1-\rho_Q}},$$

$$\left(\frac{L_j}{Q_j}\right)^{\rho_Q} = \Big(\frac{a_jP_j}{w}\Big)^{\epsilon_Q-1}Z_j^{\frac{(\epsilon_Q-1)^2}{\epsilon_Q}},$$

and the FONC for intermediates in Eq. (31)

$$P_j Z_j^{\rho_Q}\Big(\frac{Q_j}{M_j}\Big)^{1-\rho_Q}(1-a_j) = P_j^M$$

$$\Big(\frac{M_j}{Q_j}\Big)^{1-\rho_Q} = \Big(\frac{P_j}{P_j^M}\Big)(1-a_j)Z_j^{\rho_Q}$$

$$\Big(\frac{M_j}{Q_j}\Big)^{\rho_Q} = \Big(\Big(\frac{P_j}{P_j^M}\Big)(1-a_j)Z_j^{\rho_Q}\Big)^{\frac{\rho_Q}{1-\rho_Q}}$$

$$\Big(\frac{M_j}{Q_j}\Big)^{\rho_Q} = \Big(\Big(\frac{P_j}{P_j^M}\Big)(1-a_j)\Big)^{\epsilon_Q-1} Z_j^{\frac{(\epsilon_Q-1)^2}{\epsilon_Q}},$$

which we replace into (35) to obtain

$$Z_j^{-\rho_Q} = a_j\Big(\Big(\frac{a_j P_j}{w}\Big)^{\epsilon_Q-1} Z_j^{\frac{(\epsilon_Q-1)^2}{\epsilon_Q}}\Big) + (1-a_j)\Big(\Big(\Big(\frac{P_j}{P_j^M}\Big)(1-a_j)\Big)^{\epsilon_Q-1} Z_j^{\frac{(\epsilon_Q-1)^2}{\epsilon_Q}}\Big),$$

$$P_j^{1-\epsilon_Q} = Z_j^{\epsilon_Q-1} a_j^{\epsilon_Q} w^{1-\epsilon_Q} + Z_j^{\epsilon_Q-1}(1-a_j)^{\epsilon_Q}(P_j^M)^{1-\epsilon_Q}. \qquad (36)$$

The intermediate input bundle price P_j^M is obtained from the firms' cost minimizing problem.

$$\mathcal{L} = \sum_{i=1}^{N} P_i M_{ij} + \lambda_j\Big(M_j - \prod_{i=1}^{N} M_{ij}^{\omega_{ij}}\Big).$$

In competitive markets, the marginal cost of the bundle (λ_j) equals the price of the bundle (P_j^M). Taking first order conditions with respect to M_{ij}, we obtain:

$$\frac{\partial \mathcal{L}}{\partial M_{ij}} = P_i - P_j^M \omega_{ij}\frac{M_j}{M_{ij}} = 0$$

implying

$$M_{ij} = \omega_{ij} M_j \frac{P_j^M}{P_i}.$$

Replacing the previous equation into the production bundle technology

$$M_j = \prod_{i=1}^{N}\Big(\omega_{ij} M_j \frac{P_j^M}{P_i}\Big)^{\omega_{ij}},$$

$$M_j = (M_j P_j^M)^{\sum_{i=1}^{N}\omega_{ij}} \prod_{i=1}^{N}\Big(\frac{\omega_{ij}}{P_i}\Big)^{\omega_{ij}},$$

and using the fact that $\sum_{i=1}^{N} = 1$ yields

$$P_j^M = \prod_{i=1}^{N} \Big(\frac{P_i}{\omega_{ij}}\Big)^{\omega_{ij}}$$

Plugging this equation into the price equation (36) yields

$$P_j^{1-\epsilon_Q} = Z_j^{\epsilon_Q - 1} a_j^{\epsilon_Q} w^{1-\epsilon_Q} + Z_j^{\epsilon_Q - 1} (1-a_j)^{\epsilon_Q} \Big(\prod_{i=1}^{N} \Big(\frac{P_i}{\omega_{ij}}\Big)^{\omega_{ij}} \Big)^{1-\epsilon_Q},$$

which in matrix form becomes

$$(1-\epsilon_Q) \circ \log P = (\epsilon_Q - 1) \circ \log Z + \log\Big(a^{\epsilon_Q} \circ w^{1-\epsilon_Q} + (1-a)^{\epsilon_Q} \circ exp[(1-\epsilon_Q) \circ (\Omega' \log P - diag(\Omega' \log \Omega))]\Big),$$

where $diag(A)$ converts the diagonal of the square matrix A into a vector the same dimension.

Proposition 2

Volatility

Real GDP in the economy can be approximated, up to a first order,

around the steady state $\bar{Y} = Y(\bar{Z}_1, ..., \bar{Z}_N)$ as

$$Y \approx \bar{Y} + \sum_{j=1}^{N} \frac{\partial Y}{\partial Z_j} (Z_j - \bar{Z}),$$

or

$$\log Y \approx \log \bar{Y} + \sum_{j=1}^{N} \frac{\partial \log Y}{\partial \log Z_j} (\log Z_j - \log \bar{Z}).$$

To obtain GDP growth volatility take log difference

$$\log Y_t - \log Y_{t-1} \approx \sum_{j=1}^{N} \frac{\partial \log Y}{\partial \log Z_j} (\log Z_{jt} - \log Z_{jt-1}),$$

and use Hulten's theorem ($\frac{\partial \log Y}{\partial \log Z_j} = s_j$) to get

$$\log Y_t - \log Y_{t-1} \approx \sum_{j=1}^{N} s_j (\log Z_{jt} - \log Z_{jt-1}),$$

where s_j is the sales share to GDP ratio of sector j. Assuming that sectoral productivity follow a random walk $\log Z_{jt} = \log \bar{Z} + \log Z_{jt-1} + \kappa_{jt}$, in which κ_{jt} is normal i.i.d. distributed with mean zero and variance σ_j we have

$$\log Y_t - \log Y_{t-1} \approx \sum_{j=1}^{N} s_j \kappa_{jt}.$$

Taking the variance of GDP growth ($\log Y_t - \log Y_{t-1}$) we obtain

$$\sigma_{GDP}^2 \approx \sum_{j=1}^{N} s_j^2 Var(\kappa_{jt}) = \sum_{j=1}^{N} s_j^2 \sigma_j^2,$$

which combined with the results in Proposition 1 yields Proposition 2.

Appendix C

In this Appendix, I discuss the existence of a cost of complexity in standard Cobb-Douglas production technologies. When $\epsilon_M = 1$, The bundle of intermediates is:

$$M_j = \prod_{i=1}^{N} \Big(\frac{1}{\omega_{ij}}\Big)^{\omega_{ij}(1-\rho_M)} M_{ij}^{\omega_{ij}},$$

We can use the limit of $M_{ij} \rightarrow 0$ (for all non-used inputs) or simply use $M_{ij} = 0$ (for all non-used inputs) and assume that $0^0 = 1$ and $(\frac{1}{0})^0 = 1$. As before assume that the firm with one input uses M_1 units of the input, while the diversified firm uses $M_2 = \frac{1}{N} M_1 < M_1$ units of each input. In either case, we have:

$$M_j^1 = M_1$$

$$M_j^N = \Big(\frac{1}{1/N}\Big)^{(1/N)(1-\rho_M)} M_2^{1/N} ... \Big(\frac{1}{1/N}\Big)^{(1/N)(1-\rho_M)} M_2^{1/N},$$

which using the fact $M_2 = \frac{1}{N} M_1$ yields

$$M_j^N = \frac{M_1}{N^{\rho_M}}.$$

When $\rho_M = 1$ we recover the standard Cobb-Douglas production technology. In this case, the diversified bundle firm produces less ($\frac{M_1}{N}$) than the concentrated bundle firm (M_1) as a results of the complexity cost. The case in which $\rho_M = 0$ is equivalent to the case in which $\rho_M = 1/\epsilon_M$ (when $\epsilon_M \neq 1$), where there is no complexity cost.

Appendix D

In this Appendix, I describe how I obtain a global solution of the model for the quantitative section of the paper.

$\epsilon_Q \neq 1$ and $\epsilon_M = 1$

Real GDP in this economy is $\log GDP = -\log P_c$. In addition, as $\epsilon_D = 1$ we have $P_c = \prod_{j=1}^{N}\left(\frac{\beta_j}{P_j}\right)^{\beta_j}$. Assuming $\epsilon_M = 1$, Equation (27) becomes:

$$P_j^{1-\epsilon_{Q_j}} = Z_j^{\epsilon_{Q_j}-1}\left(a_j^{\epsilon_{Q_j}} w^{1-\epsilon_{Q_j}} + (1-a_j)^{\epsilon_{Q_j}}\left(\prod_{i=1}^{N}\left(\frac{P_i}{\omega_{ij}}\right)^{\omega_{ij}}\right)^{1-\epsilon_{Q_j}}\right)$$

which in matrices becomes

$$(1-\epsilon_Q)\circ\log P = (\epsilon_Q-1)\circ\log Z + \log\left(a^{\epsilon_Q} + (1-a)^{\epsilon_Q}\circ exp[(1-\epsilon_Q)\circ(\Omega'\log P - diag(\Omega'\log\Omega))]\right).$$

For a given path for $\left(\{Z_j\}_{j=1}^{N}\right)_{t=1}^{T}$ and the calibrated values of a, Ω, and ϵ_Q, I solve the non-linear system of equations for prices using the Cobb-Douglas equilibrium prices as initial values. Then, we find real GDP as:

$$\log GDP = -\log P_c = \sum_{j=1}^{N}\beta_j \log\left(\frac{\beta_j}{P_j}\right).$$

The model implied network density and diversification are calculated based on section 2, using the observed matrix Ω. The model implied service share is calculated from the model's first order conditions for labor

$$wL_j = P_j Q_j (Z_j P_j)^{\epsilon_Q - 1} a_j^{\epsilon_Q},$$

evaluated at the steady state value of $Z_j = \bar{Z}$. From Proposition 1 ii), we obtain sales $(P_j Q_j)$ and prices P_j used to obtain $wL_j = L_j$ (as wage is the numeraire).

Appendix E

We solve the cost minimization problem for firms in sector j. The Lagrangian of this problem is (max - (cost))

$$\mathcal{L} \quad = \quad -wL_j - P_j^M M_j - \lambda\left(Q_j - Z_j\left[a_j L_j^{\rho_Q} + \left(1-a_j\right)M_j^{\rho_Q}\right]^{\frac{1}{\rho_Q}}\right) \tag{37}$$

The first-order necessary and sufficient conditions for M_j is

$$-P_j^M + \lambda\frac{\partial Q_j}{\partial M_j} = 0. \tag{38}$$

Rearranging, using the fact that $\frac{\partial Q_j}{\partial M_j} = Z_j^{\rho_Q} a_j \left(\frac{Q}{M_j}\right)^{\frac{1}{\epsilon_Q}}$ and that in competitive markets the marginal cost of production in sector j (λ^1) is the price of good P_j, we have

$$P_j^M = P_j Z_j^{\rho_Q} a_j \left(\frac{Q_j}{M_j}\right)^{\frac{1}{\epsilon_Q}}. \tag{39}$$

Raising the previous equation to the power of ϵ_Q, taking logs, and rearranging we obtain

$$\log\left(\frac{P_{jt}^M M_{jt}}{P_{jt}Q_{jt}}\right) = \epsilon_Q \log\left(a_j\right) + (1-\epsilon_Q)\log\left(\frac{P_{jt}^M}{P_{jt}}\right) + (\epsilon_Q - 1)\log Z_{jt}$$

$$\Delta\log\left(\frac{P_{jt}^M M_{jt}}{P_{jt}Q_{jt}}\right) = (1-\epsilon_Q)\Delta\log\left(\frac{P_{jt}^M}{P_{jt}}\right) + (\epsilon_Q - 1)\Delta\log Z_{jt} \tag{40}$$

Now, we minimize the cost of the intermediate input bundle $\sum_{i=1}^N P_i M_{ij}$ subject to $M_j = \left(\sum_{i=1}^N \omega_{ij} M_{ij}^{\rho_M}\right)^{\frac{1}{\rho_M}}$. The Lagrangian for this problem is

$$\mathcal{L} \quad = \quad -\sum_{i=1}^N P_i M_{ij} - \lambda^2\left(M_j - \Big(\sum_{i=1}^N \omega_{ij} M_{ij}^{\rho_M}\Big)^{\frac{1}{\rho_M}}\right).$$

Taking first order conditions with respect to M_{ij}, and using the fact that in competitive markets, the marginal cost of the bundle (λ) equals its price (P_j^M)

$$-P_i + P_j^M \frac{\partial M_j}{\partial M_{ij}} = -P_i + P_j^M M_j^{1-\rho_M} M_{ij}^{\rho_M - 1}\omega_{ij} = 0. \tag{41}$$

which rearranged yields

$$\Delta\log\left(\frac{P_{it}M_{ijt}}{P_{jt}^M M_{jt}}\right) = (1-\epsilon_{M_j})\Delta\log\left(\frac{P_{it}}{P_{jt}^M}\right). \tag{42}$$

Combining Equations (40) and (42) yields

$$\Delta \log \left(\frac{P_{it} M_{ijt}}{P_{jt} Q_{jt}} \right) = (\epsilon_M - 1) \Delta \log \left(\frac{P_{jt}^M}{P_{it}} \right) + (\epsilon_Q - 1) \Delta \log \left(\frac{P_{jt}}{P_{jt}^M} \right) + (\epsilon_Q - 1) \Delta \log Z_{jt}. \tag{43}$$